

Chapter 1: Vectors

Definitions

Quantities used in Physics can be categorized as either scalars or vectors.

A scalar is a quantity that has a magnitude (size)

A vector is a quantity that has a magnitude & direction.

Vectors and scalars are mutually exclusive, i.e. no quantity can be both.

Examples: 5.0 km : Scalar
5.0 km, [North] : vector

Some common quantities used in Physics (or science in general):

Scalars	Vectors
- length	- acceleration
- temperature	- velocity
- distance	- weight
- speed	- momentum
- mass	- displacement
- work	- force
- energy	
- density	
- volume	

Representing Vectors

Arrows are used to represent vector quantities.



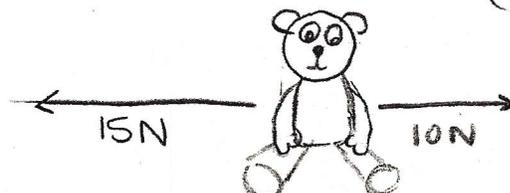
On a diagram

Length of arrow: magnitude

Where the arrow points: direction (use angles)

Example: Two kids pull on a toy in opposite directions. Draw the forces involved.

(usually to scale)



Vector Addition

When we add 2 or more vectors together, we get a **vector**.

That vector is called the **RESULTANT**.

Just like with numbers, the order in which we add vectors does not matter.

There are 2 ways to add vectors:

- Graphically
- By components (math \rightarrow triangles \rightarrow TRIG!)

Adding Vectors Graphically

We draw the vectors together in order to find our resultant.

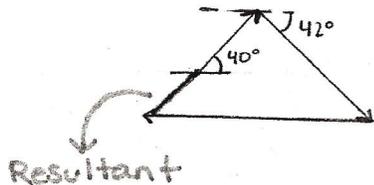
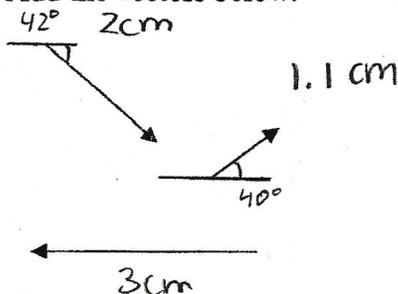
To add vectors graphically, your drawing **MUST BE TO SCALE!!!**

Graphical method #1: Head to Tail

- Draw vectors "one after the other"
- Draw magnitudes and directions (angles) to scale

Examples:

1. Add the vectors below.



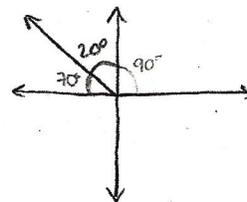
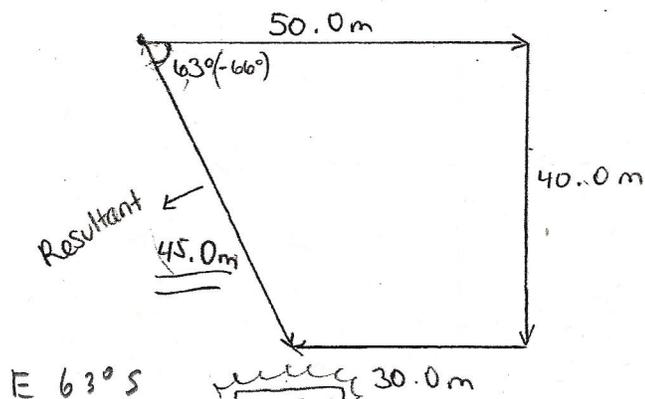
• 40°

• 42°

2. Dog runs 50.0 m to the [East], then 40.0 m [South], then 30.0 m [West]. Draw the resultant displacement of the dog.

W
N
S
E

10.0 m = 1 cm



SAME VECTOR { N 20° W
OR
W 70° N

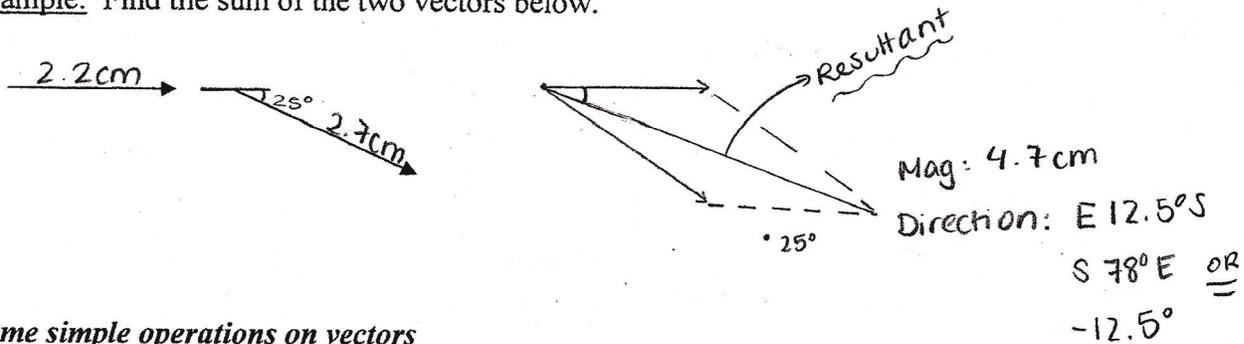
OR from +ve x-axis
= 110°

Graphical Method #2: Tail to Tail (Parallelogram) (NOT RECOMMENDED)

- Draw vectors tail to tail
- Complete the parallelogram
- Resultant is from tails of vectors to opposite corner of the parallelogram

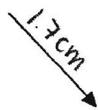
Notes: - this method only works for 2 vectors at a time.

Example: Find the sum of the two vectors below.



Some simple operations on vectors

Consider the vector \vec{A} :



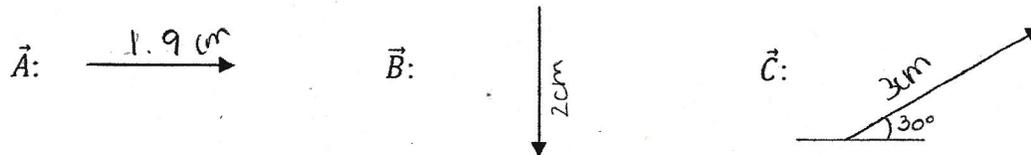
$3\vec{A}$: -3 X MAGNITUDE OF A!
- SAME DIRECTION

$\frac{1}{2}\vec{A}$ - $\frac{1}{2}$ the magnitude of A
- Same direction.

$-\vec{A}$ - same magnitude
- opposite direction
↳ 180° rotation

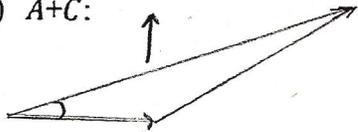
Examples:

Consider the vectors below.



Draw the sum of the following vectors.
Measure the magnitude and angle of the resultant vector.

1) $\vec{A} + \vec{C}$: Resultant



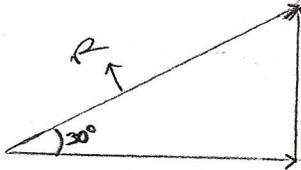
Mag: 4.9
Direction: E 20° N

N E

2) $2\vec{A} - \vec{B}$:

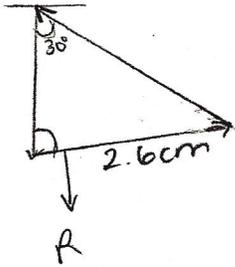
$1.9 \times 2 = 3.8$

$2A + (-B)$



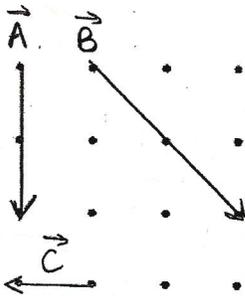
Mag: 4.8 cm
Dir: E 62° N

3) $\vec{B} - \vec{C}$:

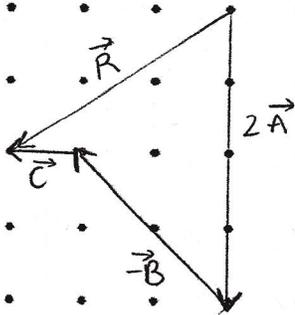


Mag: 2.6 cm
Dir: ...

And with a grid...



$2A - B + C$

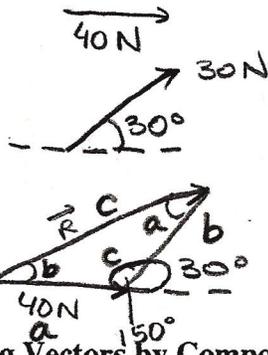


EXTRA

The "semi-graphical" method: using the cosine law

(This is especially useful if you are only looking for the magnitude.)

Cosine Law



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = (40N)^2 + (30N)^2 - 2(40N)(30N) \cos 150^\circ$$

$$c^2 = 2500N - (-2078.46)$$

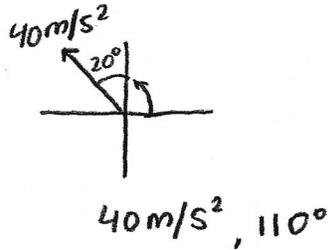
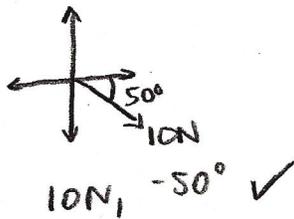
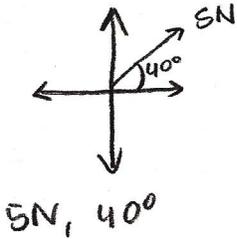
$$c^2 = \sqrt{4578.46}$$

$$c = 68 N$$

Adding Vectors by Components

There are 3 ways to express a vector (non-graphically).

1) Using magnitude and angle from the positive x-axis



10N, 310° X

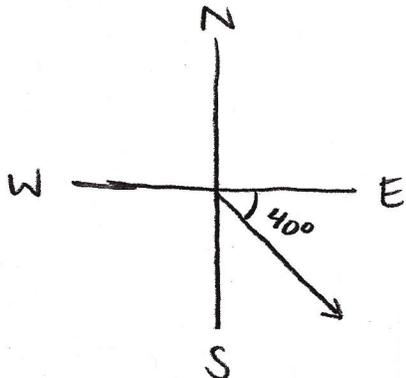
↓
still right,
but less right.

2) Using a magnitude and an angle with N, E, S, W

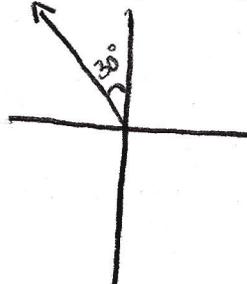
400 N E 40° S

axis starting on.

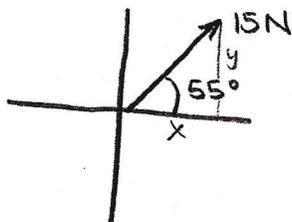
axis going towards



10N N 30° W



3) Using the x and y components of the vector (x, y) or $\begin{pmatrix} x \\ y \end{pmatrix}$



$$\cos 55^\circ = \frac{x}{15N}$$

$$x = 15N \cdot \cos 55^\circ$$

$$x = 8.6N$$

$$x = 8.6N$$

$$\sin 55^\circ = \frac{y}{15N}$$

$$y = 15N \cdot \sin 55^\circ$$

$$y = 12.3N$$

$$(8.6N, 12.3N)$$

To add vectors using components: \rightarrow most efficient

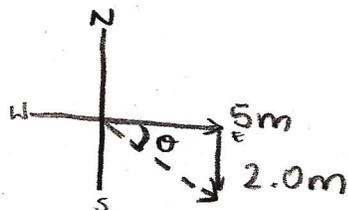
- 1) Split vectors into x and y components.
- 2) Add the x components together (this gives the x component of the resultant)
- 3) Add the y components together (this gives the y component of the resultant)
- 4) Sketch the x and y for the resultant.
- 5) Calculate the magnitude \rightarrow Pythagoras
- 6) Find the direction (angle) $\rightarrow \tan \theta$

Examples

1) Add $(3.0\text{ m}, 6.0\text{ m})$ and $(2.0\text{ m}, -8.0\text{ m})$

$$\begin{array}{r} (3.0\text{ m}, 6.0\text{ m}) \\ + (2.0\text{ m}, -8.0\text{ m}) \\ \hline (5.0\text{ m}, -2.0\text{ m}) \end{array} \rightarrow x + y \text{ of the resultant}$$

Sketch:



5.4m E 22° S
or 5.4m -22°

$$\text{Mag: } c^2 = a^2 + b^2$$

$$c^2 = (5.0\text{ m})^2 + (2.0\text{ m})^2$$

$$\sqrt{c^2} = \sqrt{29\text{ m}^2}$$

$$c = 5.4\text{ m}$$

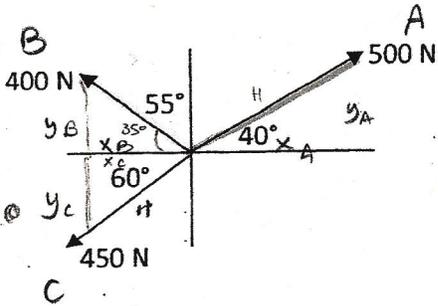
Dir:

$$\tan \theta = \frac{2.0\text{ m}}{5.0\text{ m}}$$

$$\theta = \tan^{-1} \frac{2.0\text{ m}}{5.0\text{ m}}$$

$$\theta = 22^\circ$$

2) Find the resultant of the following forces.



$$A_x = 500 \cos 40^\circ = \frac{x_A}{500 \text{ N}}$$

$$A_y = 500 \sin 40^\circ = \frac{y_A}{500 \text{ N}}$$

$$x_A = 500 \text{ N} \cdot \cos 40^\circ$$

$$y_A = 500 \text{ N} \sin 40^\circ$$

$$x_A = 383 \text{ N}$$

$$y_A = 321 \text{ N}$$

$$\vec{A} (383 \text{ N}, 321 \text{ N})$$

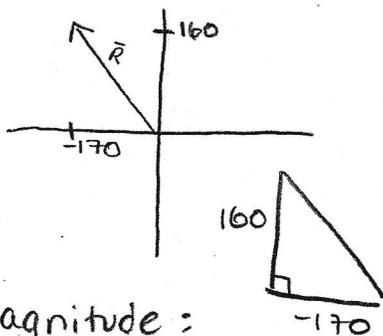
$$(383 \text{ N}, 321 \text{ N})$$

$$+ (-328 \text{ N}, 229 \text{ N})$$

$$(-225 \text{ N}, -390 \text{ N})$$

$$\hline (-170 \text{ N}, 160 \text{ N})$$

↓
resultant



Magnitude :

$$\vec{R} = \sqrt{x^2 + y^2}$$

$$\vec{R} = \sqrt{(-170 \text{ N})^2 + (160 \text{ N})^2}$$

$$\vec{R} = 233 \text{ N}$$

$$x_B =$$

$$y_B =$$

$$\cos 35^\circ = \frac{x_B}{400 \text{ N}}$$

$$\sin 35^\circ = \frac{y_B}{400 \text{ N}}$$

$$x_B = 400 \text{ N} \cos 35^\circ$$

$$y_B = 400 \text{ N} \sin 35^\circ$$

$$x_B = 328 \text{ N}$$

$$y_B = 229 \text{ N}$$

$$\vec{B} (-328 \text{ N}, 229 \text{ N})$$

$$x_C =$$

$$y_C =$$

$$\cos 60^\circ = \frac{x_C}{450 \text{ N}}$$

$$\sin 60^\circ = \frac{y_C}{450 \text{ N}}$$

$$x_C = 450 \text{ N} \cos 60^\circ$$

$$y_C = 450 \text{ N} \sin 60^\circ$$

$$x_C = 225 \text{ N}$$

$$y_C = 390 \text{ N}$$

$$\vec{C} (-225 \text{ N}, -390 \text{ N})$$

Direction : $\tan \theta = \frac{160 \text{ N}}{170 \text{ N}}$

Express :

$$233 \text{ N} \text{ W } 43^\circ \text{ N} \text{ Vectors-7}$$

$$\theta = \tan^{-1} \left(\frac{160}{170} \right)$$

$$\theta = 43^\circ$$

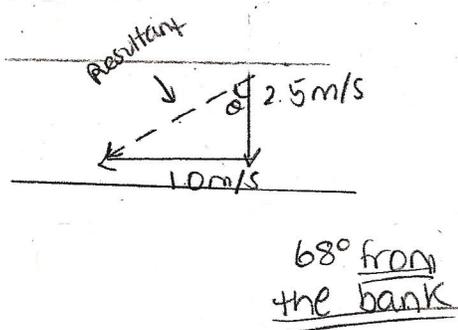
Word Problems

When solving word problems with vectors:

- Start by drawing the situation.
- Find the required vector. **(IT MAY NOT BE THE RESULTANT!)**

Examples

- 1) A swimmer wishes to cross a river that flow at a rate of 1.0 m/s. The swimmer can swim at a speed of 2.5 m/s. What is the resultant velocity of the swimmer as she crosses the river?



$$\text{mag} = \sqrt{(1.0 \text{ m/s})^2 + (2.5 \text{ m/s})^2}$$

$$\text{mag} = 2.7 \text{ m/s}$$

$$\text{Direction } \tan \theta = \frac{1.0 \text{ m/s}}{2.5 \text{ m/s}}$$

$$\theta = \tan^{-1} \left(\frac{1.0 \text{ m/s}}{2.5 \text{ m/s}} \right)$$

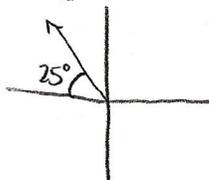
$$\theta = 22^\circ$$

- 2) During a football practice, a player sprints 30.0 m [N], then shuffles 10.0 [E] and finally runs 20.0 m [W25°N] backwards. What is the resultant displacement of the player?

$$A : (0, 30.0 \text{ m})$$

$$B : (10.0 \text{ m}, 0)$$

$$C : (-18.1 \text{ m}, 8.5 \text{ m})$$



$$\cos 25^\circ = \frac{C_x}{20.0 \text{ m}}$$

$$C_x = 20.0 \text{ m} \cos 25^\circ$$

$$C_x = 18.1 \text{ m}$$

$$\downarrow$$

$$-18.1 \text{ m}$$

$$\sin 25^\circ = \frac{C_y}{20.0 \text{ m}}$$

$$C_y = 20.0 \text{ m} \sin 25^\circ$$

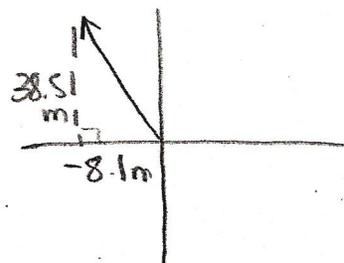
$$C_y = 8.5 \text{ m}$$

$$(0, 30.0 \text{ m})$$

$$+ (10.0 \text{ m}, 0)$$

$$(-18.1 \text{ m}, 8.5 \text{ m})$$

$$\hline (-8.1, 38.5 \text{ m})$$



$$\text{Resultant Mag} = \sqrt{(8.1)^2 + (38.5)^2}$$

$$\vec{R} = 39.3 \text{ m}$$

Direction =

$$\tan \theta = \frac{38.5 \text{ m}}{8.1 \text{ m}}$$

$$\theta = \tan^{-1} \left(\frac{38.5 \text{ m}}{8.1 \text{ m}} \right)$$

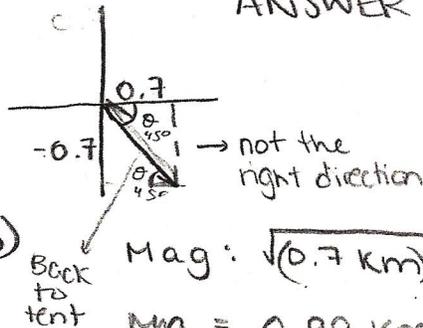
$$\{ 39.3 \text{ m } \swarrow 78^\circ \text{ N} \}$$

$$\theta = 78^\circ$$

- 3) On a camping trip, you and a friend leave the tent with your compass and walk, 1.2 km [S], then 3.6 km [W], then 0.5 km [N] and finally 4.3 km [E]. At this point, you decide you have had enough, so you walk directly back to your tent. Give the vector that would represent your displacement as you return to the tent. **ANSWER \neq RESULTANT** *

- A: (0, -1.2 km)
 B: (-3.6 km, 0)
 C: (0, 0.5 km)
 D: (4.3 km, 0)

(0.7 km, -0.7 km)



Mag: $\sqrt{(0.7 \text{ km})^2 + (0.7 \text{ km})^2}$

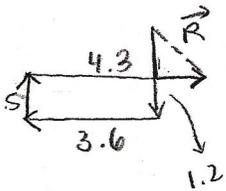
Mag = 0.99 km

$\tan \theta = \frac{0.7 \text{ km}}{0.7 \text{ km}}$

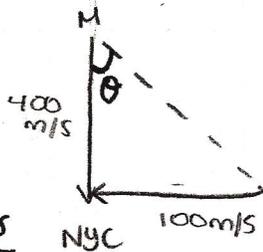
$\theta = 45^\circ$

0.99 km W 45° N

or
0.99 km northwest

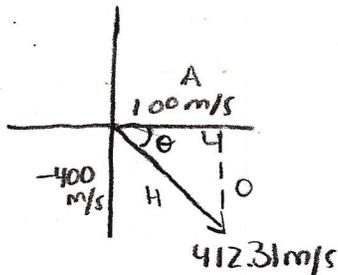


- 4) A plane can fly at a maximum speed of 400 m/s. The wind blows from the east at a speed of 100 m/s. The plane is flying from Montreal to New York City, which is located directly south of Montreal. What heading should the plane take so that it actually gets to New York?



$\sin^{-1} = \frac{100 \text{ m/s}}{400 \text{ m/s}}$

$\theta = 14.4^\circ$



A: (0, -400 m/s)

B: (100 m/s, 0)

(100 m/s, -400 m/s)

Mag: $\sqrt{(100 \text{ m/s})^2 + (400 \text{ m/s})^2}$

= $\sqrt{10000 + 160000}$

= $\sqrt{170000}$

= 412.31 m/s

$\tan \theta = \frac{400 \text{ m/s}}{100 \text{ m/s}}$

$\theta = \tan^{-1} = \left(\frac{400 \text{ m/s}}{100 \text{ m/s}} \right)$

$\theta = 76^\circ$

412.31 m/s S 76° E

- 75.6°

OR

S 14.4° E

or

E 75.6° S