RATIONAL FUNCTION (1)

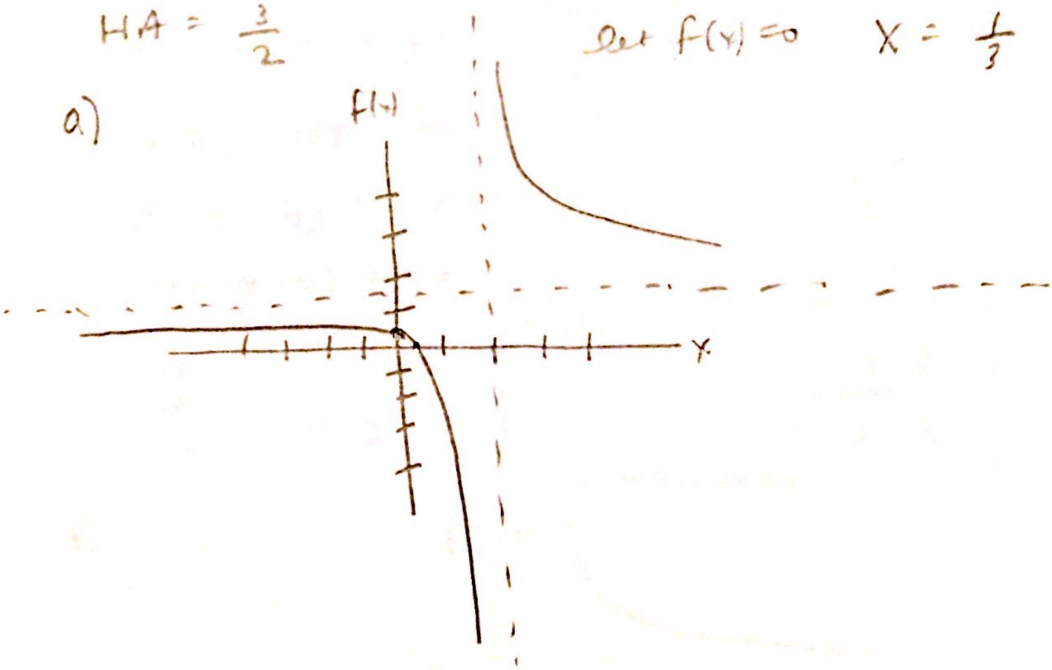
pre test key

(A) $f(x) = \dfrac{3x-1}{2x-4}$

VA = 2   let $x=0$ $f(x) = \frac{1}{4}$ $(0, \frac{1}{4})$

HA = $\frac{3}{2}$ let $f(x)=0$ $x = \frac{1}{3}$ $(\frac{1}{3}, 0)$

a)



b) domain $\mathbb{R} \setminus \{2\}$

range $\mathbb{R} \setminus \{3/2\}$

y int $\{\frac{1}{4}\}$

x int $\{\frac{1}{3}\}$

max none

min none

$+$ $]-\infty, \frac{1}{3}] \cup]2, +\infty[$

$-$ $[\frac{1}{3}, 2[$

↑ never

↓ domain

c) $y = \frac{3x-1}{2x-4}$

$x = \frac{3y-1}{2y-4}$

$x(2y-4) = 3y-1$

$2xy - 4x = 3y - 1$

$2xy - 3y = 4x - 1$

$y(2x-3) = 4x-1$

$y = \frac{4x-1}{2x-3}$ $\boxed{f^{-1}(x) = \frac{4x-1}{2x-3}}$

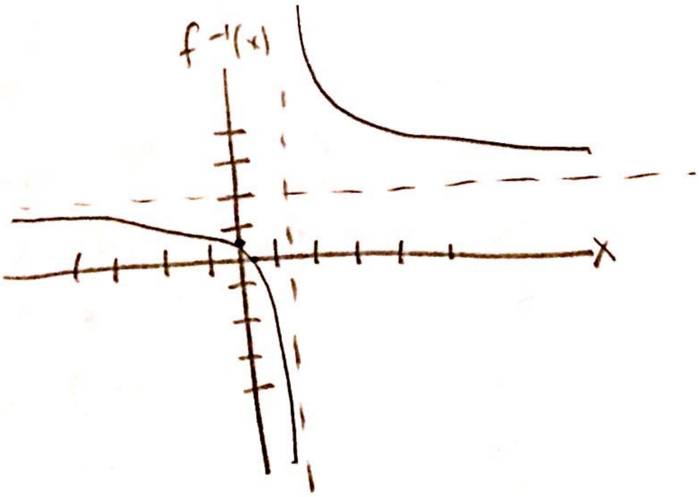
HA = 2 ✓

VA = $\frac{3}{2}$ ✓

$(0, \frac{1}{3})$ ✓

$(\frac{1}{4}, 0)$ ✓

d)



e)

$$\begin{array}{r|l} & \frac{3}{2} \\ 2x-4 & 3x-1 \\ & -\;(3x-6) \\ \hline & 5 \end{array}$$

$\frac{3}{2} + \frac{5}{2x-4}$

$\boxed{f(x) = \frac{5}{2x-4} + \frac{3}{2}}$

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A'

$$g(x) = \frac{-4}{x+5} + 1$$

VA = -5 — let x = 0, $g(x) = \frac{1}{5}$ — $(0, \frac{1}{5})$

HA = 1

let g(x) = 0 :

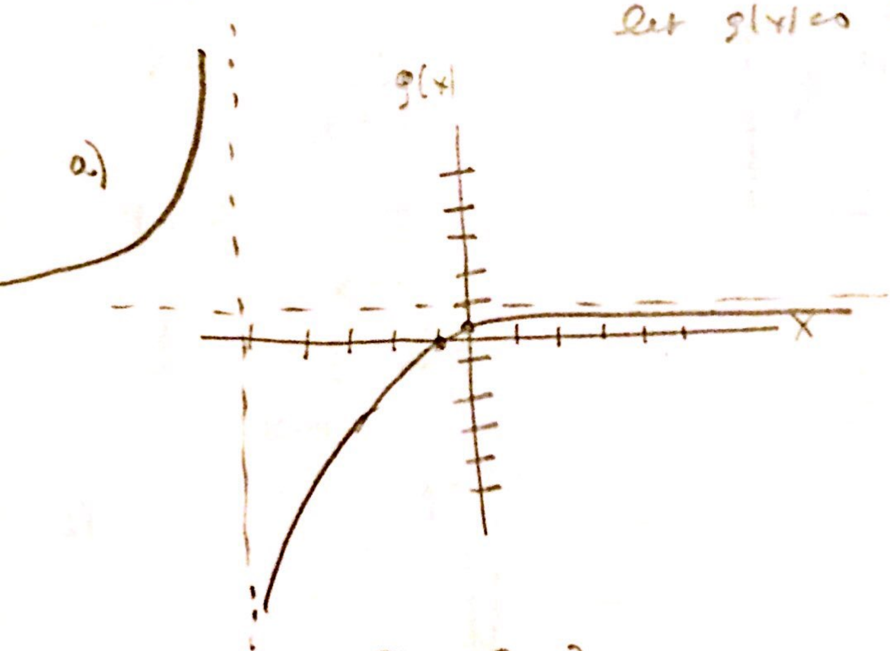
$$-1 = \frac{-4}{x+5}$$

$-x-5=-4$

$-x = 1$ — $x = -1$

$(-1, 0)$

a.)



b) domain $\mathbb{R} \setminus \{-5\}$ ✓

range $\mathbb{R} \setminus \{1\}$ ✓

y int $\{\frac{1}{5}\}$ ✓

x int $\{-1\}$ ✓

Max none ✓

min none ✓

+ $]-\infty, -5[\cup [-1, +\infty[$ ✓

- $]-5, -1]$ ✓

↑ domain ✓

↓ never ✓

c) $y = \frac{-4}{x+5} + 1$

$x = \frac{-4}{y+5} + 1$

$x - 1 = \frac{-4}{y+5}$

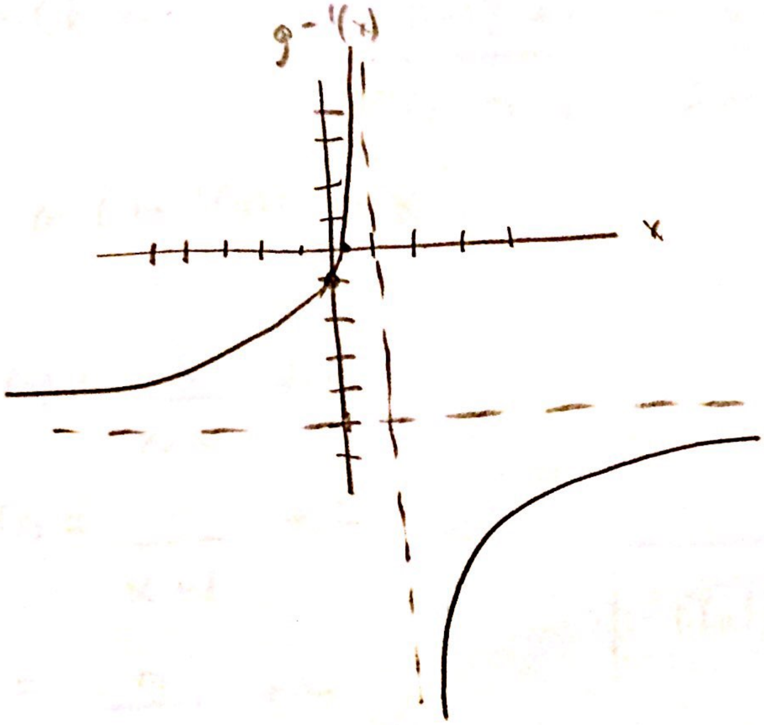
$y + 5 = \frac{-4}{x-1}$

$y = \frac{-4}{x-1} - 5$

$$\boxed{g^{-1}(x) = \frac{-4}{x-1} - 5}$$

HA = -5 ✓
VA = 1 ✓
(0, -1) ✓
($\frac{1}{5}$, 0) ✓

d)



e) $\frac{-4}{x+5} + 1 = \frac{-4}{x+5} + \frac{x+5}{x+5} = \frac{x+1}{x+5}$

$$\boxed{g(x) = \frac{x+1}{x+5}}$$

B

$$\frac{x+1}{x-5} < 2$$

Critical point

$x = 5$ (the VA)

$x+1 = 2(x-5)$

$x+1 = 2x-10$

$11 = x$ = critical point

TEST ✓ ∘ TEST ✗ ∘ TEST ✓

5 11

$]5, 11[$ ✓

C a) $\frac{h(x)}{k(x)} : \frac{2x+5}{x-5}$ ✓

b) $m(k(x)) : \frac{(x-5)+1}{5(x-5)-3} = \frac{x-4}{5x-25-3} = \frac{x-4}{5x-28}$ ✓

c) $m(m^{-1}(x)) = x$ ✓

D $i(x) = \frac{a}{x-h} + k$

$i(x) = \frac{a}{x+1} + 2$

$3 = \frac{a}{2+1} + 2$

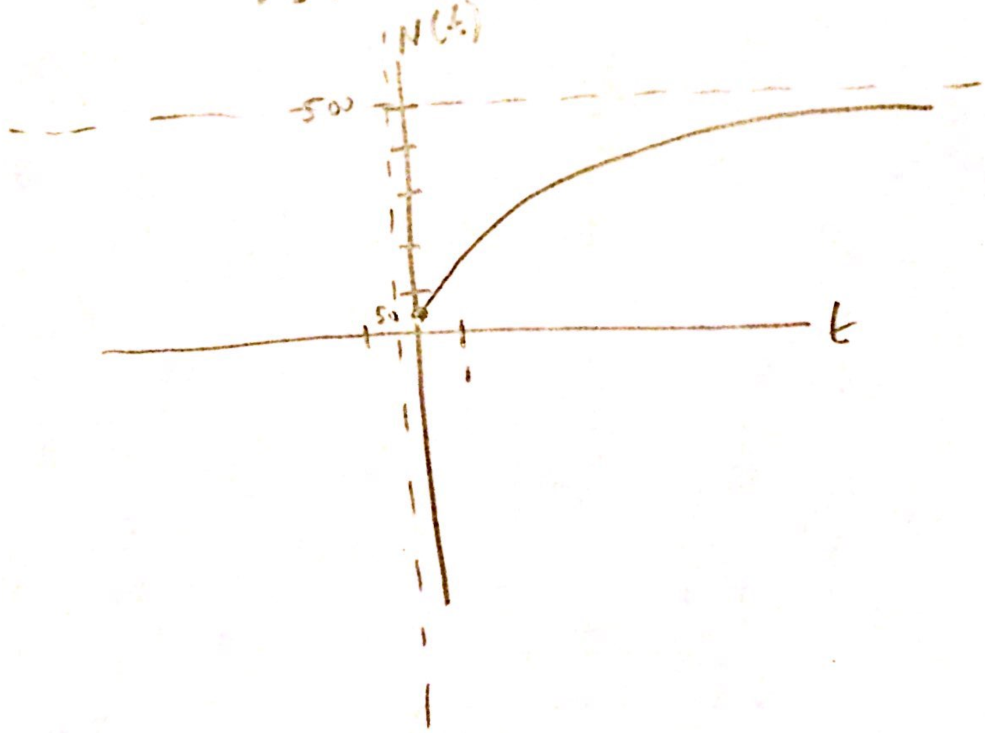
$1 = \frac{a}{3}$

$a = 3$

$$\boxed{i(x) = \frac{3}{x+1} + 2}$$ ✓

(b)

(E) $N(t) = \dfrac{-450}{3t+1} + 500$



domain $t \geq 0$

Solution.

(E)

a)

1. $N(0) = 500 - \frac{450}{1+3(0)} = 50$. This means that at the beginning of the semester, 50 students have had the flu.
2. We set $N(t) = 300$ to get $500 - \frac{450}{1+3t} = 300$ and solve. Isolating the fraction gives $\frac{450}{1+3t} = 200$. Clearing denominators gives $450 = 200(1 + 3t)$. Finally, we get $t = \frac{5}{12}$. This means it will take $\frac{5}{12}$ months, or about 13 days, for 300 students to have had the flu.
3. To determine the behavior of N as $t \to \infty$, we can use a table.

t	$N(t)$
10	≈ 485.48
100	≈ 498.50
1000	≈ 499.85
10000	≈ 499.98

we approach HA

The table suggests that as $t \to \infty$, $N(t) \to 500$. (More specifically, 500^-.) This means as time goes by, only a total of 500 students will have ever had the flu. □