Mathematics 5 SN

SINUSOIDAL GRAPHS AND WORD PROBLEMS

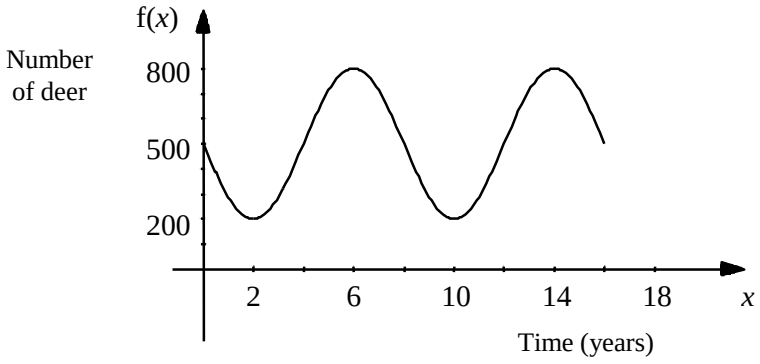
1 The tuning fork is a device used to verify the standard pitch of musical instruments. The international standard pitch has been set at a frequency of 440 cycles/second.

Write a rule in the form f(t) = A sin Bt that expresses this oscillation where t represents the number of seconds.

_____

2 Many factors influence the deer population in a given habitat: climate, hunting, predators, etc.

The following graph shows the evolution of a population of deer as a function of time.



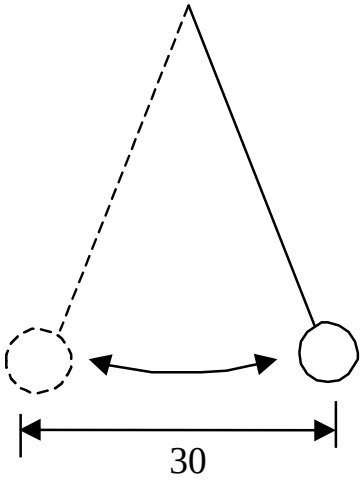
Write the rule that can be used to represent this function.

The rule of this function is _____

3 The pendulum of a Grandfather clock completes 20 cycles/minute.

It moves a distance of 30 cm.

Write a rule in the form f(t) = A sin Bt which expresses the movement of the pendulum where t represents the number of minutes.



The rule is _____.

4 An oscillatory movement is expressed by the equation

$$f(t) = 160 \sin(500\pi t - 100).$$

Find

a) its frequency

b) its period

c) its phase shift

d) its amplitude

a) _____

b) _____

c) _____

d) _____

5 A sound wave is described by a sinusoidal movement whose equation is :

$$f(t) = -230 \sin(30\pi t - 20).$$

Find

a) the amplitude

b) the period

c) the frequency

d) the phase shift

a) _____

b) _____

c) _____

d) _____

6 Radio station CKOI-FM broadcasts through a frequency of 97 KHz, or 97 000 cycles/s.

The radio's volume is set at 5, thus determining the sound amplitude.

Write a rule in the form f(t) = A sin Bt of the sinusoidal curve representing the sound waves transmitted.

The rule is _____.

7 For each of the following graphs, find

1) the amplitude

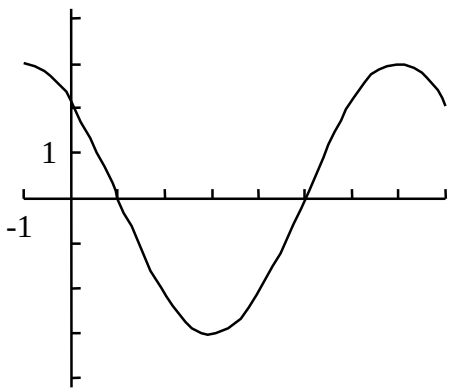
2) the period

3) the frequency

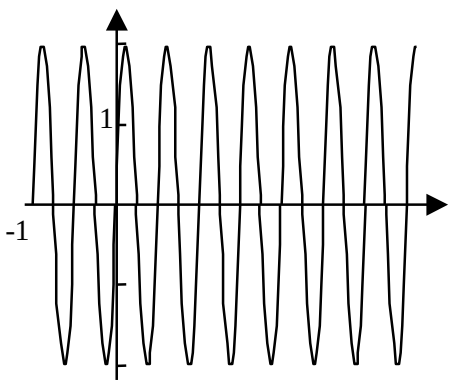
4) the phase shift

5) the equation

a)



b)



a) 1) amplitude = _____

2) period = _____

3) frequency = _____

4) phase shift = _____

5) f(x) = _____

b) 1) amplitude = _____

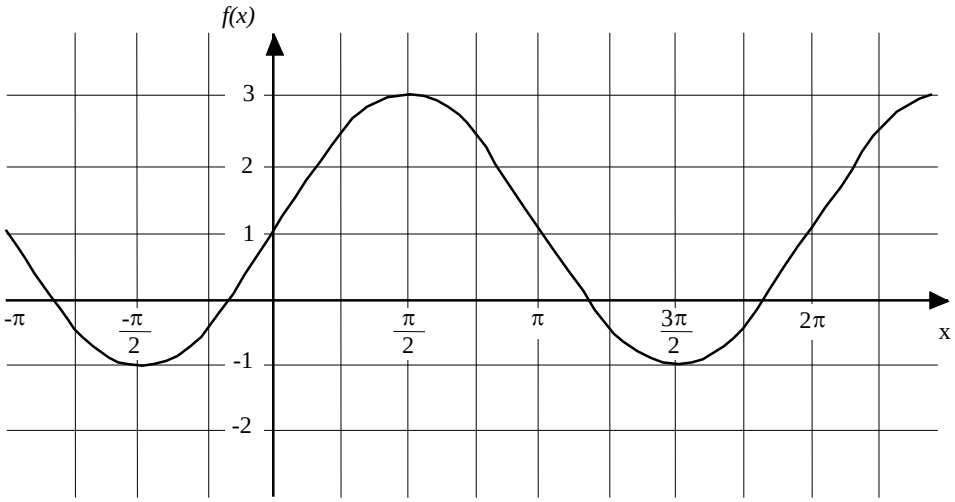
2) period = _____

3) frequency = _____

4) phase shift = _____

5) f(x) = _____

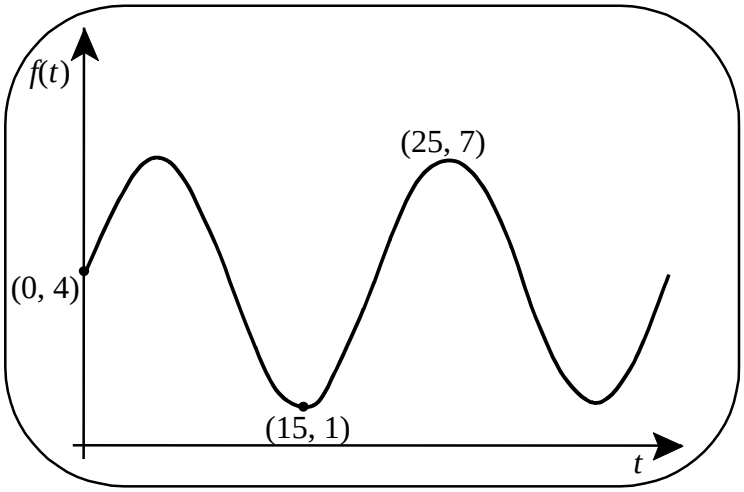
8 Jan is doing research on the phenomena of vibrations. She compiles a series of results and obtains the following graph on the computer screen.



What rule represents this function?

The rule representing this function is f(*x*) = _____.

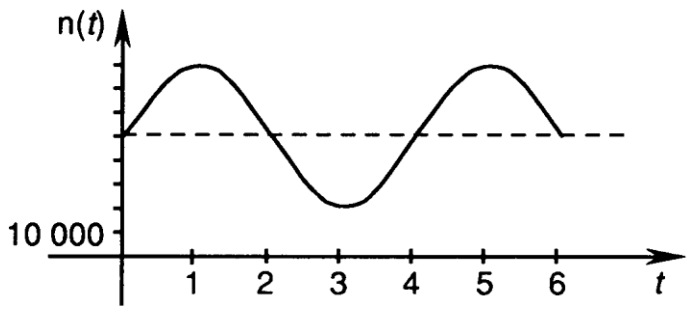
9 The screen of the oscilloscope below illustrates a sinusoidal function f, representing the amplitude of the vibration of a guitar string as a function of time t, in seconds.



What is the rule of this sinusoidal function?

The rule of the sinusoidal function is _____.

10 The number of pairs of shoes manufactured by a factory from December to June is associated with the sinusoidal function illustrated below.



where *t* is the number of months elapsed since December and n(*t*), the number of pairs of shoes.

What is the rule of function n?

The rule that corresponds to function n is _____

11 Given the following trigonometric function: $g(x) = -\frac{1}{2}\sin\left(2x - \frac{\pi}{2}\right) + 3$

State

a) the period of this function.

b) the amplitude of this function.

c) the phase shift of this function.

d) the y-intercept of this function.

a) The period of this function is _____.

b) The amplitude of this function is _____.

c) The phase shift of this function is _____.

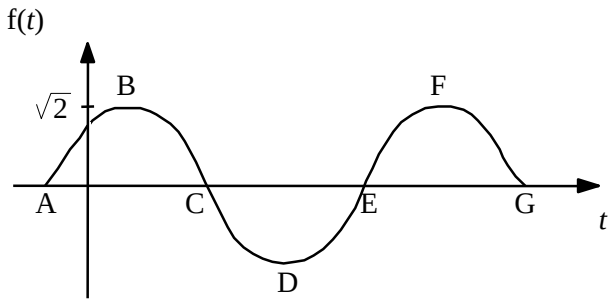
d) The y-intercept of this function is _____.

12 The temperature of an oven in a laboratory rises until it reaches a temperature of 22°C. The oven then stops heating and reactivates only when the temperature goes down to 18°C. The rule that defines the temperature of the oven at time *t*, in minutes, is the following: $f(t) = 2\sin \pi t + 20$.

During the first minute, when is the temperature of the oven 21°C?

These times are _____.

13 The following is a graph of a sinusoidal function f :



The coordinates of several points on the graph are given :

$$A\left(\frac{-\pi}{4}, 0\right),\ B\left(\frac{\pi}{4}, \sqrt{2}\right),\ C\left(\frac{3\pi}{4}, 0\right),\ D\left(\frac{5\pi}{4}, -\sqrt{2}\right),\ E\left(\frac{7\pi}{4}, 0\right),\ F\left(\frac{9\pi}{4}, \sqrt{2}\right),\ G\left(\frac{11\pi}{4}, 0\right).$$

What is the equation of this sinusoidal function?

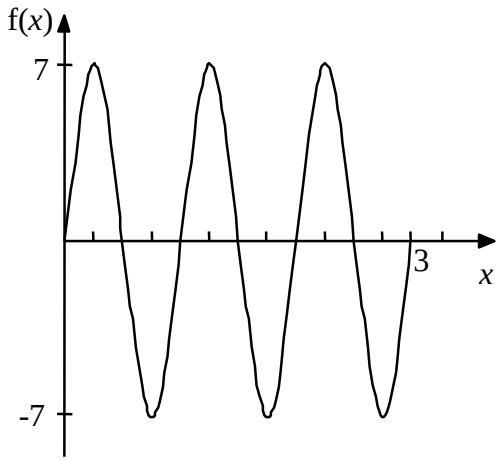
A) $f(t) = \sqrt{2}\sin\frac{4}{3}\left(t - \frac{\pi}{4}\right)$

B) $f(t) = \sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$

C) $f(t) = \sqrt{2}\sin\left(t + \frac{\pi}{4}\right)$

D) $f(t) = \sqrt{2}\sin\frac{4}{3}\left(t + \frac{\pi}{4}\right)$

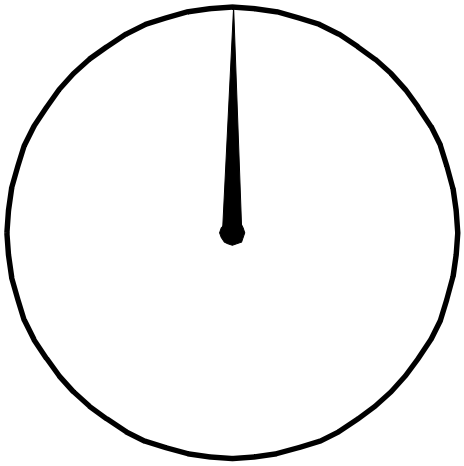
14 A team studies variations of certain physical phenomena. As you can see from the diagram on the right, for three seconds, the graph appearing on the oscilloscope is that of a sinusoidal function.



Write the rule of correspondence for this sinusoidal function.

The rule of correspondence for this sinusoidal function is : f(x) = _____.

15 The given clock only has a second hand.

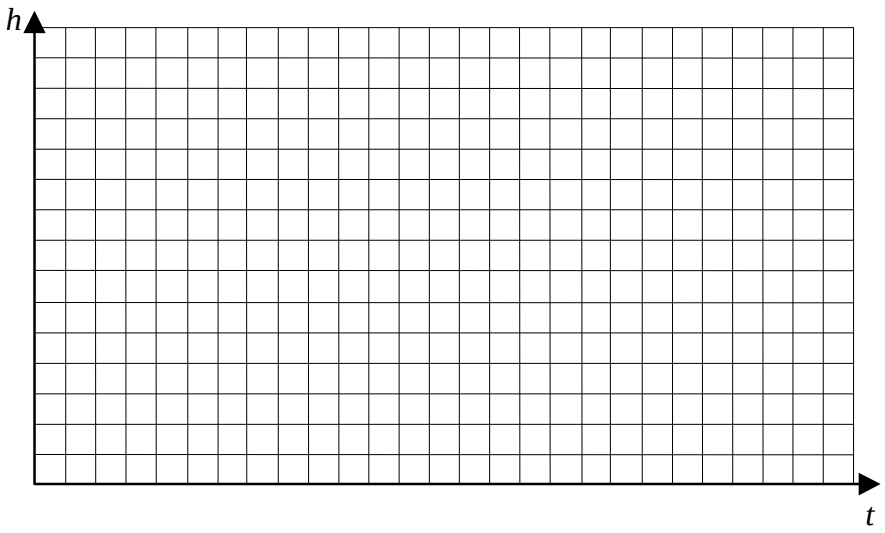


At noon, Maria sees the tip of the second hand at the top of the clock and notes the height of the tip of second hand in relation to the bottom of the clock. The second hand is 10 cm long.

What is the function rule describing the height (h) in cm of the tip of the second hand compared to the time (t) elapsed in seconds since noon?

Show all your work.

Show all your work.



Answer The function rule describing the height (h) of the tip of the second hand in relation to the time (t) elapsed since noon is:

$h(t)$ = _____

16 A kayaker is drifting on the Atlantic. The ocean is relatively calm and the movements of the waves can be represented by the equation below,

$$h(t) = 2\sin\frac{2\pi}{9}\left(t - \frac{\pi}{3}\right)$$

where t represents the time in seconds and $h(t)$ represents the height in metres.

In one minute, how many times did the kayaker reach the top of a wave?

A) 14 times

B) 9 times

C) 7 times

D) 6 times

17 During an experiment, the intensity i*(t)* of the electric current of a device as a function of time *t* elapsed since the beginning of the experiment is given by:

$$i(t) = 6 \sin\left(\frac{\pi t}{12} + \frac{2\pi}{3}\right) + 6$$ where *t* is expressed in seconds.

The device emits a sound signal each time the current's intensity is equal to 9.

The experiment lasts 120 seconds.

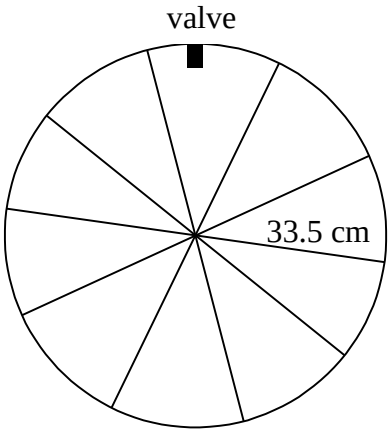
How many sound signals does the device emit during the experiment?

Show all your work.

Show all your work.

Answer Within 120 seconds, the device emitted _____ sound signals.

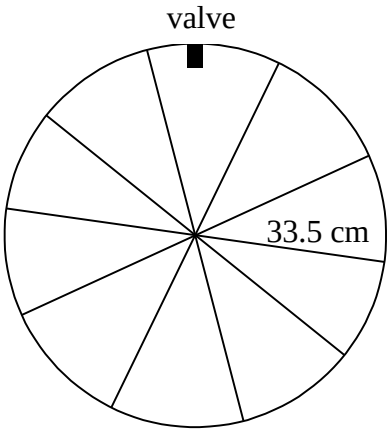
18 Emma is going for a ride on her unicycle. The radius of the wheel is 33.5 cm. When she gets on, the valve of the wheel is at its maximum height. She pedals along a path at a speed of 20 km/h.



At what height will the valve be located after she has cycled for 3 minutes?

Show all your work.

Show your work.



Answer After 3 minutes, the valve will be situated at a height of _____ cm.

19 The average temperature of a fictitious town is given by the equation

$$f(x) = 20 \sin\left(\frac{\pi}{13}x\right) + 2$$

where x represents the number of weeks since the beginning of the year.

During the course of the year, for how many weeks was the average temperature higher than 15 °C?

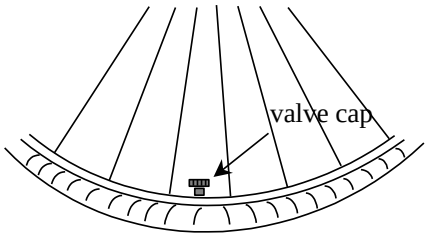
Show all your work.

Show your work.

Answer During the course of the year, the average temperature is higher than 15 °C for _____ weeks.

20 Sara is riding her bicycle. The front wheel makes a complete turn in 2 seconds. The diameter of the front wheel, including the tire, is 46 cm. The thickness of the tire is 4 cm. Moreover, the valve cap is 4 cm away from the tire rim.

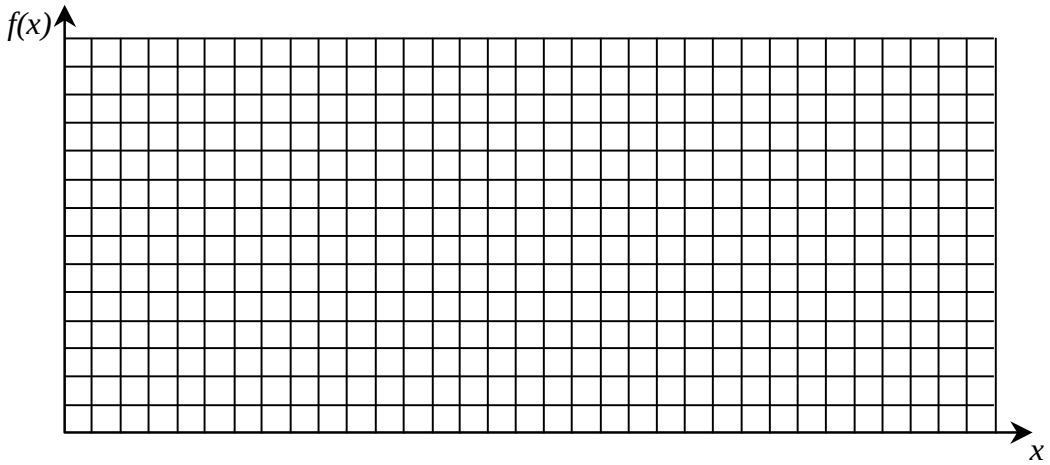
When the wheel starts moving, the cap is at its shortest distance from the ground.



At what times, during the first 4 seconds of the ride, will the valve cap be at 33 cm from the ground?

Show all your work.

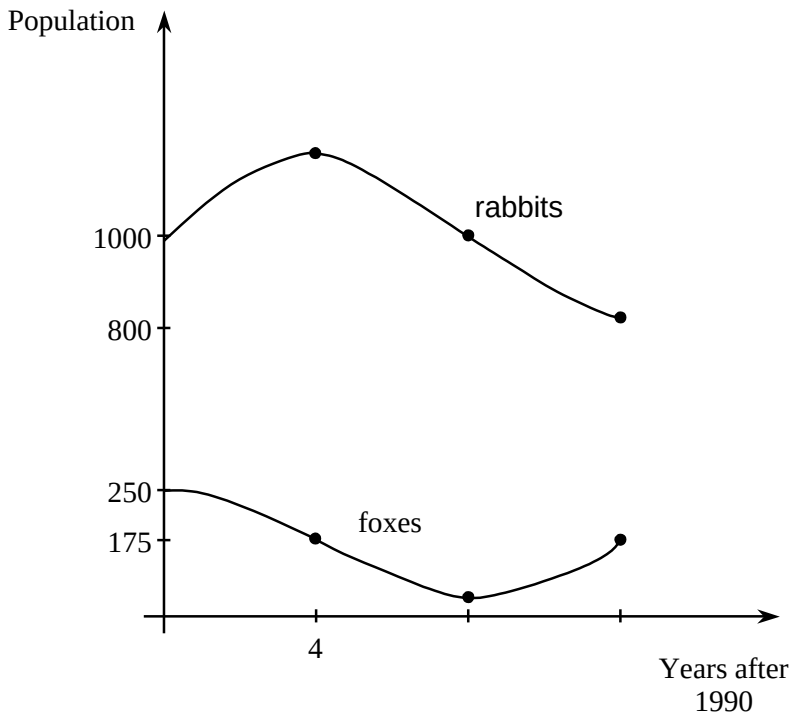
Show all your work.



Answer: The times which the valve cap will be 33 cm above the ground are:

_____ .

21 The following diagram (not drawn to scale) represents a predator-prey situation of the population of rabbits and foxes in a region of Mt.Tremblant Park, both of which follow a sinusoidal model.



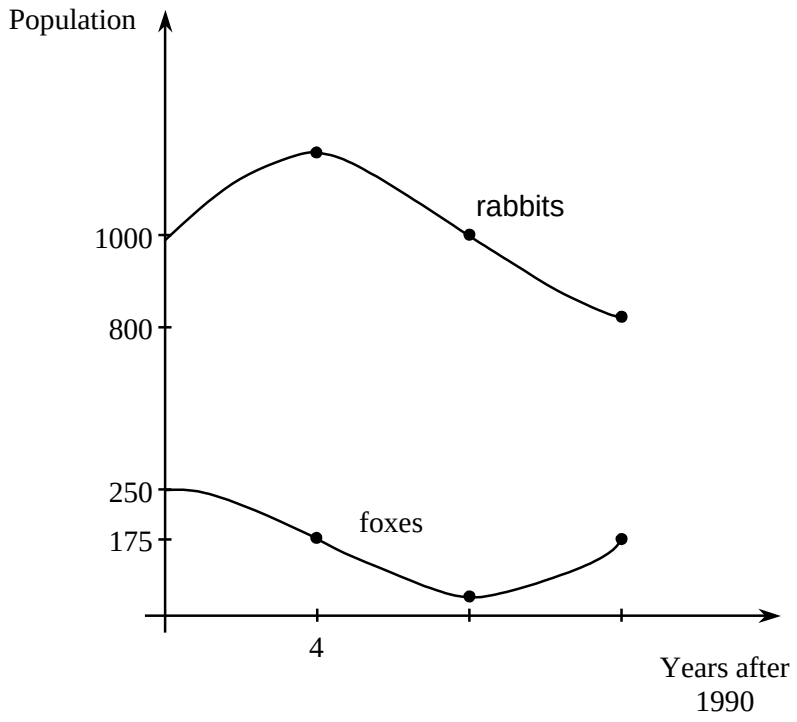
Consider $t = 0$ to be 1990. Initially there are 1000 rabbits and 250 foxes. The graph shows the rabbit population increasing for 4 years then decreasing to its minimum population of 800 in 2002.

Over the same period of time the fox population starting from its maximum decreases, reaches its minimum, then increases to a population of 175 in 2002.

If the model continues as shown, what is the difference between the rabbit and fox populations in 2004? (Answer to the nearest integer.)

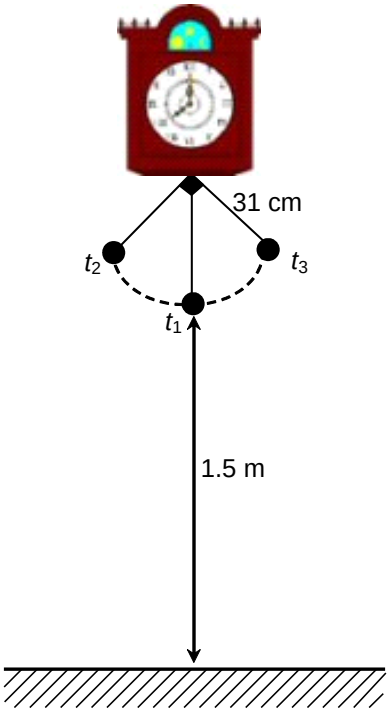
Show all your work.

Show all your work.



Answer: The difference between the rabbit and the fox populations is _____.

22 A cuckoo clock uses a pendulum to keep time. The movement of the pendulum can be described by a sinusoidal function.



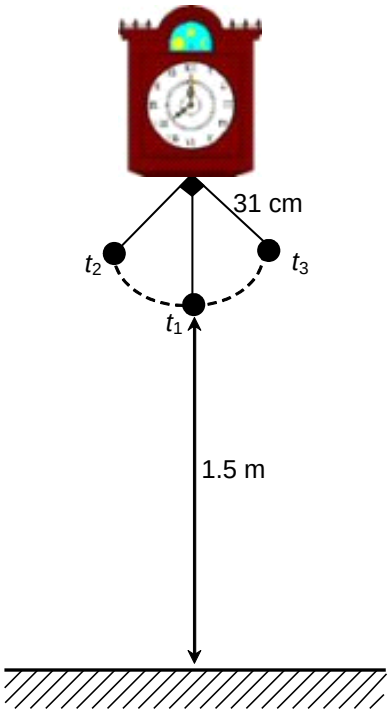
The length of the pendulum is 31 cm. At its lowest point, the pendulum is 1.5 m from the ground.

The pendulum starts its movement at t_1. The interior angle between t_2 and t_3 is 90° and it takes the pendulum 0.875 second to go from t_2 to t_3.

What is the height of the pendulum relative to the ground after 1 hour? Round your answer to the nearest centimetre.

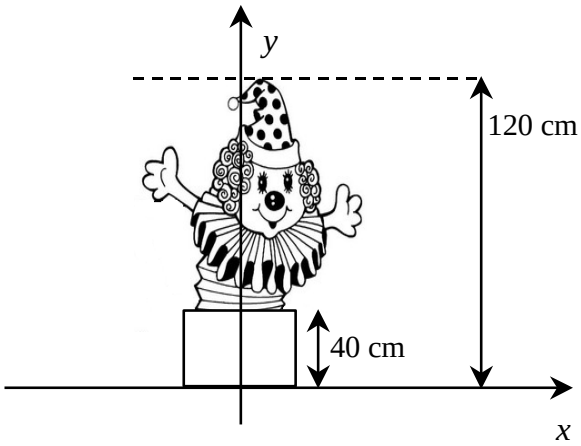
Show all your work.

Show all your work.



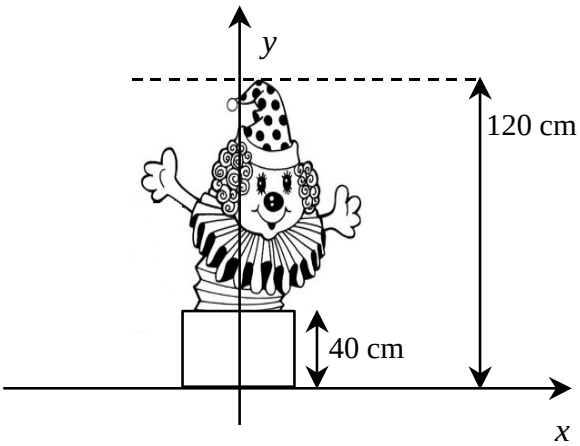
Answer: To the nearest cm, the pendulum will be _____ cm above the ground after 1 hour.

23 The diagram below depicts the head of a Jack-in-a-box used in the display window of a department store. The head is connected to a motor, and its up-and-down movement follows a sinusoidal curve. The head is compressed to 40 cm at $t = 0$ and it reaches a maximum height of 120 cm. It bounces with a frequency of 10 cycles per minute.



At what height is the head, 5 seconds after it is released?

Show all your work.



Answer: Five seconds after it is released, the head is at the height of _____ cm.

2- Correction key

1 f(t) = A sin 880 πt or f(t) = A sin 2764.6t

2 The rule of this function is

$$f(x) = -300 \sin \frac{\pi}{4} x + 500$$

or

$$f(x) = -300 \sin \frac{\pi}{4}(x - 4) + 500$$

or

$$f(x) = -300 \cos \frac{\pi}{4}(x - 2) + 500$$

3 The rule is f(t) = 15 sin 40πt or f(t) = 15 sin 125.66t.

4 a) 250 b) $\frac{1}{250}$ = 0.004 c) $\frac{1}{5\pi}$ = 0.06366 d) 160

5

a) 230 b) $\frac{1}{15}$ = 0.0667 c) 15 d) $\frac{2}{3\pi}$ = 0.2122

6

The rule is f(t) = 5 sin 194 000πt or f(t) = 5 sin 609 469t.

Note : Since f = 97 000 then $P = \frac{1}{f} = \frac{1}{97\ 000}$

and $B = \frac{2\pi}{P} = \frac{2\pi}{\frac{1}{97\ 000}} = 194\ 000\pi$ or 609 469

A = 5

Therefore, the rule is f(t) = 5 sin 194 000πt or f(t) = 5 sin 609 469t.

7

a) 1) amplitude = 3

2) period = 8

3) frequency = $\frac{1}{8}$

4) phase shift = 5 or phase shift = 1

5) $f(x) = 3\sin \frac{\pi}{4}(x - 5)$ or $f(x) = -3\sin \frac{\pi}{4}(x - 1)$

b) 1) amplitude = 2

2) period = $\frac{1}{2}$

3) frequency = 2

4) phase shift = 0

5) $f(x) = 2 \sin 4\pi x$

8

The rule representing this function is $f(x) = 2 \sin x + 1$

or any equivalent rule.

9 The rule of the sinusoidal function is $f(t) = 3 \sin \frac{\pi t}{10} + 4$.

10 The rule that corresponds to function n is

$$n(t) = 30\,000 \sin \frac{\pi}{2} t + 50\,000$$

11 a) The period of this function is π.

b) The amplitude of this function is 0.5.

c) The phase shift of this function is $\frac{\pi}{4}$

d) The y-intercept of this function is 3.5.

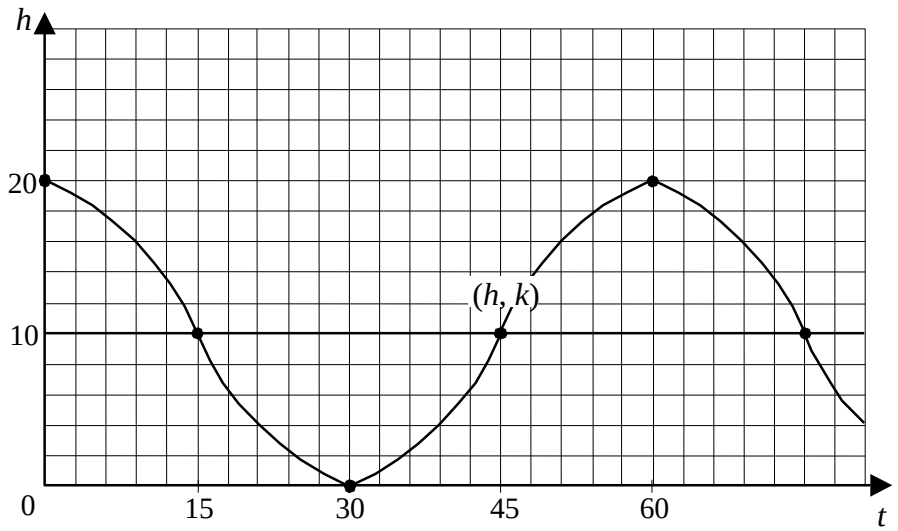
12 These times are 1/6 of a minute (or 10 sec) and 5/6 of a minute (or 50 sec).

13 C

14 The rule of correspondence for this sinusoidal function is : $f(x) = 7 \sin 2\pi x$

15 Example of an appropriate method

Graphic representation of the situation



The function rule is

$h(t) = A \sin B(t - h) + k$ or $h(t) = A \cos B(t - h) + k$

Value of B

The period is 60 seconds.

$$\frac{2\pi}{B} = 60 \quad \text{and} \quad B = \frac{\pi}{30}$$

Value of (h, k)

For a sine graph, there is a horizontal shift of 45 to right, $h = 45$

axis of symmetry is at 10 $k = 10$

Answer The rule describing the height (h) of the needle at time (t) elapsed since noon is:

$$h(t) = 10 \sin \frac{\pi}{30}(t - 45) + 10$$

or $$h(t) = 10 \cos \frac{\pi}{30}(t) + 10.$$

16 C

17 Example of an appropriate method

Solving the equation for *i(t)* = 9

$$6\sin\left(\frac{\pi t}{12}+\frac{2\pi}{3}\right)+6=9$$

$$6\sin\left(\frac{\pi t}{12}+\frac{2\pi}{3}\right)=3$$

$$\sin\left(\frac{\pi t}{12}+\frac{2\pi}{3}\right)=\frac{1}{2}$$

$$\text{arc}\sin\left(\sin\left(\frac{\pi t}{12}+\frac{2\pi}{3}\right)\right)=\text{arc}\sin\frac{1}{2}$$

$$\frac{\pi t}{12}+\frac{2\pi}{3}=\frac{\pi}{6}$$

$$\frac{\pi t}{12}=\frac{\pi}{6}-\frac{4\pi}{6}$$

$$\frac{\pi t}{12}=\frac{-3\pi}{6}$$

t = -6

or

$$\frac{\pi t}{12}+\frac{2\pi}{3}=\frac{5\pi}{6}$$

$$\frac{\pi t}{12}=\frac{5\pi}{6}-\frac{4\pi}{6}$$

$$\frac{\pi t}{12}=\frac{\pi}{6}$$

t = 2

Period of the function

$$\frac{2\pi}{b}=\frac{2\pi}{\frac{\pi}{12}}=24$$

(24 seconds elapse between each sound signal)

Number of seconds that have elapsed since the start of the experiment

2	26	50	74	98	~~122~~
~~-6~~	18	42	66	90	114

~~ Answers not within the given interval

Answer: During the 120 seconds, the apparatus emitted 10 sound signals

18 Example of an acceptable solution

Circumference of the wheel

$$C = 2\pi r$$

$$= 2\pi(33.5)$$

$$= 67\pi \approx 210.49$$

Speed in cm/sec

$$20 \text{ km/h} = 2\,000\,000 \text{ cm}/3600 \text{ s} = \frac{5000}{9} \text{ cm/s} \approx 555.\bar{5} \text{ cm/s}$$

Period

$$p = \frac{67\pi}{\frac{5000}{9}} = \frac{603\pi}{5000}$$

Equation that represents the height of the valve as a function of time in seconds

$$h(t) = a\cos(bt) + k$$

$$a = 33.5 \quad k = 33.5 \quad b = \frac{2\pi}{603\pi/5000} = \frac{10\,000}{603}$$

$$h(t) = 33.5\cos\left(\frac{10\,000}{603}t\right) + 33.5$$

after 3 minutes $t = 180$

$$h(180) = 33.5\cos\left(\frac{10\,000}{603} \bullet 180\right) + 33.5 = 61.85$$

Answer: After 3 minutes, the valve will be situated at a height of 61.85 cm.

19 Example of an acceptable solution

Temperature of 15 °C

$$20 \sin\left(\frac{\pi}{13}x\right) + 2 = 15$$

$$\Leftrightarrow 20 \sin\left(\frac{\pi}{13}x\right) = 13$$

$$\Leftrightarrow \sin\left(\frac{\pi}{13}x\right) = \frac{13}{20}$$

$$\Leftrightarrow \sin\left(\frac{\pi}{13}x\right) = 0.65$$

Period = $p = \frac{2\pi}{|b|} \Leftrightarrow p = \frac{2\pi}{\left|\frac{\pi}{13}\right|} \Leftrightarrow p = 26$

$\sin\theta = 0.65 \Leftrightarrow \theta_1 \approx 0.71$ and $\theta_2 \approx 2.43$

$\frac{\pi x}{13} = 0.71 \Leftrightarrow x_1 \approx 2.93$

$\frac{\pi x}{13} = 2.43 \Leftrightarrow x_2 \approx 10.07$

$x_1 \approx 2.93$

$x_2 \approx 10.07$

Given that the period is 26 weeks

$x_3 = 2.93 + 26 = 28.93$

$x_4 = 10.07 + 26 = 36.07$

Intervals in which the temperature is greater than 15 °C

$x_2 - x_1 =$ 7.14

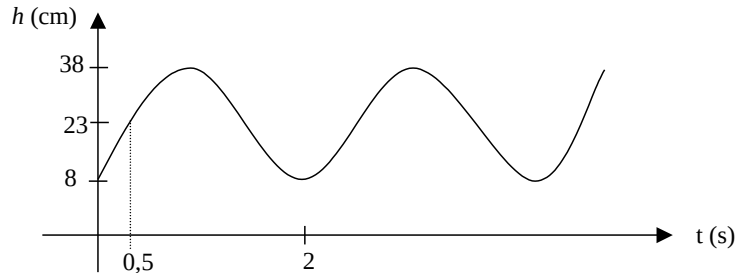
$x_4 - x_3 =$ 7.14

Answer: During the course of the year, the average temperature is higher than 15 °C for 14.28 weeks.

20 Example of an appropriate solution

Equation

$f(x) = a \sin b(x - h) + k$



Parameter *b*

The period is 2 seconds.

$$2 = \frac{2\pi}{|b|}$$

$$|b| = \pi$$

Parameter *a*

The minimum height of the valve is 8 cm and the maximum height is 38 cm.

The amplitude *a* is

$$a = \frac{30}{2}$$

$$= 15$$

Parameter *k*

Minimum value + value of *a*

8 + 15 = 23

Parameter *h*

The phase shift is one-quarter of a period.

Thus $h = \frac{2}{4} = 0.5$.

Equation

$f(x) = 15 \sin \pi(x - 0.5) + 23$ (accept any equivalent form)

By replacing $f(x)$ by 33, we obtain

$$\begin{aligned}
33 &= 15 \sin \pi(x - 0.5) + 23 \\
10 &= 15 \sin \pi(x - 0.5) \\
\frac{10}{15} &= \sin \pi (x - 0.5) \\
\pi (x_1 - 0.5) &= \sin^{-1}\left(\frac{10}{15}\right) \\
\pi (x_1 - 0.5) &= 0.7297 \text{ rad} \\
x_1 &= 0.73 \text{ s} \\
\pi (x_2 - 0{,}5) &= \pi - 0.7297 \text{ rad} \\
&= 2.41 \\
x_2 &= 1.26 \text{ s}
\end{aligned}$$

The first 4 seconds: By adding the period to x_1 and x_2, we obtain

0.73 s; 1.26 s; 2.73 s; 3.26 s.

Answer: The times when the cap of the valve is at 33 cm from the ground are:

0.73 s; 1.26 s; 2.73 s; 3.26 s.

21 Example of an appropriate solution

Rabbit population is of the form

$$R(x) = a \sin b(x - h) + k$$

For simplicity

$h = 0$

$k = 1000$ (vertical shift)

$a = 200$

$\frac{3}{4}$ of the period is given by the 12 years

$$\therefore \frac{3}{4} \bullet \frac{2\pi}{b} = 12 \Rightarrow b = \frac{\pi}{8} \text{ and } R(x) = 200 \sin \frac{\pi}{8}x + 1000$$

Likewise, $f(x)$ is of the form

$$F(x) = a \cos b(x - h) + k$$

$h = 0$, $a = 75$, $k = 175$, $b = \frac{\pi}{8}$

$$F(x) = 75 \cos \frac{\pi}{8}x + 175$$

In 2004

$x = 14$ and $R(14) - F(14) \approx 630.545$

Answer: The difference between the rabbit and the fox populations is **631**.

Accept 630 or 631.

Note: Students who have determined the rule for one of the populations have shown they have a partial understanding of the problem.

22 Example of an appropriate solution

The minimum height of the pendulum is 150 cm.

The maximum height of the pendulum is 150 + (31 – 31 sin 45°) = 159.08 cm.

$$y = a \cos b(x - h) + k$$

$$a = \frac{159.08 - 150}{2}$$
$$= 4.54$$

$$b = \frac{2\pi}{0.875}$$
$$= \frac{16\pi}{7}$$

$$k = \frac{159.08 + 150}{2}$$
$$= 154.54$$

$$h = 0 \text{ if } a \text{ is negative}$$

$$f(t) = -4.54 \cos\left(\frac{16\pi t}{7}\right) + 154.54$$

$$1 \text{ hour} = 3600 \text{ seconds}$$
$$f(3600) = 156 \text{ cm}$$

Answer: To the nearest cm, the pendulum will be **156** cm above the ground after 1 hour.

Note: Students who use an appropriate method in order to determine parameters *a*, *b* and *k* have shown they have a partial understanding of the problem.

Students' answers may vary considerably, depending on the rounding they applied in the course of their work.

23 Example of an appropriate solution

Rule of distance of head to the ground

$$\text{Frequency} = \frac{10 \text{ cycles}}{60 \text{ seconds}}$$
$$\text{Period} = 6 \text{ seconds/cy cle}$$
$$\text{Amplitude} = 40$$

$$P = \frac{2\pi}{|b|} \qquad |b| = \frac{2\pi}{6}$$
$$= \frac{\pi}{3}$$

$$y = a \cos b\left(x - h\right) + k$$

$$k = \frac{\left|\max + \min\right|}{2}$$

$$= \frac{120 + 40}{2}$$

$$= 80$$

$$y = -40 \cos \frac{\pi}{3} t + 80$$

Distance to ground at $t = 5$

$$y = -40 \cos \frac{\pi}{3}\left(5\right) + 80$$

$$y = 60 \text{ cm}$$

Answer: Five seconds after it is released, the head is at the height of **60 cm**.

Note: To account for rounding at different places, accept answers in the range of 59 to 61.

Students who use an equivalent rule to arrive at the same answer should not be penalized.

Students who use an appropriate method to determine any 2 parameters have shown they have a partial understanding of the problem.