Rationalizing Numerators

In application, we may also need to rationalize the numerator of an expression. The process is similar to rationalizing denominators as described on p.939 in the text book.

Example Rationalize the numerators in the following expressions:

$$1. \frac{\sqrt{x} - 4}{x - 16} = \frac{(\sqrt{x} - 4)(\sqrt{x} + 4)}{(x - 16)(\sqrt{x} + 4)} = \frac{(x - 16)}{(x - 16)(\sqrt{x} + 4)} = \frac{1}{(\sqrt{x} + 4)}$$

$$2. \frac{\sqrt{a + x} - \sqrt{a}}{x} = \frac{(\sqrt{a + x} - \sqrt{a})(\sqrt{a + x} + \sqrt{a})}{x(\sqrt{a + x} + \sqrt{a})} = \frac{(a + x - a)}{x(\sqrt{a + x} + \sqrt{a})} = \frac{1}{(\sqrt{a + x} + \sqrt{a})}$$

$$3. \frac{\sqrt{6x} + \sqrt{3}}{3} = \frac{(\sqrt{6x} + \sqrt{3})(\sqrt{6x} - \sqrt{3})}{3(\sqrt{6x} - \sqrt{3})} = \frac{(6x - 3)}{3(\sqrt{6x} - \sqrt{3})} = \frac{2x - 1}{(\sqrt{6x} - \sqrt{3})}$$

$$4. \frac{\sqrt{x} - \sqrt{5}}{5 - x} = \frac{(\sqrt{x} - \sqrt{5})(\sqrt{x} + \sqrt{5})}{(5 - x)(\sqrt{x} + \sqrt{5})} = \frac{(x - 5)}{(5 - x)(\sqrt{x} + \sqrt{5})} = \frac{-1}{(\sqrt{x} + \sqrt{5})}$$