Worksheet on Rational Functions

Steps for graphing rational functions

1. Factor numerator and denominator completely and put in lowest terms. Identify any holes.

2. Find all intercepts.

3. Find all asymptotes.

4. Determine the behavior around the vertical asymptotes using a table of signs.

5. Find the places where the function crosses the horizontal asymptote/oblique asymptote.

6. Sketch a smooth graph based on the above information.

Graph.

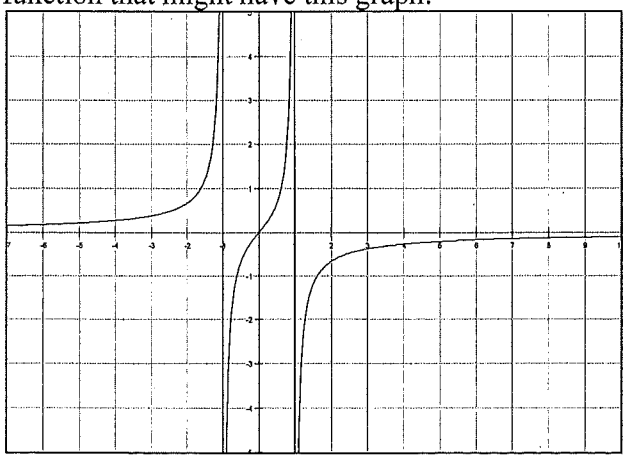
1.) $r(x) = \dfrac{6x+2}{x-2}$ 2.) $r(x) = \dfrac{x-1}{x^2-4}$ 3.) $r(x) = \dfrac{x^2+1}{x^2}$

Graph the following using the above steps.

4.) $r(x) = \dfrac{x^3+3x^2}{x^3+3x^2-4x-12}$ 5.) $r(x) = \dfrac{x^2-x-12}{x+1}$ 6.) $r(x) = \dfrac{2(x+2)^2(x-5)}{(x+5)(x-2)^2}$

7.) Make up a rational function that has all these features: x-intercepts at -1 and 2, vertical asymptotes $x = -5$ and $x = 6$, and horizontal asymptote $y = 3$.

8.) Make up a rational function that might have this graph:



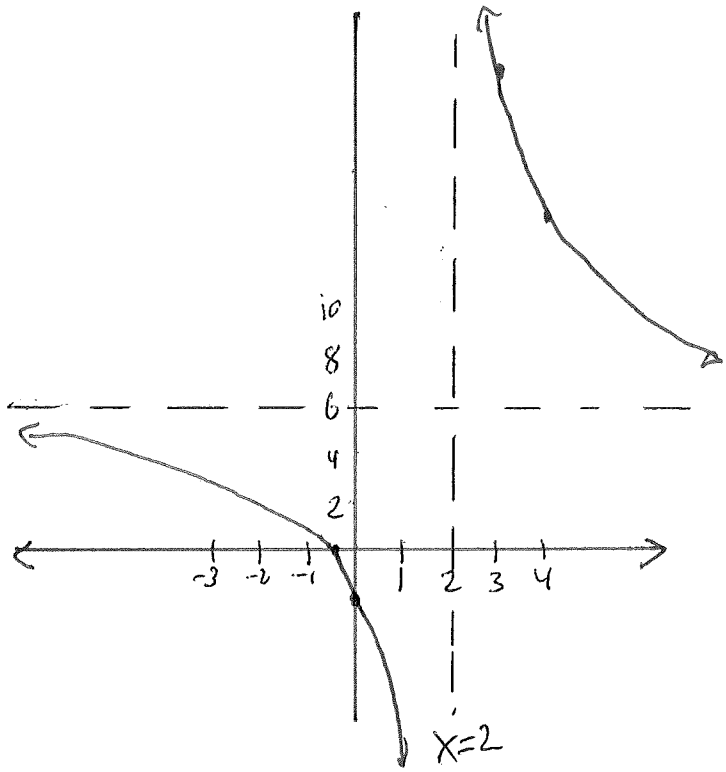
Worksheet on Rational Functions

1. $r(x) = \dfrac{6x+2}{x-2}$

Domain: $x \neq 2$
Range: $r(x) \neq 6$
Zeroes: $(-\frac{1}{3}, 0)$
Holes: None
END Behavior: $x \to \pm\infty$ $y \to 6$
Horiz Asy: $y=6$
Vert Asy: $x=2$
y-int: $(0,-1)$

$\dfrac{6x}{x}$

Test $x=3$
$(3, 20)$



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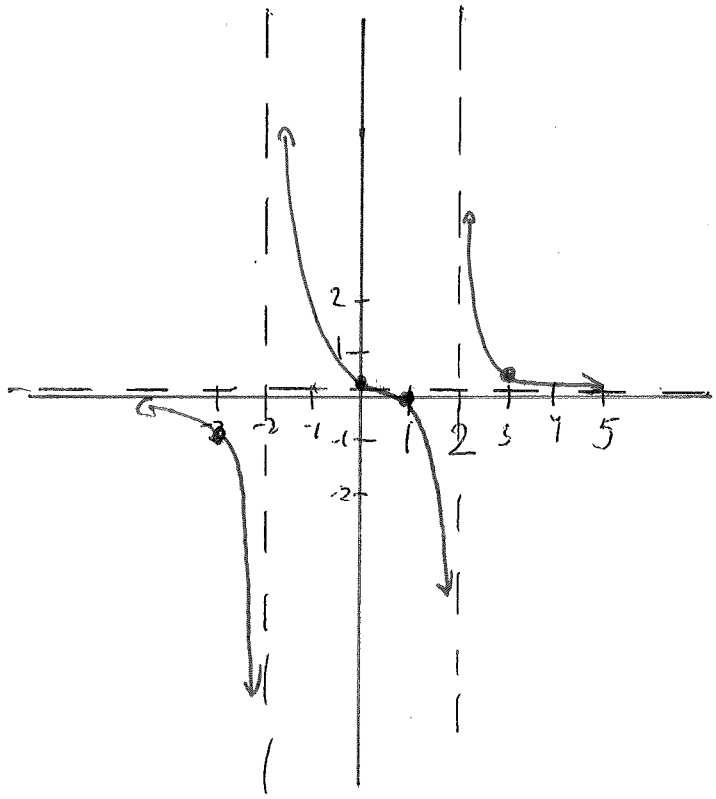
2. $r(x) = \dfrac{x-1}{x^2-4} = \dfrac{x-1}{(x+2)(x-2)}$

Domain: $x \neq -2, 2$
Range: $y \in \mathbb{R}$
Zeroes: $(1,0)$
Holes: None
END Behavior: $x \to \pm\infty$ $y \to 0$
Horiz Asym: $y=0$
Vert Asym: $x=2$ $x=-2$
y-int: $(0, \frac{1}{4})$

$\dfrac{x}{x^2} = \dfrac{1}{x}$

Test

x	y
1.5	- *
3	$\frac{2}{5}$
-3	$-\frac{4}{5}$

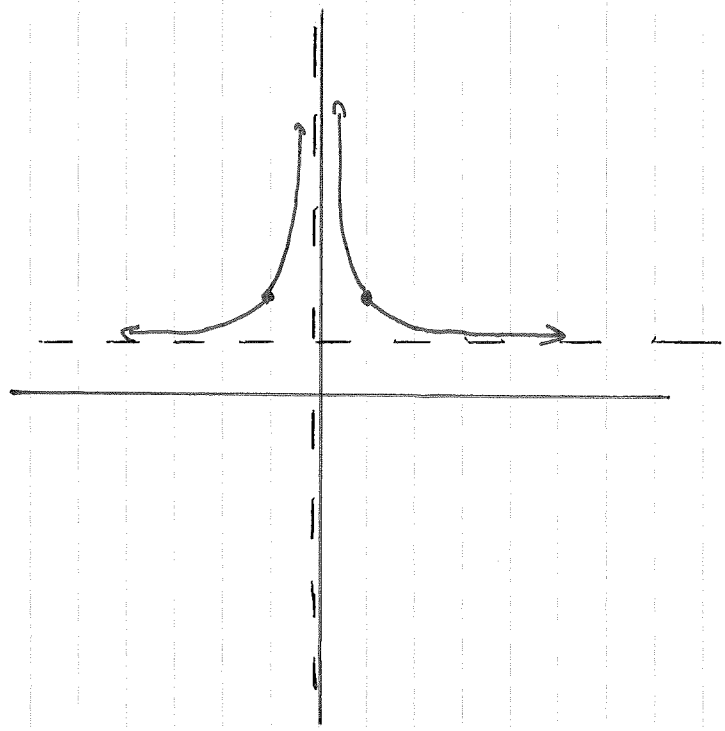


3. $r(x) = \frac{x^2+1}{x^2}$

Domain: $x \neq 0$
Range: $y > 1$ or $(1, \infty)$ $\frac{x^2}{x^2}$
Zeroes: None
Holes: None
END Behavior: $x \to \pm\infty$ $y \to 1$
Horiz Asym: $y = 1$
Vert Asym: $x = 0$
y-Int: (None

Test

x	y
1	2
-1	2



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4. $r(x) = \frac{x^3+3x^2}{x^3+3x^2-4x-12} = \frac{x^2(x+3)}{(x+2)(x+3)(x-2)}$

Domain: $x \neq -2, 2, -3$
Range: $(-\infty, 0] \cup (1, \infty)$
Zeroes: $(0, 0)$
Hole: $(-3, \frac{9}{5})$ $\frac{x^3}{x^3}$
END Behav: $x \to \pm\infty$ $y \to 1$
Horiz Asym: $y = 1$
Vert Asym: $x = -2$ $x = 2$
y-Int: $(0, 0)$

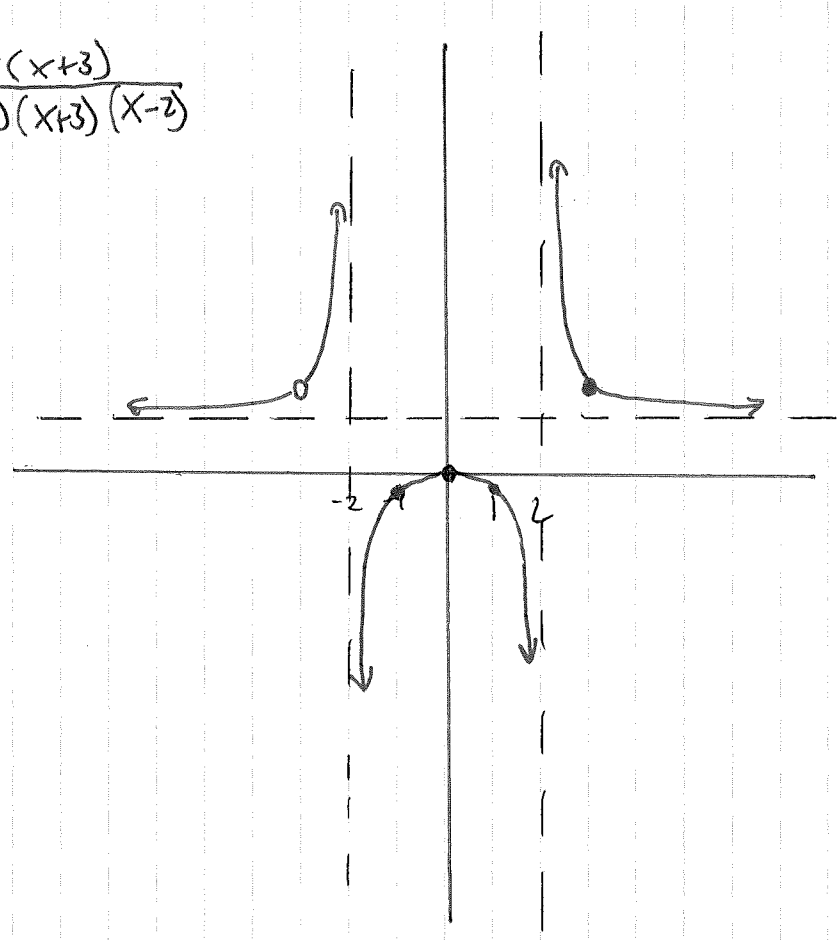
Test

x	y
-1	$-\frac{1}{3}$
1	$-\frac{1}{3}$
3	$\frac{9}{5}$

$\frac{1}{1 \cdot (-3)}$

$\frac{1}{3(-1)}$

$\frac{9}{5 \cdot 1}$



6. $r(x) = \dfrac{2(x+2)^2(x-5)}{(x+5)(x-2)^2}$

Domain: $x \neq -5, 2$
Range: $y \neq 2$
Zeroes: $(-2,0)$ $(5,0)$
Holes: None
End Behav: $x \to \pm\infty$ $\quad \dfrac{2x^3}{x^3}$ $\quad y \to 2$
Horiz Asy: $y=2$
Vert Asy: $x=-5$, $x=2$
y-int: $(0, -2)$

Test

x	y
-6	$\frac{11}{2}$
-3	$-\frac{8}{25}$
3	$-\frac{25}{2}$

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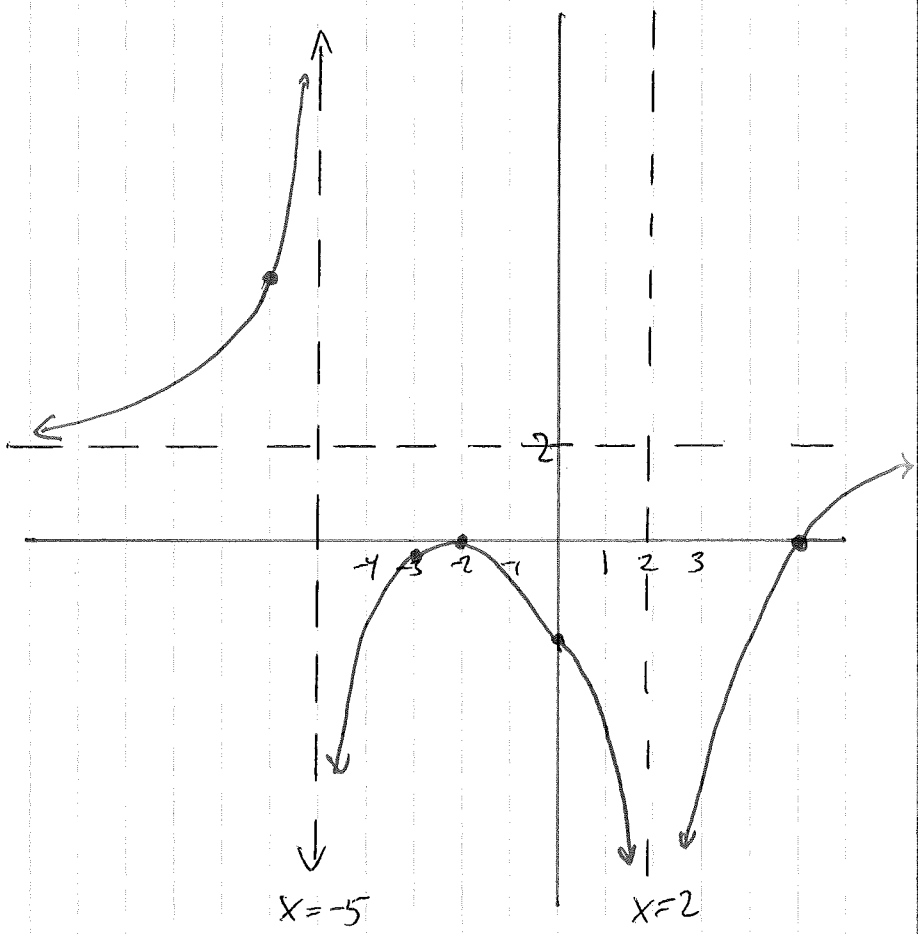
$\dfrac{2(-4)^2(-11)}{-1(-8)^2}$

$\dfrac{\cancel{2}(-1)^2(-8)}{\cancel{2}(-5)^2}$

$\dfrac{\cancel{2}(5)^2(-\cancel{2})}{\cancel{8}(1)^2}$

$\cancel{4}\ 4$



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7. $\dfrac{3(x+1)(x-2)}{(x+5)(x-6)}$

8. $\dfrac{x}{(x+1)(x-1)}$