

Examples: If  $\vec{u} = (4,3)$  and  $\vec{v} = (1,3)$  then

- $\vec{u} + \vec{v} = (4,3) + (1,3) = (4+1, 3+3) = (5,6)$
- $\vec{u} - \vec{v} = (4,3) - (1,3) = (4-1, 3-3) = (3,0)$
- $2\vec{v} = 2(1,3) = (2 \cdot 1, 2 \cdot 3) = (2,6)$
- $-\vec{u} = -(4,3) = (-4,-3)$

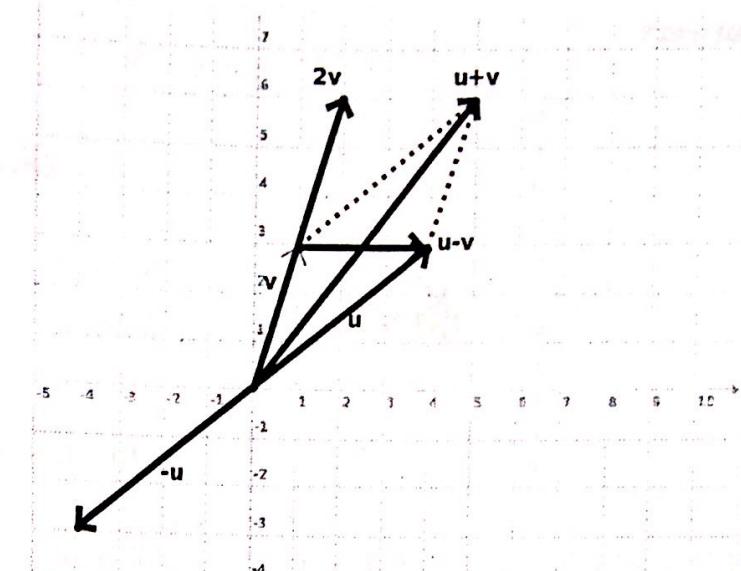


Figure 4: Geometric interpretation of vector operations

Given vector  $\vec{u} = (u_1, u_2)$  its length can be calculated easily with the formula

$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2}$$

## Exercises

1. Sketch each one of the following vectors having initial point  $P_1$  and terminal point  $P_2$

- |                         |                            |                           |
|-------------------------|----------------------------|---------------------------|
| a) $P_1(4,8), P_2(3,7)$ | b) $P_1(3,-5), P_2(-4,-7)$ | c) $P_1(-5,0), P_2(-3,1)$ |
| d) $P_1(0,0), P_2(2,3)$ | e) $P_1(3,-7), P_2(-2,5)$  | f) $P_1(-1,0), P_2(0,-1)$ |

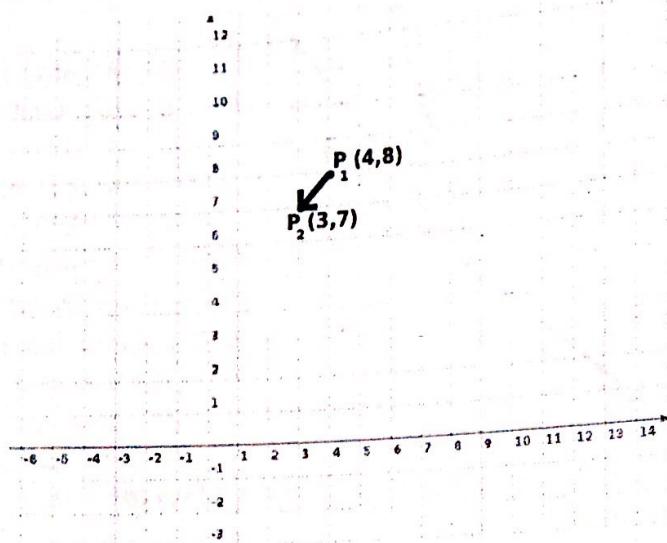
2. Find the length of each one of the vectors  $\overrightarrow{P_1 P_2}$  in problem 1a-f.

3. Find the vector  $\vec{u}$  in standard position that is equal to vector  $\overrightarrow{P_1 P_2}$  (for each vector in 1a-f). Calculate  $\|\vec{u}\|$  for each  $\vec{u}$ .

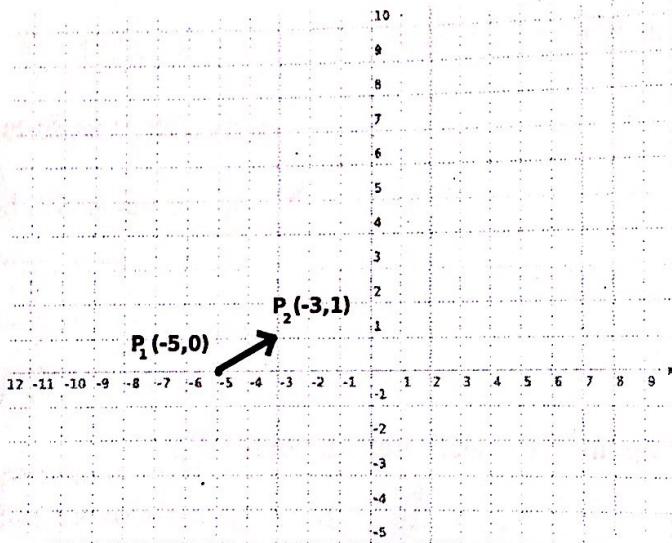
4. Let  $\vec{u} = (-3,1)$ ,  $\vec{v} = (4,0)$  and  $\vec{w} = (6,-1)$ . Find the following vectors

- |                            |                             |   |
|----------------------------|-----------------------------|---|
| a) $\vec{v} - \vec{w}$     | b) $6\vec{u} + 2\vec{v}$    | c) $-\vec{v} + \vec{u}$                           |
| d) $5(\vec{v} - 4\vec{u})$ | e) $-3(\vec{v} - 3\vec{w})$ | f) $(2\vec{u} - 7\vec{w}) - (8\vec{v} + \vec{u})$ |

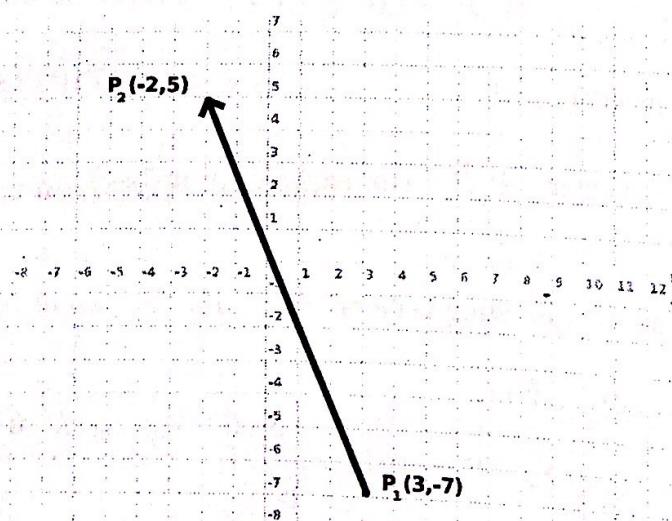
## Answers



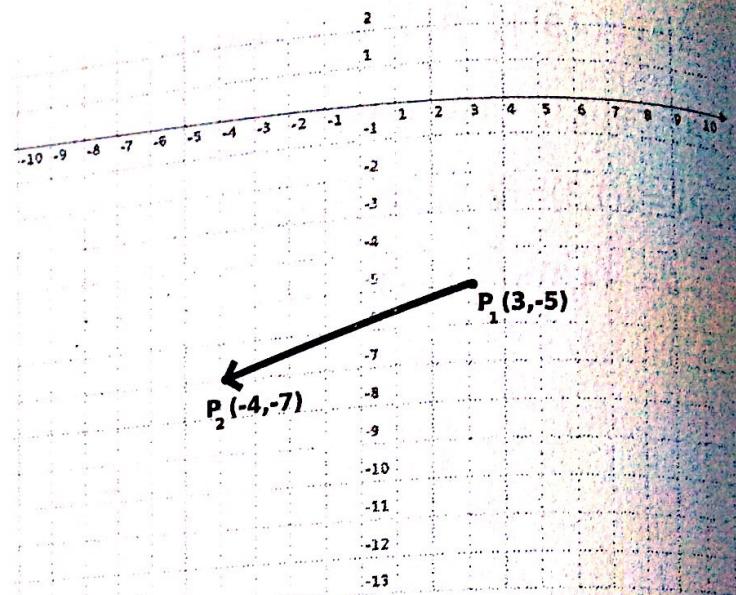
Answer 1a: Vector  $\overrightarrow{P_1 P_2}$



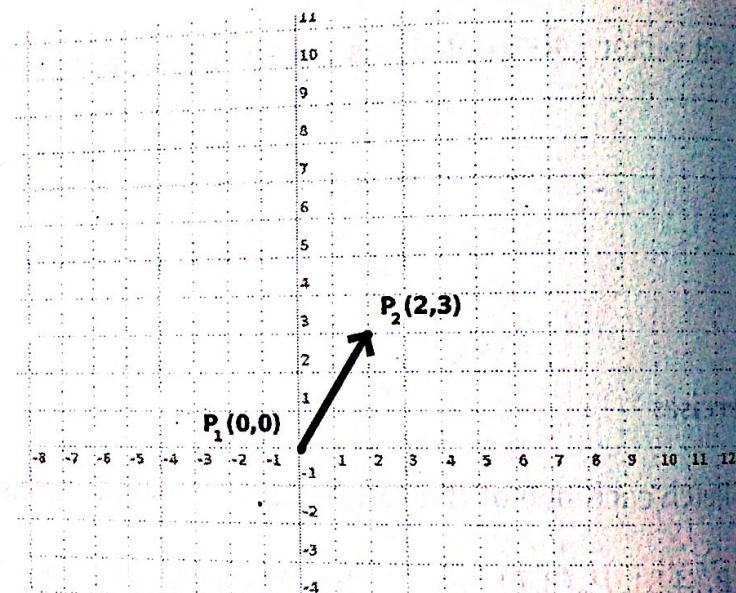
Answer 1c: Vector  $\overrightarrow{P_1 P_2}$



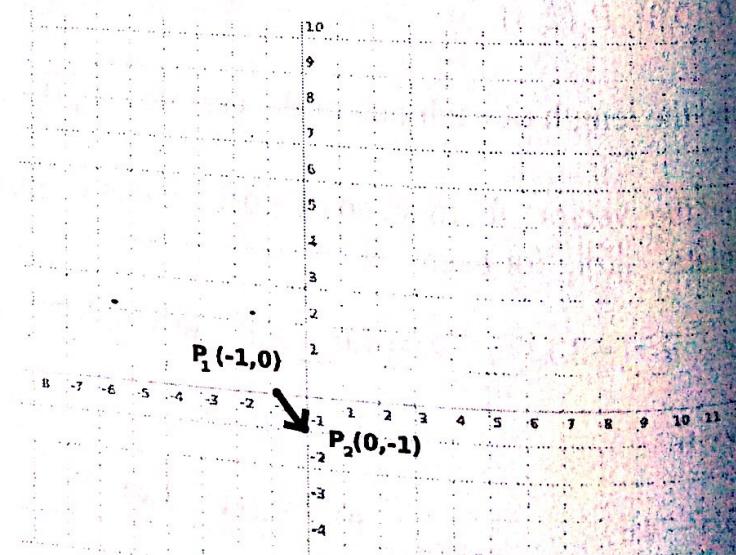
Answer 1e: Vector  $\overrightarrow{P_1 P_2}$



Answer 1b: Vector  $\overrightarrow{P_1 P_2}$



Answer 1d: Vector  $\overrightarrow{P_1 P_2}$



Answer 1f: Vector  $\overrightarrow{P_1 P_2}$

2.

a)  $\|\overrightarrow{P_0 P_1}\| = \sqrt{(3-4)^2 + (7-8)^2} = \sqrt{1+1} = \sqrt{2}$

b)  $\|\overrightarrow{P_0 P_1}\| = \sqrt{(-4-3)^2 + (-7-(-5))^2} = \sqrt{49+4} = \sqrt{53}$

c)  $\|\overrightarrow{P_0 P_1}\| = \sqrt{(-3-(-5))^2 + (1-0)^2} = \sqrt{4+1} = \sqrt{5}$

d)  $\|\overrightarrow{P_0 P_1}\| = \sqrt{(2-0)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$

e)  $\|\overrightarrow{P_0 P_1}\| = \sqrt{(-2-3)^2 + (5-(-7))^2} = \sqrt{25+144} = \sqrt{169} = 13$

f)  $\|\overrightarrow{P_0 P_1}\| = \sqrt{(0-(-1))^2 + (-1-0)^2} = \sqrt{1+1} = \sqrt{2}$

3.

a)  $\vec{u} = (3-4, 7-8) = (-1, -1)$

$$\|\vec{u}\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

b)  $\vec{u} = (-4-3, -7-(-5)) = (-7, -2)$

$$\|\vec{u}\| = \sqrt{(-7)^2 + (-2)^2} = \sqrt{53}$$

c)  $\vec{u} = (-3-(-5), 1-0) = (2, 1)$

$$\|\vec{u}\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

d)  $\vec{u} = (2-0, 3-0) = (2, 3)$

$$\|\vec{u}\| = \sqrt{2^2 + 3^2} = \sqrt{13}$$

e)  $\vec{u} = (-2-3, 5-(-7)) = (-5, 12)$

$$\|\vec{u}\| = \sqrt{(-5)^2 + (12)^2} = \sqrt{169} = 13$$

f)  $\vec{u} = (0-(-1), -1-0) = (1, -1)$

$$\|\vec{u}\| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

4.

a)  $\vec{v} - \vec{w} = (4, 0) - (6, -1) = (4-6, 0-(-1)) = (-2, 1)$

b)  $6\vec{u} + 2\vec{v} = 6(-3, 1) + 2(4, 0) = (-18, 6) + (8, 0) = (-10, 6)$

c)  $-\vec{v} + \vec{u} = -(4, 0) + (-3, 1) = (-4, 0) + (-3, 1) = (-7, 1)$

d)  $5(\vec{v} - 4\vec{u}) = 5\vec{v} - 20\vec{u} = 5(4, 0) - 20(-3, 1) = (20, 0) - (-60, 20) = (80, -20)$

e)  $-3(\vec{v} - 3\vec{w}) = -3\vec{v} + 9\vec{w} = -3(4, 0) + 9(6, -1) = (-12, 0) + (54, -9) = (42, -9)$

f)  $2\vec{u} - 7\vec{w} - (8\vec{v} + \vec{u}) = 2\vec{u} - 7\vec{w} - 8\vec{v} - \vec{u} = \vec{u} - 7\vec{w} - 8\vec{v} = (-3, 1) - 7(6, -1) - 8(4, 0)$

$$= (-3, 1) - (42, -7) - (32, 0) = (-3-42-32, 1-(-7)-0) = (-77, 8)$$