

## Solutions to “More Optimization Problems”

1. Let  $x = \#$ chickens  
 Let  $y = \#$ quails

$$x \geq 0$$

$$y \geq 0$$

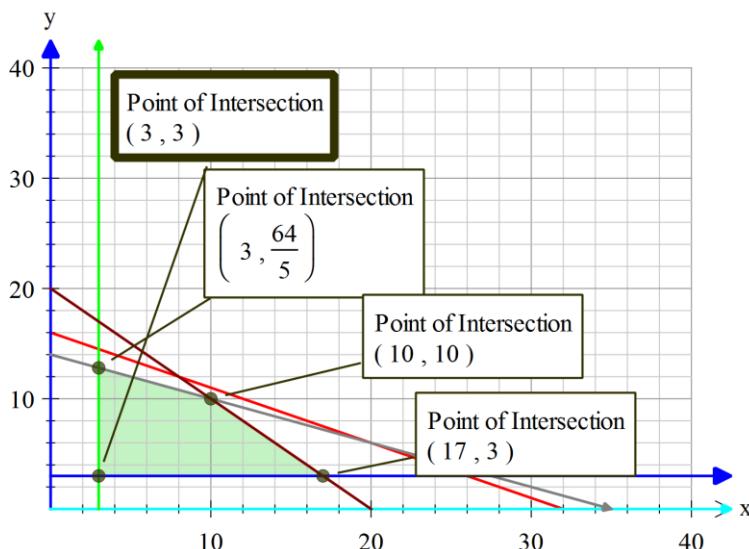
$$2x + 5y \leq 70$$

$$x \geq 3$$

$$y \geq 3$$

$$x + y \leq 20$$

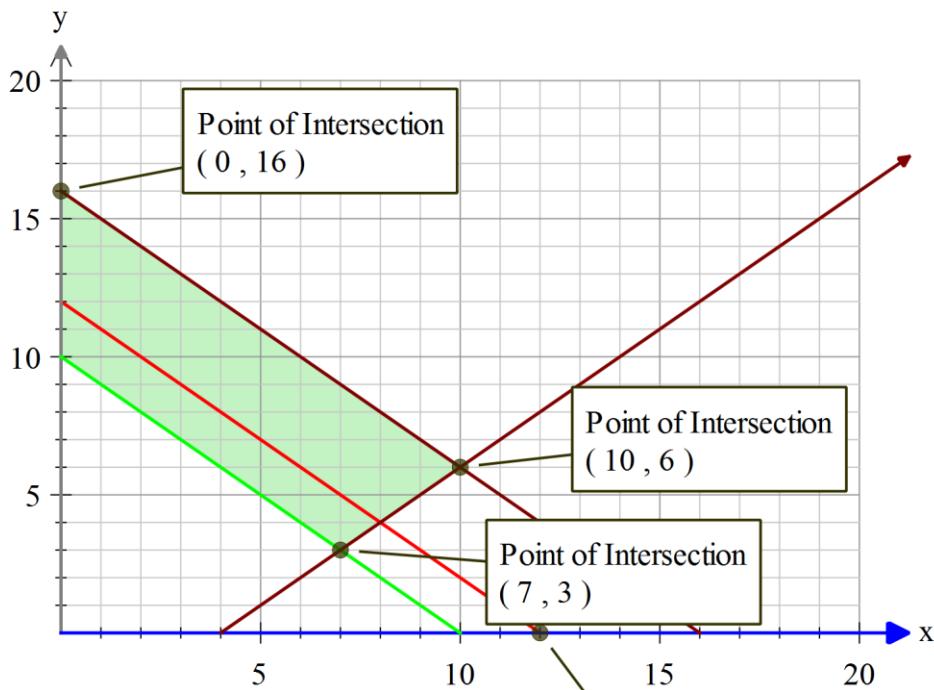
Objective function: Maximize  $P = 2x + 4y$



$(x,y)$	$P = 2x + 4y$
$(3,3)$	18
$(3,12.8)$	57.2
$(10,10)$	60 ← Max
$(17,3)$	46

Solution : 10 chickens and 10 quails will give him a maximum profit of \$60.

2. Let  $x = \#$ females  
 Let  $y = \#$ males  
 $x \geq 0$   
 $y \geq 0$   
 $x + y \geq 10$   
 $x + y \leq 16$       Objective: Minimize  $x + y$   
 $x \leq y + 4$



$(x,y)$	$S = x + y$
(7,3)	10 ← Min
(10,6)	16
(0,16)	16
(0,10)	10 ← Min

Since both (7,3) and (0,10) minimize the objective function, then all of the integral points in between also do so. The possible solutions, therefore, are:

- 7 females and 3 males
- 6 females and 4 males
- 5 females and 5 males
- 4 females and 6 males
- 3 females and 7 males
- 2 females and 8 males
- 1 female and 9 males
- 0 females and 10 males

3. Let  $x$  = # litres of oil  
 Let  $y$  = # litres of gas

$$x \geq 0$$

$$y \geq 0$$

$$x + y \leq 30$$

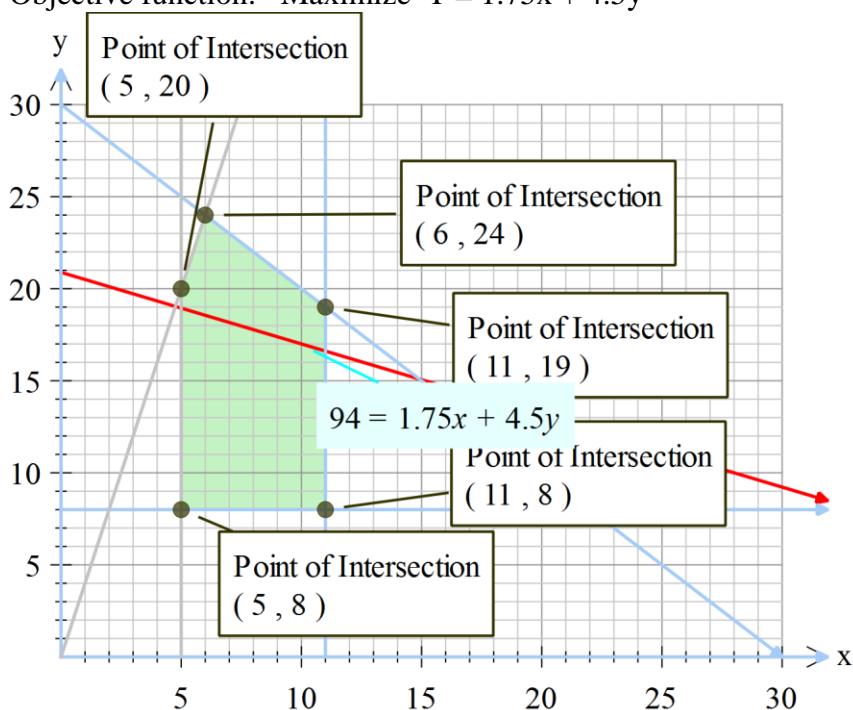
$$y \geq 8$$

$$x \geq 5$$

$$x \leq 11$$

$$y \leq 4x$$

Objective function: Maximize  $T = 1.75x + 4.5y$

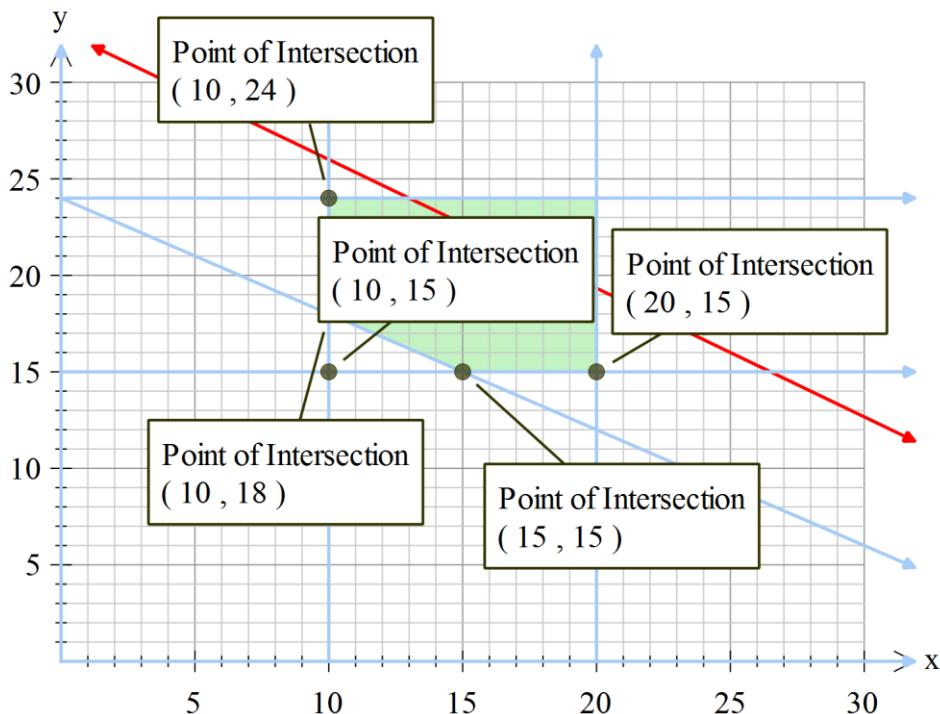


$(x,y)$	$T = 1.75x + 4.5y$
(5,20)	98.75
(6,24)	118.50 ← Max
(11,19)	104.75
(11,8)	55.25
(5,8)	44.75

Solution: A maximum running time of 118.5 mininutes will be obtained by adding 6 L of oil and 24 L of gas.

4. Let  $x$  = # units of nitrogen  
 Let  $y$  = # units of phosphorous  
 $x \geq 10$   
 $x \geq 0$        $x \leq 20$   
 $y \geq 0$        $y \geq 15$   
 $y \leq 24$   
 $3x + 5y \geq 120$

Objective function: Minimize  $C = 4x + 6y$



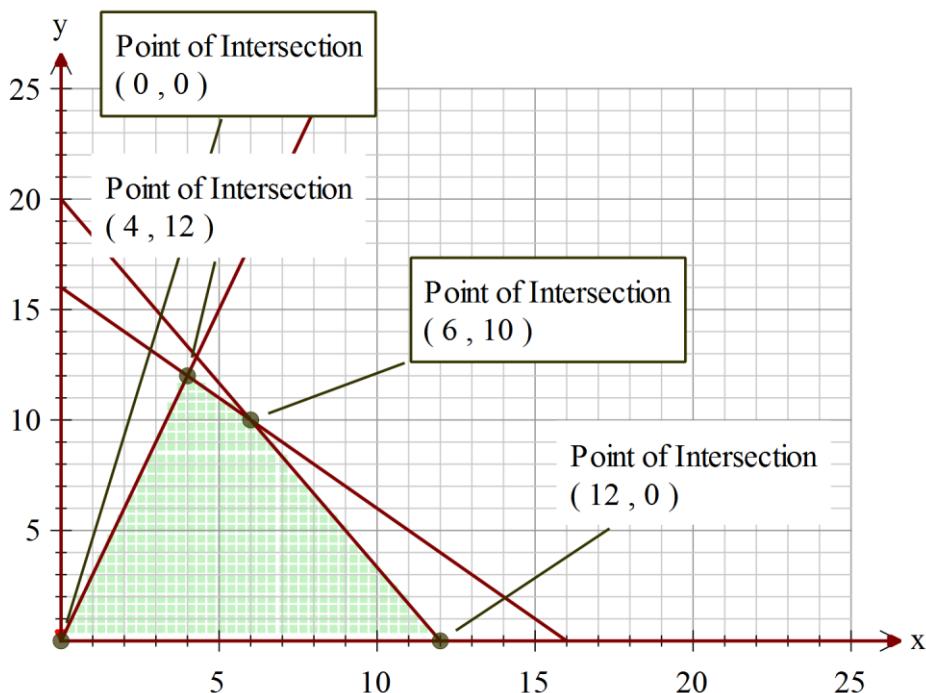
$(x,y)$	$C = 4x + 6y$
(10,18)	148 ← Min
(10,24)	184
(15,15)	150
(20,24)	224
(20,15)	170

Solution : 10 units of nitrogen and 18 units of phosphorous will result in a minimum cost of \$148

5. Let  $x$  = #hectares of cherries  
 Let  $y$  = #hectares of raspberries

$$\begin{array}{ll} x \geq 0 & x + y \leq 16 \\ y \geq 0 & 5x + 3y \leq 60 \\ & y \leq 3x \end{array}$$

Objective Max  $R = 3000x + 2500y$



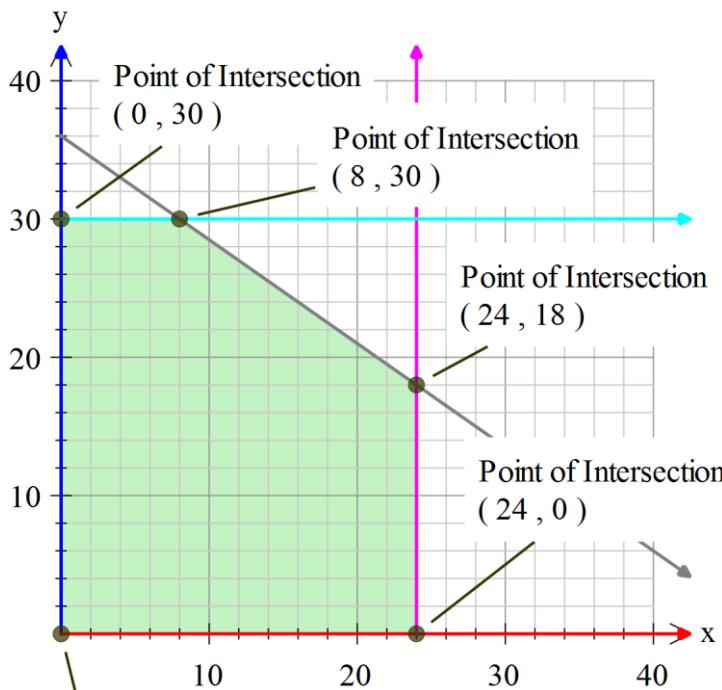
$(x,y)$	$R = 3000x + 2500y$
(0,0)	0
(4,12)	42 000
(6,10)	43 000 ← Max
(12,0)	36 000

Solution: The farmer should plant 6 hectares of cherries and 10 hectares of raspberries for a maximum revenue of \$43 000.

6. Let  $x$  = # cords of birch  
 Let  $y$  = # cords of maple

$$\begin{array}{ll} x \geq 0 & x \leq 24 \\ y \geq 0 & y \leq 30 \\ \frac{x}{4} + \frac{y}{3} \leq 12 & \end{array}$$

Objective: Max  $R = 100x + 150y$



$(x,y)$	$R = 100x + 150y$
(0,0)	0
(0,30)	4500
(8,30)	5300 ← Max
(24,0)	2400
(24,18)	5100

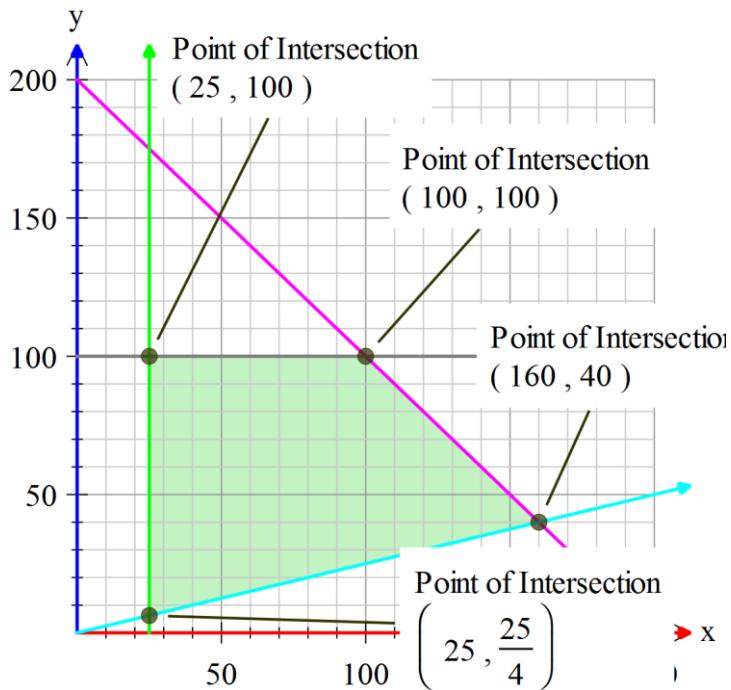
Solution: 8 cords of birch and 30 cords of maple will result in a maximum revenue of \$5300

7. Let  $x$  = # of enriched loaves  
 Let  $y$  = # of whole wheat loaves

$$x + y \leq 200$$

$$\begin{array}{ll} x \geq 0 & y \geq \frac{1}{4}x \\ y \geq 0 & y \leq 100 \\ & x \geq 25 \end{array}$$

$$\text{Max } P = 0.35x + 0.28y$$



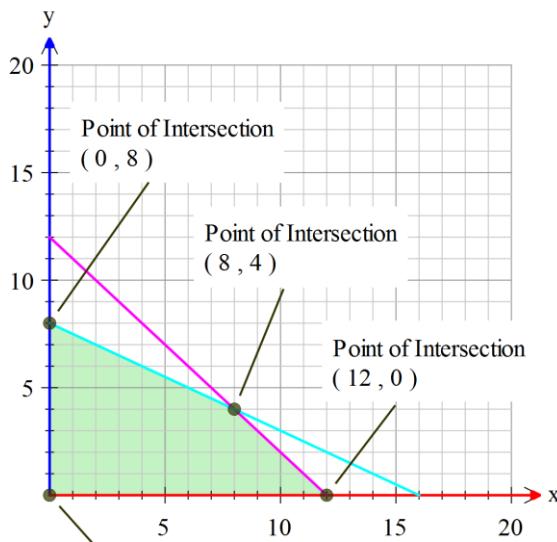
$(x,y)$	$P = 0.35x + 0.28y$
(25,100)	36.75
(100,100)	63
(160,40)	67.20 ← Max
(25,6.25)	10.50

Solution: 8 enriched loaves and 30 whole wheat loaves will result in a maximum profit of \$67.20

8. Let  $x$  = #children  
 Let  $y$  = #adults

$$\begin{array}{l} x \geq 0 \\ y \geq 0 \end{array} \quad \begin{array}{l} x + y \leq 12 \\ \frac{1}{2}x + y \leq 8 \end{array}$$

Objective: Max  $12x + 20y$



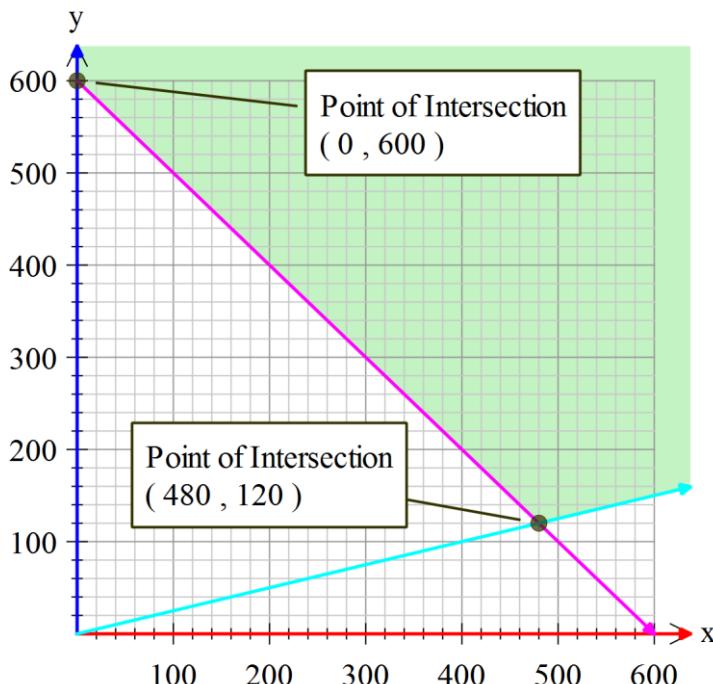
$(x,y)$	$12x + 20y$
(0,8)	160
(12,0)	144
(8,4)	176 ← Max
(0,0)	0

Solution: 8 child lessons and 4 adult lessons will result in a maximum revenue of \$176

9. Let  $x = \#$  kg of sand  
 Let  $y = \#$  kg of cement

$$\begin{array}{ll} x \geq 0 & x + y \geq 600 \\ y \geq 0 & y \geq \frac{1}{4}x \end{array}$$

$$\text{Min } 0.01x + 1y$$



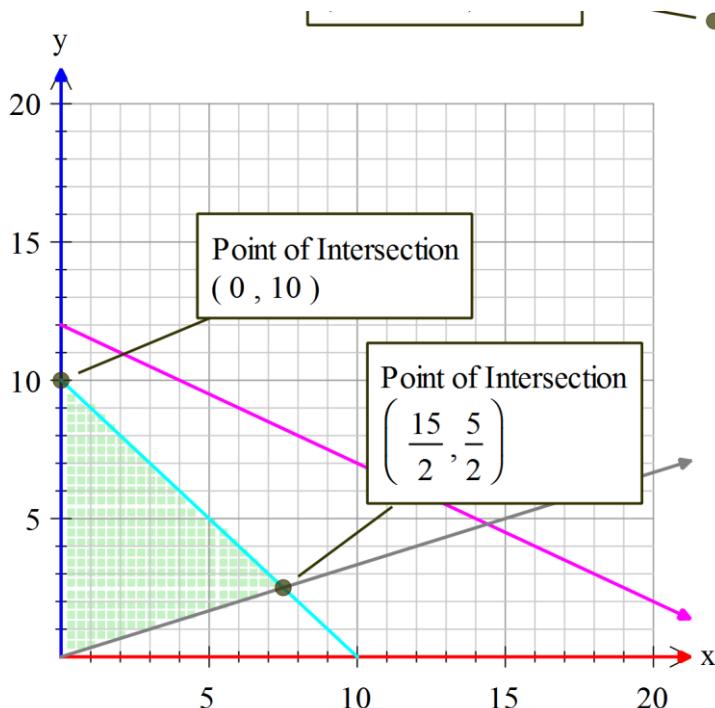
$(x,y)$	$0.01x + 1y$
$(0,600)$	600
$(480,120)$	124.80 ← Min

Solution: 480 kg of sand and 120 kg of cement will result in a minimum cost of \$124.80

10. Let  $x$  = # pages of text  
 Let  $y$  = # pages of graphics

$$\begin{array}{ll} x \geq 0 & 0.5x + y \leq 12 \\ y \geq 0 & x + y \leq 10 \\ & x \leq 3y \end{array}$$

$$\text{Max } 2.50x + 4y$$



$(x,y)$	$2.50x + 4y$
(0,0)	0
(0,10)	40 ← Max
(7.5,2.5)	28.75

Solution: she should prepare 0 pages of text and 10 pages of graphics to make \$40

11. Let  $x = \#$  days in France  
 Let  $y = \#$  days in Italy

$$x \geq 0$$

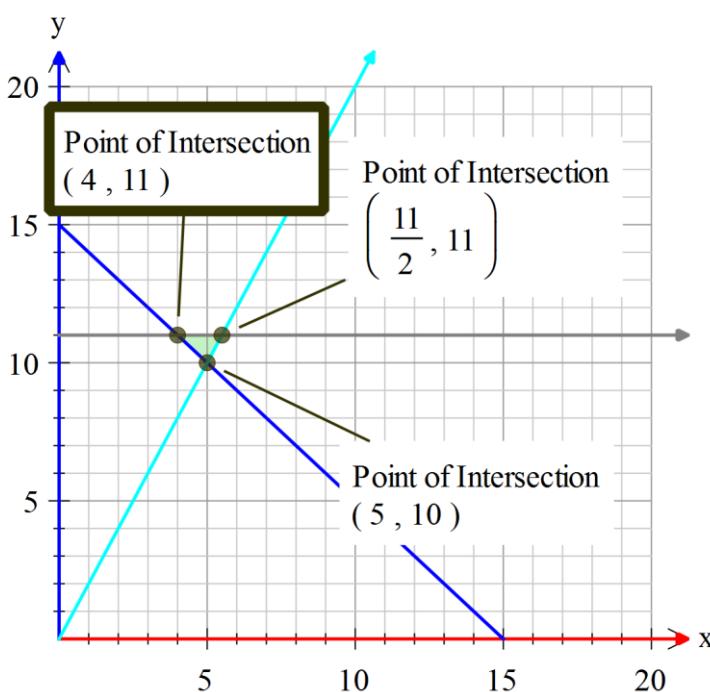
$$y \geq 0$$

$$x + y \geq 15$$

$$y \geq 2x$$

$$y \leq 11$$

$(x,y)$	$\text{Min } 80x + 150y$
(5,10)	1900 ← Min
(4,11)	1970
(5.5,11)	2090



Solution: They should spend 5 days in France and 10 days in Italy to spend as little money as possible (\$1900)

12. Let  $x$  = # pages in black & white  
 Let  $y$  = # pages in colour

$$x \geq 0$$

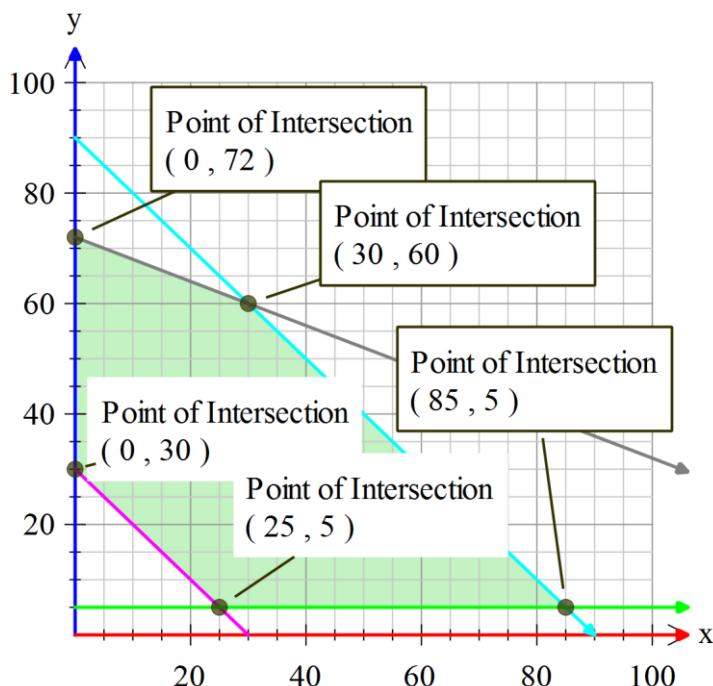
$$y \geq 0$$

$$x + y \geq 30$$

$$x + y \leq 90$$

$$2x + 5y \leq 360$$

$$y \geq 5$$



$(x,y)$	Min $0.10x + 0.25y + 3$
(0,72)	21.00
(30,60)	21.00
(85,5)	12.75
(25,5)	6.75 ← Min
(0,30)	10.50

There should be 25 black and white pages and 2 colour pages so the cost of the book is a minimum (\$6.75)

13. Let  $x = \#$  round pills

Let  $y = \#$  oval pills

$$x \geq 0$$

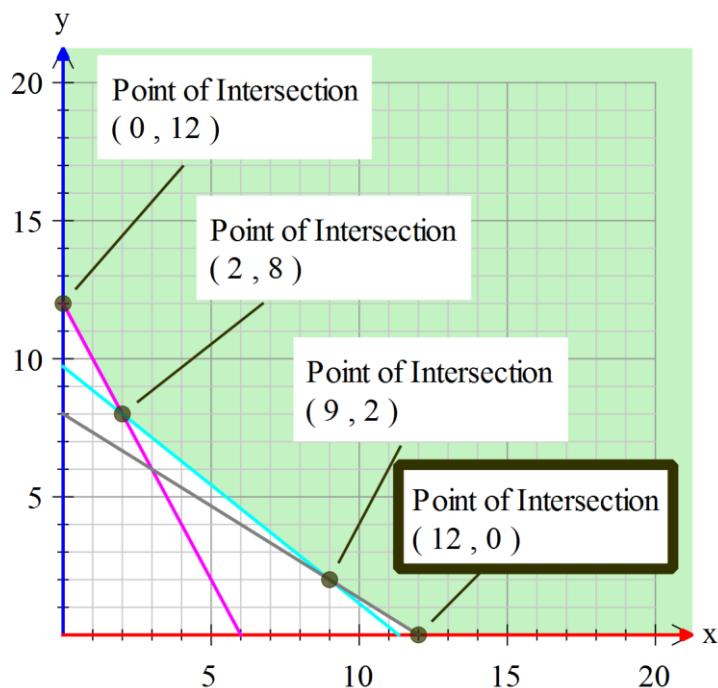
$$y \geq 0$$

$$2x + y \geq 12$$

$$6x + 7y \geq 68$$

$$2x + 3y \geq 24$$

$(x,y)$	Min $x + y$
$(0,12)$	12
$(2,8)$	$10 \leftarrow$ Min
$(9,2)$	11
$(12,0)$	12



The minimum number of round pills is 2 and oval pills is 8