## Mathematics 5 SN RATIONAL FUNCTIONS

Given $f(x)=3 x+4$ and $g(x)=5 x-7$. The function $h$ is defined by

$$
h(x)=\frac{f(x)}{g(x)} \text { where } g(x) \square 0 .
$$

What is the domain and range of function $h$ ?
A) $\operatorname{Dom} h=\mathfrak{R} \backslash\left\{\frac{3}{5}\right\}$
C) $\operatorname{Dom} h=\mathfrak{R} \backslash\left\{\frac{7}{5}\right\}$

$$
\text { Ima } h=\mathfrak{R} \backslash\left\{\frac{-7}{5}\right\}
$$

$$
\text { Ima } h=\mathfrak{R} \backslash\left\{\frac{-3}{5}\right\}
$$

B) $\operatorname{Dom} h=\mathfrak{R} \backslash\left\{\frac{3}{5}\right\}$
D) $\operatorname{Dom} h=\mathfrak{R} \backslash\left\{\frac{7}{5}\right\}$
Ima $h=\mathfrak{R} \backslash\left\{\frac{7}{5}\right\}$
Ima $h=\mathfrak{R} \backslash\left\{\frac{-3}{5}\right\}$

The function $g$ is defined by the following rule:

$$
g(x)=\frac{x+2}{4 x+20}
$$

What is the rule of its inverse $g^{-1}$ ?
A) $\quad g^{-1}(x)=\frac{-20 x+2}{4 x-1}$
B) $\quad g^{-1}(x)=\frac{4 x+20}{x+2}$
C) $\quad g^{-1}(x)=\frac{20 x-2}{-4 x}$
D) $\quad g^{-1}(x)=\frac{-18}{4 x}$

Max's New Year's resolution was to lose weight by paying strict attention to the food he ate. Since then, his weight has varied according to the function whose rule is:

$$
M(t)=\frac{500}{t+50}+80
$$

where $t$ represents the number of days gone by since January $1^{\text {st }}$, and $m(t)$ represents Max's weight in kilograms.

According to the rule of this function, what minimum weight, rounded to the nearest unit, can Max hope to reach?

Max can hope to reach the minimum weight of $\qquad$ kg.

Given the function $f(x)=\frac{-10}{x+1}+3$.

What is the rule of correspondence of the inverse of this function?

The rule of correspondence of the inverse of this function is $f^{-1}(x)=$ $\qquad$ -.

Given the function $f$ defined by $f(x)=\frac{8 x-2}{3 x}$ and the function $h$ defined by $h(x)=\frac{12 x+4}{2 x+1}$.

Which of the following can be used to obtain the function $g$ defined by $g(x)=\frac{52 x^{2}+16 x-2}{6 x^{2}+3 x}$, given the rules of $f$ and $h$ ?
A) $(f \circ h)(x)=g(x)$
B) $(f+h)(x)=g(x)$
C) $(f \bullet h)(x)=g(x)$
D) $(f-h)(x)=g(x)$

Given the rational function $f(x)=\frac{3 x+5}{2 x-1}$.

What are the equations of the asymptotes of this function?
A) $x=\frac{1}{2} \quad y=\frac{3}{2}$
B) $x=\frac{1}{2} \quad y=-5$
C) $x=\frac{3}{2} \quad y=\frac{1}{2}$
D) $x=-\frac{1}{2} \quad y=\frac{3}{2}$

Given the rational function $g(x)=\frac{3}{4 x+6}-8$.

What are the equations of the asymptotes of $\mathrm{g}(x)$ ?
$x=$ $\qquad$
$y=$ $\qquad$

A mathematical relationship exists between the number of students who participate in the end-of-year school activities and the amount of money students will be charged to participate.

The greater the number of participants, the less each has to pay. This relation is represented by the following function:

$$
\mathrm{N}(p)=\frac{87453}{3 p-300}+1522
$$

where $\quad p$ is the price of the activities;
$N(p)$ is the number of students participating.

At least $70 \%$ of the 1230 students in County High must agree to participate, before the end-of-year activities will be approved.

What is the maximum price the school can charge for the end-of-year activities?

Show all your work.

Answer: The maximum price the school can charge is $\$$ $\qquad$ .

9 Given the function $f(x)=\frac{4 x+3}{2 x-1}$.

What are the coordinates of the point of intersection of its vertical and horizontal asymptotes?
A) $\left(\frac{1}{2}, \frac{-3}{4}\right)$
B) $\left(\frac{1}{2}, 0\right)$
C) $\left(\frac{1}{2}, \frac{3}{4}\right)$
D) $\left(\frac{1}{2}, 2\right)$

10 A function is represented by the rule $f(x)=\frac{-6 x+1}{2 x+3}$.

Which of the following graphs represents $f^{-1}(x)$ ?
A)

C)

B)

D)


Given the function $f(x)=\frac{-4(x-2)}{x+1}$

What are the asymptotes of the inverse $f^{-1}$ ?
A) $x=-1$
$y=2$
C) $\quad x=-1$
$y=-4$
B) $x=-4$
$y=2$
D) $\quad x=-4$
$y=-1$

A function is represented by the equation $f(x)=\frac{x+8}{x-2}$.

What equation can be used to represent $f^{-1}(x)$ ?
$f^{-1}(x)=$ $\qquad$

Given: $f(x)=\frac{x+2}{x+3}$ and $g(x)=2 x+5$.

What are the equations of the vertical and horizontal asymptotes of $(f \circ g)(x)$ ?

The equations of the asymptotes are $x=$ $\qquad$ and $y=$ $\qquad$ .

Given the rational function $f(x)=\frac{-3(x-2)}{2 x-6}$.

What are the domain and range of $f(x)$ ?

The domain of $f(x)$ is $\qquad$ -
$\qquad$ -.

Given the function $f(x)=\frac{a}{b(x-h)}+k, a>0, b<0$.

Which of the following describes the function throughout its domain?
A) Decreasing
C) Decreasing, then increasing
B) Increasing
D) Increasing, then decreasing

Given the rational function $f(x)=\frac{c x+d}{p x+t}$, in which $c>0, d>0, p>0$ and $t>0$.

What is the product of the parameters that define its vertical and horizontal asymptotes?
A) $\frac{c d}{p t}$
B) $\frac{-c t}{p^{2}}$
C) $\frac{d}{p}$
D) $\frac{t d}{p c}$

$$
f(x)=\frac{2 x-1}{4-x}
$$

What is the solution set of the inequality $f(x) \geq 0$ ?

Show all your work.
$\qquad$

Given the function $f(x)=\frac{7}{-3 x-12}-5$.

What is the rule for $f^{-1}(x)$ ?

The rule for $f^{-1}(x)$ is

## 2- Correction key

D

Max can hope to reach the minimum weight of 80 kg .

The inverse of this function is $f^{-1}(x)=\frac{-10}{x-3}-1$.


B


A
$7 \quad x=-\frac{3}{2}$

$$
y=-8
$$

Example of an appropriate solution

$$
\begin{aligned}
\mathrm{N}(p) & =\frac{29151}{p-100}+1522 \\
\mathrm{~N}(p) & =1230\left(\frac{70}{100}\right) \\
& =861 \\
p & =? \\
861 & =\frac{29151}{p-100}+1522 \\
-661 & =\frac{29151}{p-100} \\
-661(p-100) & =29151 \\
-661 p+66100 & =29151 \\
-661 p & =-36949 \\
p & =55.8986 \ldots \\
p & \approx \$ 55.90
\end{aligned}
$$

Answer: $\quad$ The maximum price the school can charge is $\$ \mathbf{5 5 . 9 0}$.

D

C

D

$$
f^{-1}(x)=\frac{8+2 x}{x-1} \quad \text { or } \quad f^{-1}(x)=\frac{10}{x-1}+2
$$

The equations of the asymptotes are $\boldsymbol{x}=\mathbf{- 4}$ and $\boldsymbol{y}=\mathbf{1}$.

The domain of $f(x)$ is $]-\infty, 3[\cup] 3,+\infty[$.
The range of $f(x)$ is $]-\infty,-1.5[\cup]-1.5,+\infty[$.

$$
\frac{2 x-1}{4-x} \geq 0 \quad x=\frac{1}{2} \quad x \neq 4
$$

sub 0 sub 3 sub 5
$\begin{array}{lll}- & \frac{+}{+}=- & \frac{+}{-}=- \\ + & +\end{array}$


Answer: The solution set is $\left[\frac{1}{2}, 4[\right.$.

Alternate solution
$y=\frac{2 x-1}{-x+4}$

Vertical asymptote

$$
x=4
$$

Horizontal asymptote

$$
y=-2
$$

$y$-intercept

$$
-\frac{1}{4}
$$


$x$-intercept

$$
\frac{1}{2}
$$

Solution

$$
\frac{2 x-1}{-x+4} \geq 0
$$

Answer: $\quad$ The solution set is $\left[\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{4}[\right.$

Answer: $f^{-1}(x)=\frac{7}{-3(x+5)}-4$

Accept any other equivalent rule.

