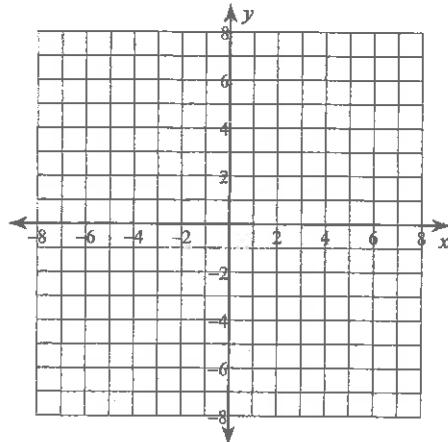


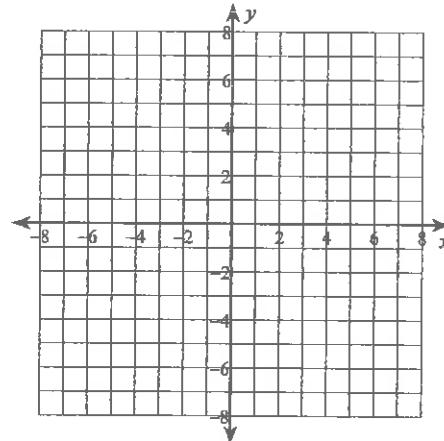
PRETEST LOGARITHMIC FUNCTIONS PART 2

Sketch the graph of each function and perform a complete study, including equation of asymptote.

1) $f(x) = \log_5(4x + 14) - 2$



2) $f(x) = \log_{\frac{1}{4}}(4x - 11) + 5$



Solve each equation.

3) $\log_3(n+9) = \log_3(4n+6)$

4) $\log(-3n+1) = \log -4n$

5) $\log_{16}(42 - 3p^2) = \log_{16}(-2p^2 + p)$

6) $\log_{11}(p^2 + 1) = \log_{11}(4p - 2)$

7) $\log_6 2 + \log_6 3x^2 = 2$

8) $\log_5(x+5) - \log_5 x = \log_5 74$

9) $\log_8 2 + \log_8(x^2 - 9) = 4$

10) $\log_8 x - \log_8(x+4) = \log_8 75$

11) $2^x = 5^{x+2}$

12) $3^x = 7^{x-1}$

13) If $\log_c 5 = x$ and $\log_c 2 = y$, what is $\log_c(100c^5)$? (in terms of x and y)

14) Simplify $\log_4 8 + \log_5 625 - (\log_8 8^8 + \log_{\frac{1}{2}} 8)$

15) What is the rule of a logarithmic function ($y = \log_a(x-h)+k$) if $h=1$, passing through $(4, 2)$ and $(10, 7)$?

16) How long will it take a \$2000 investment to triple its value, if its interest rate is 6%, compounded monthly?

17) A car purchased for \$15000 depreciates by 30% yearly. In how many years will it be worth \$500?

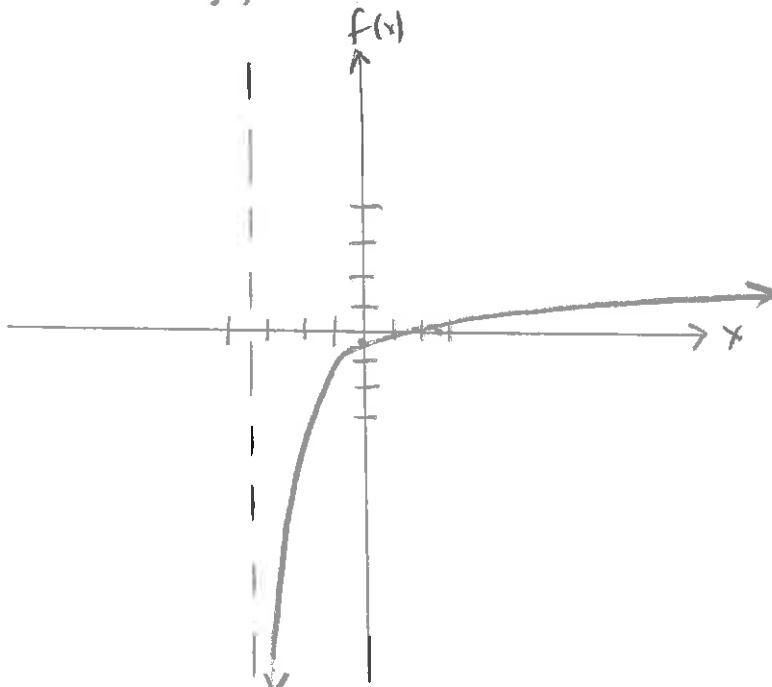
18) A town's population increases from 11211 people to 96627 at an annual rate of 4.18%. How many years did this take?

Logarithmic Functions
pretest - Part 2

①

$$1) f(x) = \log_5(4x+14) - 2$$

$$= \log_5 4(x + \frac{7}{2}) - 2$$



let $x = 0$

$$f(x) = \log_5 14 - 2$$

$$\approx -0.36$$

let $f(x) = 0$

$$\log_5(4x+14) - 2 = 0$$

$$\log_5(4x+14) = 2$$

$$4x+14 = 2^5$$

$$4x = 11$$

$$x = \frac{11}{4}$$

domain $[-\frac{7}{2}, +\infty]$

range $[-2, +\infty]$

Vertical asymptote $x = -\frac{7}{2}$

y int $\approx \{-0.36\}$

x int $\{\frac{11}{4}\}$

max none

min none

↑ domain

↓ never

+ $[\frac{11}{4}, +\infty]$

- $[-\frac{7}{2}, \frac{11}{4}]$



②

$$\begin{aligned} 2) \quad f(x) &= \log_4 (4x-11) + 5 \\ &= -\log_4 (4x-11) + 5 \\ &= -\log_4 4(x - \frac{11}{4}) + 5 \end{aligned}$$

let $x = 0$

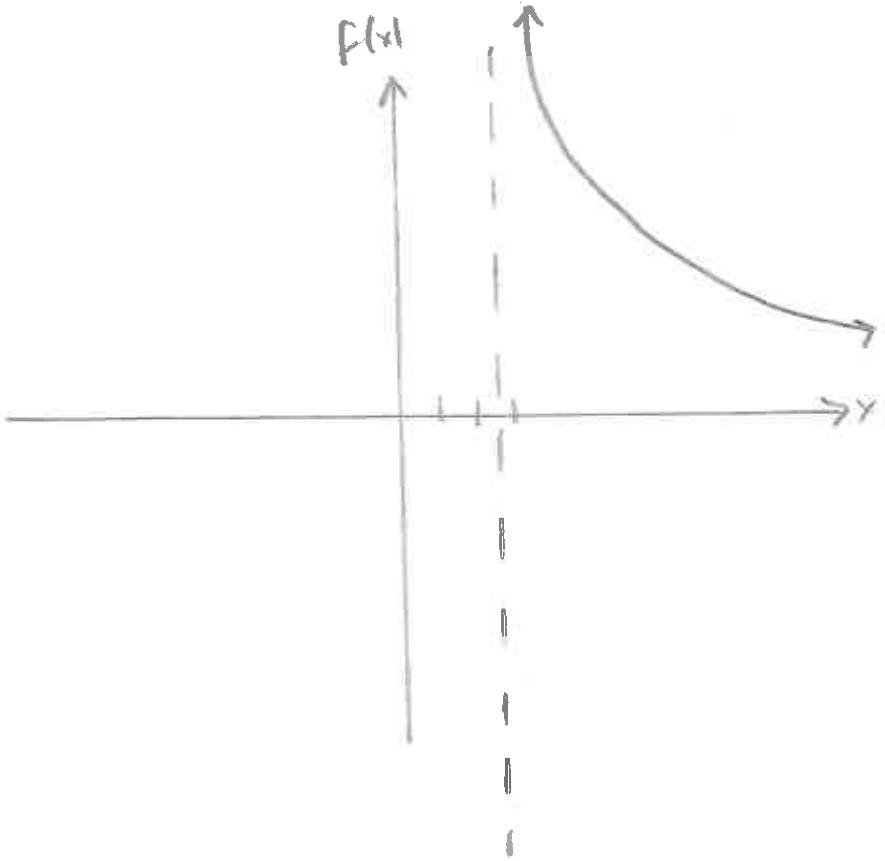
$$f(x) = -\log_4 (-11) + 5$$

no solution

let $f(x) = 0$

$$\begin{aligned} -\log_4 (4x-11) + 5 &= 0 \\ -\log_4 (4x-11) &= -5 \\ +\log_4 (4x-11) &= +5 \\ 4x-11 &= 4^5 \\ 4x &= 11 + 4^5 \\ x &= \frac{11 + 4^5}{4} \end{aligned}$$

$$= 258.75$$

domain $[\frac{11}{4}, +\infty]$ range $]-\infty, +\infty[$ Vertical asymptote $x = \frac{11}{4}$

y-int - none

x-int - $\{258.75\}$

max none

min none

↑ new

↓ domain

+ $[\frac{11}{4}, 258.75]$ - $[258.75, +\infty[$

(3)

$$3) \log_3(n+9) = \log_3(4n+6)$$

$$n+9 = 4n+6$$

$$3 = 3n$$

$$1 = n \quad \text{verify} \quad \checkmark$$

✓

$$4) \log(-3n+1) = \log -4n$$

$$-3n+1 = -4n$$

$$n = -1 \quad \text{verify} \quad \checkmark$$

✓

$$5) \log_{16}(42-3p^2) = \log_{16}(-2p^2+7)$$

$$42-3p^2 = -2p^2 + 7$$

$$0 = p^2 + p - 42$$

$$(p+7)(p-6) = 0$$

$$p = -7 \quad \text{verify} \quad \text{No}$$

$$p = 6 \quad \text{verify} \quad \text{No}$$

No solution

✓

$$6) \log_{11}(p^2+1) = \log_{11}(4p-4)$$

$$p^2+1 = 4p-4$$

$$p^2 - 4p + 3 = 0$$

$$(p-3)(p-1) = 0$$

$$p = 3 \quad \text{verify} \quad \checkmark$$

$$p = 1 \quad \text{verify} \quad \checkmark$$

✓

$$\{1, 3\}$$

④

$$7) \log_6 2 + \log_6 3x^2 = 2$$

$$\log_6 2 \cdot 3x^2 = 2$$

$$6x^2 = 6^2$$

$$6x^2 = 36$$

$$x^2 = 6$$

$$x = \sqrt{6} \quad \text{verify } \checkmark$$

$$x = -\sqrt{6} \quad \text{verify } \checkmark$$

$$\{-\sqrt{6}, \sqrt{6}\}$$

✓

$$8) \log_5(x+5) - \log_5 x = \log_5 74$$

$$\log_5 \frac{x+5}{x} = \log_5 74$$

$$\frac{x+5}{x} = 74$$

$$x+5 = 74x$$

$$5 = 73x$$

$$x = \frac{5}{73} \quad \text{verify } \checkmark$$

$$\{\frac{5}{73}\}$$

✓

$$9) \log_8 2 + \log_8 (x^2 - 9) = 4$$

$$2(x^2 - 9) = 8^4$$

$$x^2 - 9 = \frac{8^4}{2}$$

$$x^2 = 9 + \frac{8^4}{2} = 2057$$

$$x = \sqrt{2057} = \sqrt{121} \sqrt{17} = 11\sqrt{17}$$

$$x = -\sqrt{2057} = -11\sqrt{17}$$

$$\text{verify } \checkmark \quad \{-11\sqrt{17}, 11\sqrt{17}\}$$

$$10) \log_3 x - \log_3 (x+4) = \log_3 75$$

$$\log_3 \frac{x}{x+4} = \log_3 75$$

$$\frac{x}{x+4} = 75$$

$$x = 75x + 300$$

$$-300 = 74x$$

$$\frac{-300}{74} = x \quad \text{verify } \text{N}$$

no solution

/

✓

⑤

$$11) \quad 2^x = 5^x + 2$$

$$2^x = 5^x \cdot 5^{-2}$$

$$\left(\frac{2}{5}\right)^x = 25$$

$$\left(\frac{2}{5}\right)^x = 25 \quad \longrightarrow \quad \log_{\frac{2}{5}} 25 = x$$

$$12) \quad 3^x = 7^{x-1}$$

$$\frac{3^x}{1} \frac{7^x}{7^1}$$

$$7 \cdot 3^x = 7^x$$

$$7 = \frac{7^x}{3^x}$$

$$\begin{aligned} & \left(\frac{7}{3}\right)^x = 7 \\ & \log_{\frac{7}{3}} 7 = x \end{aligned}$$

$$13) \quad \log_c(100c^r) = \log_c 100 + \log_c c^r$$

$$= \log_c(25 \cdot 4) + \log_c c^r$$

$$= \log_c 5^2 + \log_c 2^2 + \log_c c^r$$

$$= 2 \log_c 5 + 2 \log_c 2 + r \log_c c$$

$$= 2x + 2y + r$$

$$14) \quad \log_4(4^3)^{\frac{1}{2}} + \log_5 5^4 - \log_8 8^8 + \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-3}$$

$$\frac{3}{2} + 4 - 8 - 3 = \frac{3+8-16-6}{2} = \frac{-11}{2}$$

$$⑤ y = \log_c(x-1) + k$$

⑥

$$2 = \log_c(4-1) + k \rightarrow k = 2 - \log_c(3)$$

$$3 = \log_c(10-1) + k \rightarrow k = 3 - \log_c 9$$

$$2 - \log_c 3 = 3 - \log_c 9$$

$$\log_c 9 - \log_c 3 = 3 - 2$$

$$\log_c\left(\frac{9}{3}\right) = 1$$

$$c = 3 \quad \boxed{c=3}$$

$$k = 2 - \log_3 3 = 2 - 1 = 1 \quad \boxed{k=1}$$

$$\boxed{y = \log_3(x-1) + 1}$$

$$⑥ A = P \left(1 + \frac{r}{n}\right)^{nt} \quad \begin{aligned} 3(2000) &= 2000 \left(1 + \frac{0.06}{12}\right)^{nt} \\ 3 &= (1.005)^{12t} \end{aligned}$$

$$\log_{1.005} 3 = 12t$$

$$t = \frac{1}{12} \left(\frac{\log 3}{\log 1.005} \right) = \boxed{18.4 \text{ years}}$$

$$⑦ y = a e^x \rightarrow 15000 e^{0.7x} = 500$$

$$0.7^x = 0.0333$$

$$\log_{0.7} 0.0333 = x$$

$$\boxed{x = 9.53 \text{ years}}$$

$$⑧ y = a e^x \rightarrow (11211)(1.0418)^x = 96627$$

$$1.0418^x = 8.6189$$

$$\log_{1.0418} 8.6189 = x$$

$$\boxed{x = 52.6 \text{ years}}$$