

Chapter 10: Refraction

What is refraction?

Refraction: The phenomenon that occurs at the boundary of two translucent substances. Because light travels at different speeds in different media, it will slightly change direction ("bend") when it goes from one medium to another.

Medium: Substance material through which light travels.

Index of refraction (n): Factor by which the speed of light is decreased in a given medium, compared to the speed of light in a vacuum.

Note: $n = \frac{c}{v}$

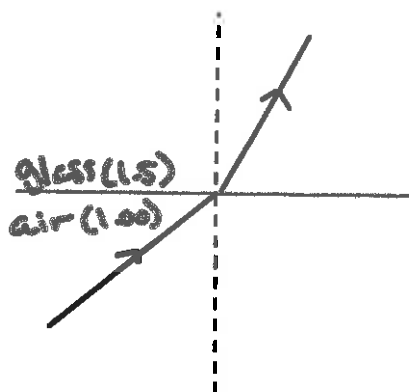
where n : index of refraction of medium
 c : speed of light in a vacuum ($3.0 \times 10^8 \text{ m/s}$)
 v : speed of light in medium

$n \geq 1.00$

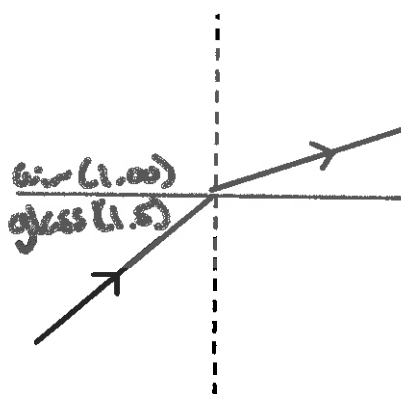
Some indexes of refraction:

Vacuum:	1.00 (Exactly!)
Air:	1.00029 (Basically 1.00)
Crystal:	2.00
Glass:	1.50
Water:	1.33
Ice:	1.31

When light enters a medium that is more optically dense (higher value of n), the ray bends **TOWARDS** the normal

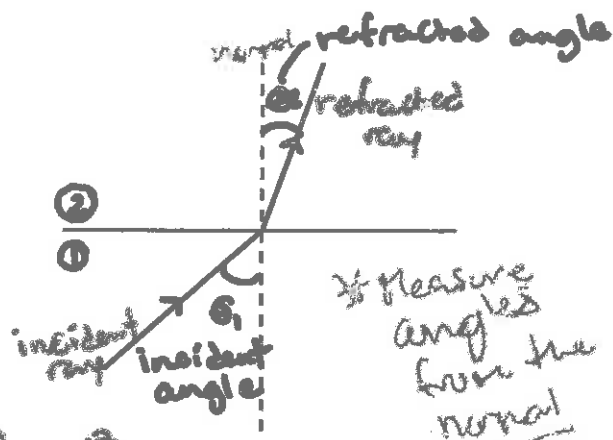


When light enters a medium that is less optically dense (lower value of n), the ray bends **AWAY** from the normal



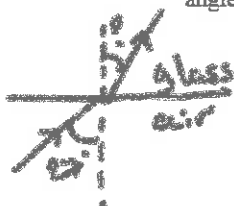
Law of Refraction (Snell's Law)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



Examples:

1. A light ray traveling through air enters glass ($n = 1.50$) at an angle of 27° . What is the angle at which the light ray travels through the glass?



$$\begin{aligned} n_1 &= 1.00 \\ \theta_1 &= 27^\circ \\ n_2 &= 1.50 \\ \theta_2 &= ? \end{aligned}$$

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ \sin \theta_2 &= \frac{n_1 \sin \theta_1}{n_2} \\ &= \frac{(1.00)(\sin 27^\circ)}{1.5} \end{aligned}$$

$$\begin{aligned} \sin \theta_2 &= 0.3027 \\ \theta_2 &= \sin^{-1}(0.3027) \\ &= \underline{\underline{18^\circ}} \end{aligned}$$

2. A light ray travels through air at an angle of 29° enters an unidentified substance where it then travels at an angle of 25° . What is the index of refraction of the unidentified substance?



$$\begin{aligned} n_1 &= 1.00 \\ \theta_1 &= 29^\circ \\ n_2 &= ? \\ \theta_2 &= 25^\circ \end{aligned}$$

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ n_2 &= \frac{n_1 \sin \theta_1}{\sin \theta_2} \\ &= \frac{(1.00)(\sin 29^\circ)}{\sin 25^\circ} \end{aligned}$$

$$\underline{\underline{n_2 = 1.15}}$$

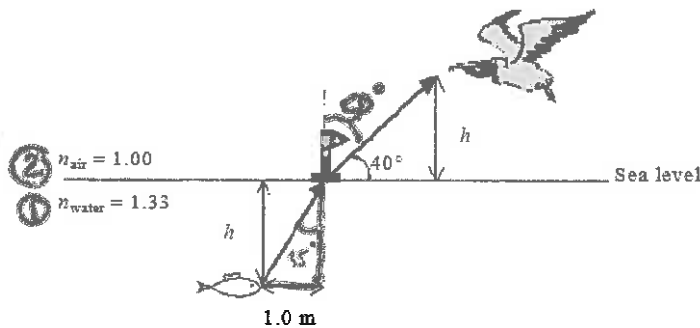
3. Since the refractive index of water is 1.33, what is the speed of light in water?

$$n = \frac{c}{v} \Rightarrow v = \frac{c}{n}$$

$$= \frac{3.0 \times 10^8 \text{ m/s}}{1.33}$$

$$v = 2.26 \times 10^8 \text{ m/s}$$

4. A pelican is flying above sea level in search of its next snack. It sees a fish at an angle of 40° to the horizontal, as shown below. The fish is located 1.0 m away, horizontally, from the buoy. The height of the pelican above the water is equal to the depth of the fish.



What is the height of the pelican above sea level?

$$n_1 = 1.33 \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\theta_1 = ? \quad \sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1}$$

$$n_2 = 1.00 \quad = \frac{(1.00) \sin 50^\circ}{1.33}$$

$$\theta_2 = 50^\circ$$

$$\sin \theta_1 = 0.576$$

$$\theta_1 = \sin^{-1}(0.576)$$

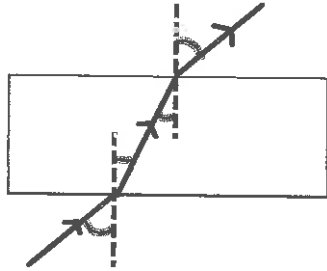
$$= 35^\circ$$

A right-angled triangle is shown with a horizontal base of 1.0 m and a vertical height of h . The angle at the base is 35° .

$$\tan 35^\circ = \frac{1.0 \text{ m}}{h}$$

$$h = \frac{1.0 \text{ m}}{\tan 35^\circ} = 1.4 \text{ m}$$

Light Through a Medium with Parallel Sides



The ray that goes in towards the glass is parallel to the ray that exits the glass.
(Shifted)

Critical Angle and Total Internal Reflection

- Critical angle (θ_c):
- Maximum angle for which refraction will occur
 - Minimum angle for which total internal reflection will occur
 - Is an incident angle
 - Depends on both medium involved
 - Only happens when light goes from a medium with higher index of refraction to one with a lower index of refraction.

Ex:

refraction

refraction

When the incident ray hits the surface at an angle greater or equal to the critical angle, all the light is reflected. When this occurs inside a closed medium, this is called total internal reflection.

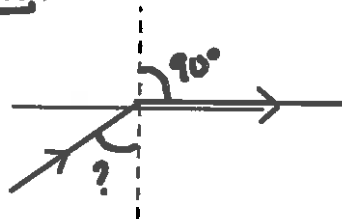


*fiber optics
cable*

How to find the critical angle

→ when the angle of refraction is 90° .

→ θ *incidence*



Examples:

1. A light ray inside a glass block ($n = 1.50$) hits the surface as if it was to enter in air. For what angle of incidence will total internal reflection occur?



$$\begin{aligned} n_1 &= 1.50 \\ \theta_1 &= ? \\ n_2 &= 1.00 \\ \theta_2 &= 90^\circ \end{aligned}$$

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ \sin \theta_1 &= \frac{n_2 \sin \theta_2}{n_1} \end{aligned}$$

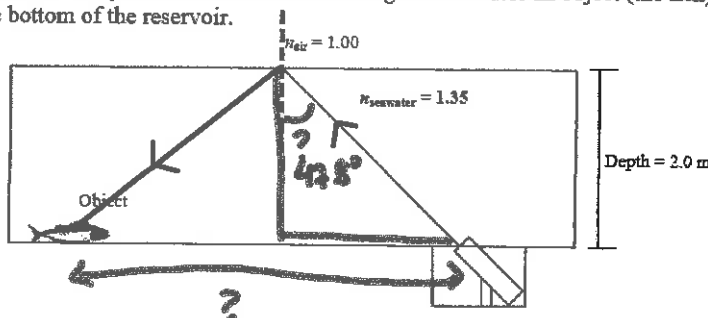
$$= \frac{(1.00) \sin 90^\circ}{1.5}$$

$$\sin \theta_1 = 0.6667$$

$$\theta_1 = 41.8^\circ \leftarrow \text{critical angle}$$

Ans: for angles $> 41.8^\circ$

2. An adjustable laser is attached to the bottom of a reservoir containing seawater ($n = 1.35$). The laser is adjusted so that the reflected light illuminates an object (the fish) placed on the bottom of the reservoir.



What minimum distance must there be between the laser light source and the object so that the ray of light does not exit the surface of the seawater?

Critical angle sea water \rightarrow air

$$\begin{aligned} n_1 &= 1.35 \\ \theta_1 &= ? \\ n_2 &= 1.00 \\ \theta_2 &= 90^\circ \end{aligned}$$

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ \sin \theta_1 &= \frac{n_2 \sin \theta_2}{n_1} \end{aligned}$$

$$= \frac{(1.00) \sin 90^\circ}{1.35}$$

$$= 0.7407$$

$$\theta_1 = 47.8^\circ$$



$$\tan 47.8^\circ = \frac{x}{2.0\text{m}}$$

$$x = 2.0 \tan 47.8^\circ = 2.21\text{m}$$

$$\times 2$$

$$\boxed{= 4.42\text{m}}$$

Lenses

Lenses are made of transparent materials that have an index of refraction greater than the index of refraction of air (usually plastic or glass).

Because light enters and leaves the lens at an angle, it changes direction. ^{refracts!} By changing the shape of the lens, we can control exactly how the light will be deviated.

Some uses of lenses:

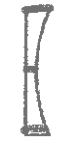
- Eyeglasses
- Microscope
- Telescope
- Overhead projector

Types of lenses

Diverging Lenses Concave



Biconcave



Planoconcave



Diverging meniscus
(negative meniscus)

Center thinner
than edges

Converging Lenses Convex



Biconvex



Planoconvex



Converging meniscus
(positive meniscus)

Center thicker
than edges

Note: When light travels through a lens, it changes direction twice: once when it enters the lens, and once when it leaves the lens.

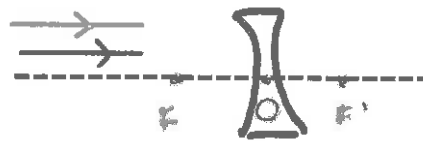
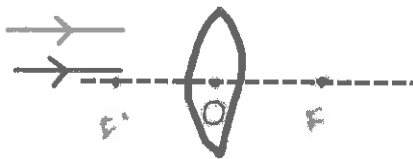
Parts of a lens

Principle Axis: same as for reflection

Optical Center (O): geometrical center of lens

Primary Focus (F): - where the rays converge (converging / convex lens)
- where the rays appear to come from (diverging / concave lens)

Secondary Focus (F'): same distance on other side of lens

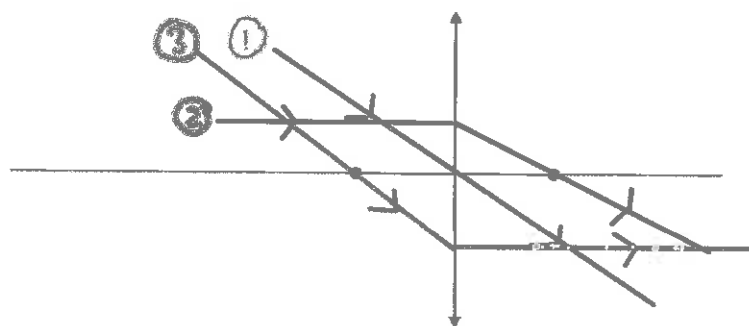


Drawing the principle rays going through a lens

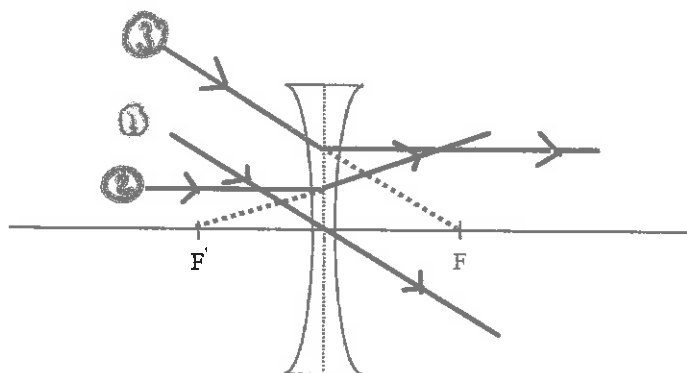
3 principal rays:

- ① Through the optical center. The refracted ray comes out undeviated
- ② Parallel to principal axis:
 - Converging (convex) lens*
The ray is refracted through the focal point
 - Diverging (concave) lens*
The ray is refracted "as if" it came from the focal point
- ③ Through the focus:
Or "as if" from
(*Converging lens*)
The ray is refracted parallel to the principal axis
- * As if FO focus on other side: The ray is refracted parallel to the principal axis
(*Diverging lens*)

Convex (converging lens)



Concave (diverging lens)



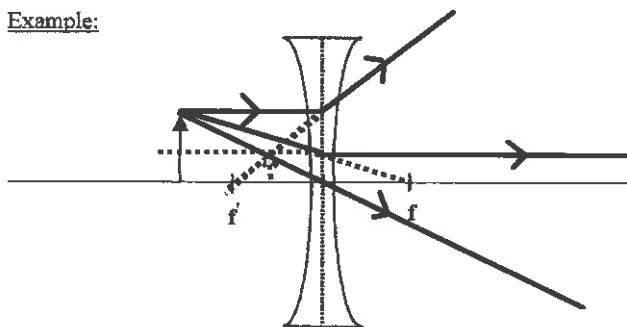
Images Formed by Lenses

Images formed by DIVERGING lenses (concave) = same as a diverging (concave) mirror

Always

- Virtual
- Upright
- Smaller than image
- Located between F and O

Example:



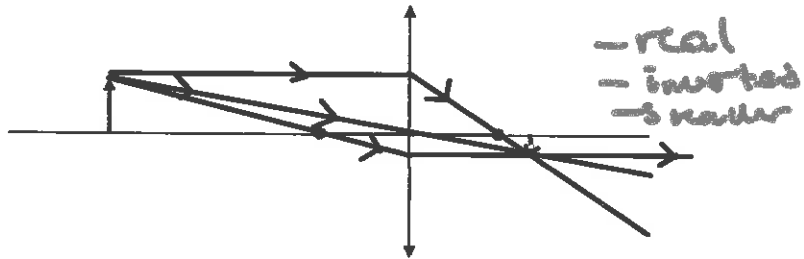
- Virtual
- Smaller
- Upright

Images formed by CONVERGING lenses (convex) = same as converging (convex) mirror

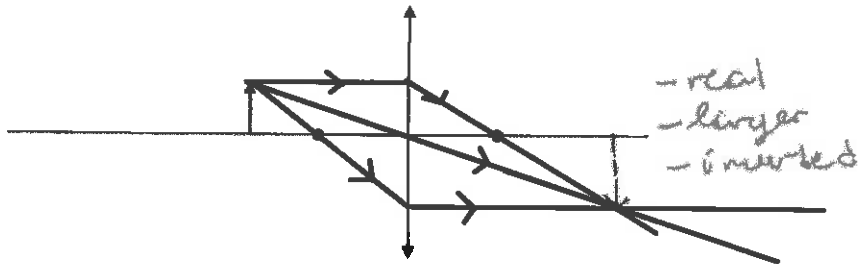
Object \ Image	Real or Virtual?	Upright or Inverted?	Smaller or bigger than object?	Where?
Far beyond 2F	Real	Inverted	Smaller	At F
Beyond 2F	Real	Inverted	Smaller	Between F and 2F
At 2F	Real	Inverted	Same size	At 2F
Between 2F and F	Real	Inverted	Larger	Beyond 2F
At F	NO IMAGE			
Between F and O	Virtual	Upright	Larger	Same side of lens as object

Example:

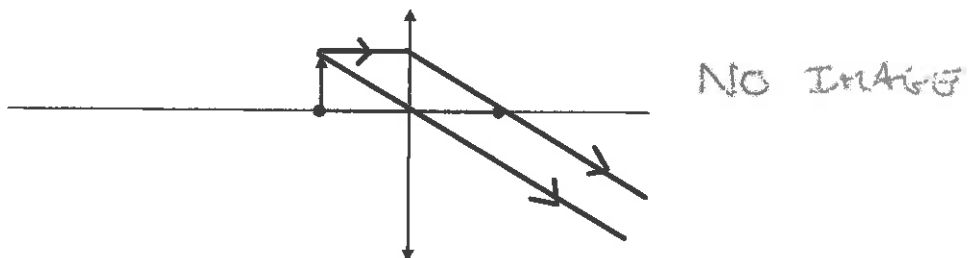
1.



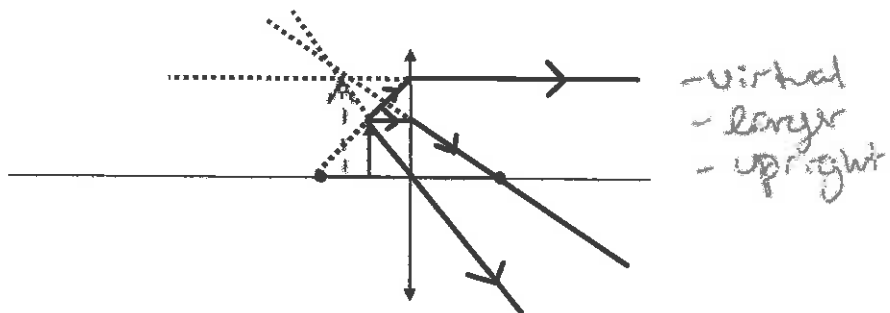
2.



3.



4.



Locating images using formulas (lenses)

Just as for mirrors, for lenses we have:

$$M \quad h_o \quad h_i \quad d_o \quad d_i$$

The meaning of positive and negative signs is the same as for mirrors. \oplus virtual

f : focal length of the lens (distance between focal point and vertex)

Note: If f is positive, the lens is converging (convex).

If f is negative, the lens is diverging (concave).

* Distances are always measured from the optical center of the lens.

$$M = -\frac{d_i}{d_o}$$

$$M = \frac{h_i}{h_o}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Examples:

1. A rabbit 0.10 m in height is located 1.5 m from a converging lens with a focal length of 0.50 m. What is the height of the rabbit's image?

$f = 0.50 \text{ m}$ ① Find d_i

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{0.5} = \frac{1}{1.5} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{0.5} - \frac{1}{1.5}$$

$$\frac{1}{d_i} = \frac{2}{1.5} - \frac{1}{1.5}$$

$$\frac{1}{d_i} = \frac{1}{1.5}$$

$$d_i = 1.5 \text{ m}$$

② Find h_i

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{d_i h_o}{d_o}$$

$$h_i = -\frac{(1.5 \text{ m})(0.10 \text{ m})}{1.5 \text{ m}}$$

$$h_i = -0.10 \text{ m}$$

2. A manufacturer of slide projectors wants to produce images 1.5 m in height on a screen that is placed 5.0 m from the lens of the projector. If the height of the slides is 30 mm, calculate the focal length of the lens.

$d_o = 5.0 \text{ m}$ ① Find d_i

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$d_i = -\frac{d_o h_i}{h_o}$$

$$d_i = -\frac{(5.0 \text{ m})(-1.5 \text{ m})}{0.03 \text{ m}}$$

$$d_i = 250 \text{ m}$$

② Find f

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{f} = \frac{1}{5.0 \text{ m}} + \frac{1}{250 \text{ m}}$$

$$\frac{1}{f} = \frac{50}{250 \text{ m}} + \frac{1}{250 \text{ m}}$$

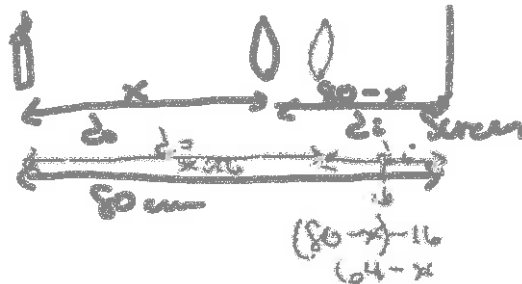
$$\frac{1}{f} = \frac{51}{250 \text{ m}}$$

$$f = \frac{250 \text{ m}}{51}$$

$$f = 4.9 \text{ m}$$

→ The lens is then moved 16cm closer to the screen, and an image is still formed on screen.

3. An object and a screen are fixed at a distance of 80 cm apart. A convex lens forms an image of the object on the screen. Find the focal length of the lens.



$$\textcircled{1} \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{80-x}$$

$$\textcircled{2} \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{x+16} + \frac{1}{64-x}$$

$$\textcircled{1} \frac{1}{x} + \frac{1}{80-x} = \frac{1}{x+16} + \frac{1}{64-x}$$

$$\frac{80-x+x}{x(80-x)} = \frac{64-x+x+16}{(x+16)(64-x)}$$

$$\frac{80}{80x-x^2} = \frac{80}{64x-x^2+1024-16x}$$

$$80(48x-x^2+1024) = 80(80x-x^2)$$

$$48x \times 1024 = 80x$$

$$1024 = 32x$$

$$x = 32 \text{ cm}$$

③ Find f

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{80-x}$$

$$\frac{1}{f} = \frac{1}{32 \text{ cm}} + \frac{1}{80-32}$$

$$\frac{1}{f} = \frac{1}{32 \text{ cm}} + \frac{1}{48 \text{ cm}}$$

$$\frac{1}{f} = \frac{48}{1536 \text{ cm}} + \frac{32}{1536 \text{ cm}}$$

$$\frac{1}{f} = \frac{80}{1536 \text{ cm}}$$

$$f = 1536 \text{ cm}$$

$$80$$

$$f = 19.2 \text{ cm}$$