

1. Perform the division. List the quotient and remainder.

(a) $\frac{3x^2-11x+5}{x-4}$

Answer

$$\begin{array}{r|rrrr} [4] & 3 & -11 & 5 & \\ & 0 & 12 & 4 & \\ \hline & 3 & 1 & [9] & \end{array}$$

Therefore, $3x^2 - 11x + 5 = (x - 4)(3x + 1) + 9$ where $3x + 1$ is the quotient and 9 is the remainder.

(b) $\frac{5x^5+3x^3+1}{x+2}$

Answer

$$\begin{array}{r|rrrrrrr} [-2] & 5 & 0 & 3 & 0 & 0 & 1 & \\ & 0 & -10 & 20 & -46 & 92 & -184 & \\ \hline & 5 & -10 & 23 & -46 & 92 & [-183] & \end{array}$$

Therefore, $5x^5 + 3x^3 + 1 = (5x^4 - 10x^3 + 23x^2 - 46x + 92)(x + 2) - 183$ where $5x^4 - 10x^3 + 23x^2 - 46x + 92$ is the quotient and -183 is the remainder.

(c) $\frac{9x^3+14x-6}{3x-2}$

Answer

$$\begin{array}{r|rrrr} [\frac{2}{3}] & 9 & 0 & 14 & -6 \\ & 0 & 6 & 4 & 12 \\ \hline & 9 & 6 & 18 & [6] \end{array}$$

Therefore, $9x^3 + 14x - 6 = (9x^2 + 6x + 18)(x - \frac{2}{3}) + 6$ where $9x^2 + 6x + 18$ is the quotient and 6 is the remainder.

2. What is the remainder of the division of $p(x)$ by $x - 3$ if:

(a) $p(x) = 3x^4 + 3x - 1$

Answer

$$\begin{array}{r}
 [3] \quad 3 \quad 0 \quad 0 \quad 3 \quad -1 \\
 \quad \quad 0 \quad 9 \quad 27 \quad 81 \quad 252 \\
 \hline
 \quad \quad 3 \quad 9 \quad 27 \quad 84 \quad [251]
 \end{array}$$

Therefore, the remainder is $p(3) = 251$.

(b) $p(x) = 7x^5 - 500x + 3$

Answer

$$\begin{array}{r}
 [3] \quad 7 \quad 0 \quad 0 \quad 0 \quad -500 \quad 3 \\
 \quad \quad 0 \quad 21 \quad 63 \quad 189 \quad 567 \quad 201 \\
 \hline
 \quad \quad 7 \quad 21 \quad 63 \quad 189 \quad 67 \quad [204]
 \end{array}$$

Therefore, the remainder is $p(3) = 204$.

(c) $p(x) = 4x^4 + x$

Answer

$$\begin{array}{r}
 [3] \quad 4 \quad 0 \quad 0 \quad 1 \quad 0 \\
 \quad \quad 0 \quad 12 \quad 36 \quad 108 \quad 327 \\
 \hline
 \quad \quad 4 \quad 12 \quad 36 \quad 109 \quad [327]
 \end{array}$$

Therefore, the remainder is $p(3) = 327$.

3. Find all roots.

(a) $x^3 - 2x^2 - 5x + 6$

Answer

$$x^3 - 2x^2 - 5x + 6 = (x - 1)(x + 2)(x - 3)$$

and therefore the roots are $\{-2, 1, 3\}$.

(b) $x^4 + 2x^3 - 9x^2 - 2x + 8$

Answer

$$x^4 + 2x^3 - 9x^2 - 2x + 8 = (x - 1)(x + 1)(x - 2)(x + 4)$$

and therefore the roots are $\{-4, -1, 1, 2\}$.