Mathematics 5 SN

OPTIMIZATION

1 Jonathan works at a golf club during his summer vacation. He sometimes cleans the premises and sometimes works in the kitchen at the club's restaurant.

Jonathan makes \$8 per hour when cleaning the premises and \$9.50 per hour when working in the kitchen.

There are certain constraints on the number of hours he can devote to each job every week. This situation is represented by the system of inequalities and the polygon of constraints given below.

$$x \geq 0$$

$$y \geq 0$$

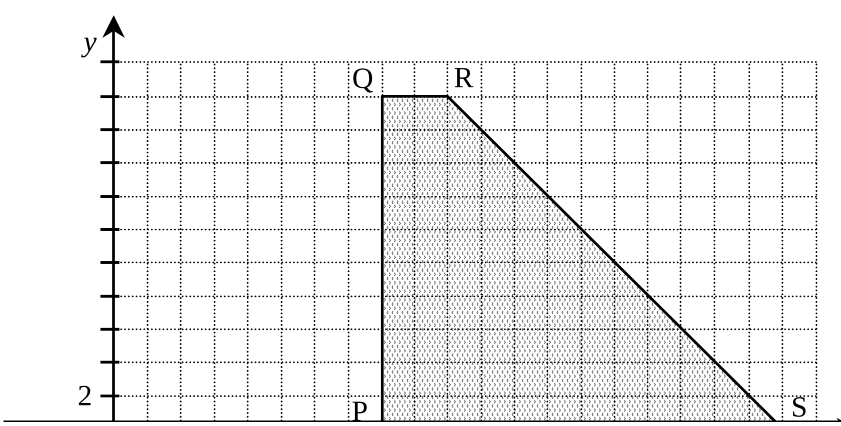
$$x + y \leq 40$$

$$x \geq 16$$

$$y \leq 20$$

x : the number of hours spent cleaning the premises

y : the number of hours spent working in the kitchen



Coordinates of the vertices of the polygon
P(16, 0)
Q(16, 20)

R(20, 20)
S(40, 0)

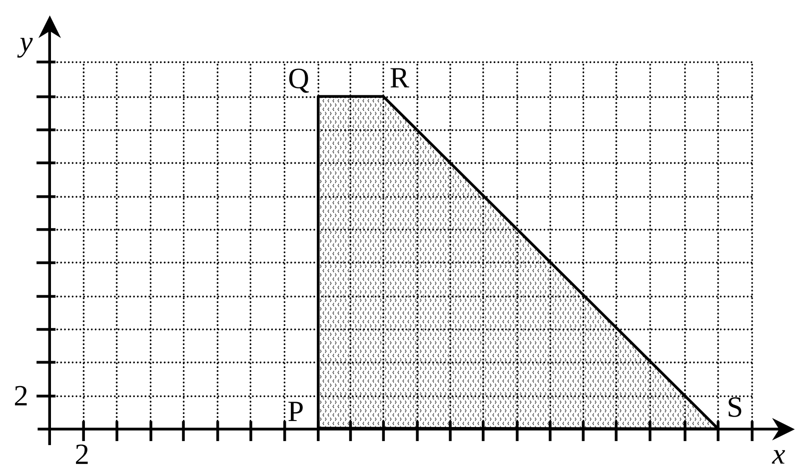
This week, Jonathan’s employer informed him that there would be an additional constraint. This new constraint is represented by the following inequality:

$$x \geq y + 20$$

With this new constraint, by how much will Jonathan's maximum possible income decrease?

Show all your work.

Show all your work

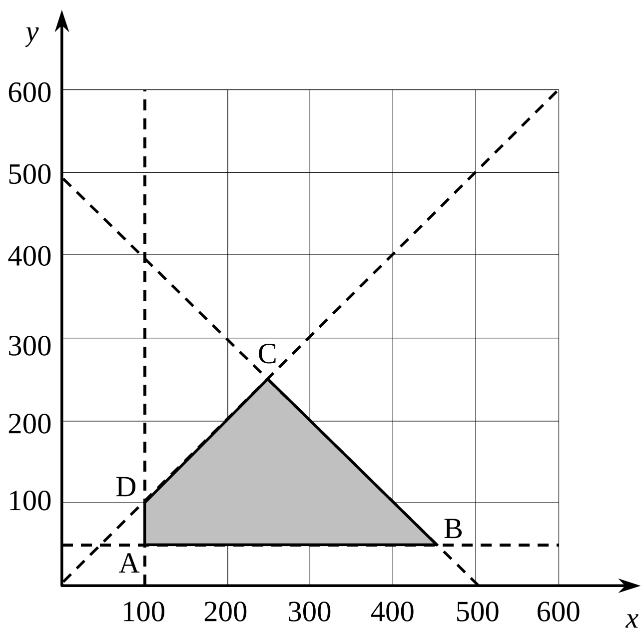


Answer: With this new constraint, Jonathan's maximum possible income will decrease by $__________.

2

To raise money, the Graduation Committee decides to sell cases of fruit. The following polygon represents the constraints that must be respected.

If x represents the number of cases of oranges for sale and y, the number of cases of grapefruit for sale, the constraints are:



① $x \leq y$

② $x + y \leq 500$

③ $x \geq 100$

④ $y \geq 50$

For each case of oranges and grapefruit sold, the Graduation Committee makes a profit of $1.00 and $1.50, respectively.

Yesterday, the head of the committee received a call from the supplier. Because of a recent flood, the supplier can deliver a maximum of 400 cases of fruit.

By how much will the maximum possible revenue decrease because of the flood?

Show all your work.

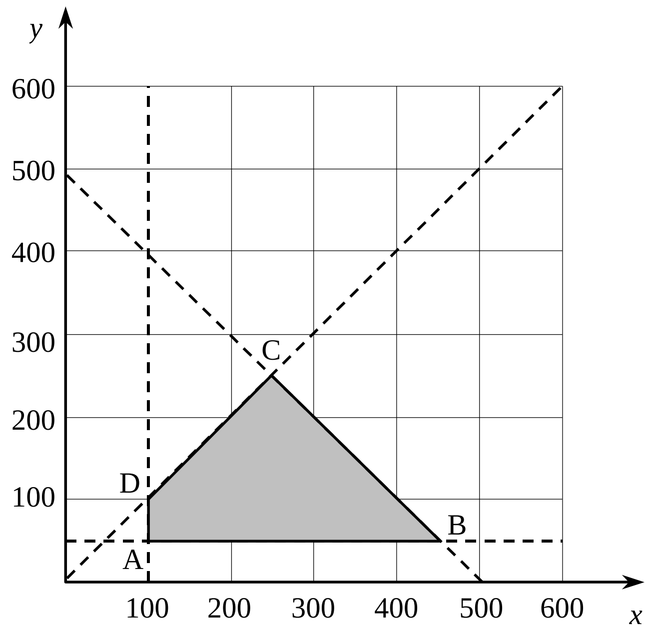
Show all your work.

① $x \leq y$

② $x + y \leq 500$

③ $x \geq 100$

④ $y \geq 50$



Answer: The decrease in revenue caused by the flood is $_________________ .

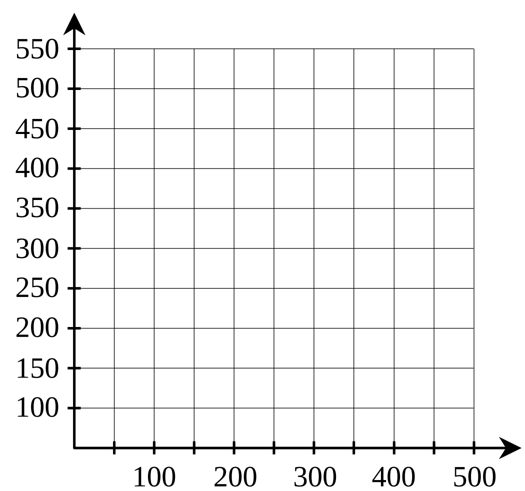
3

To raise money, the rugby team sells caps and sweaters. In order to avoid saturating the market, the team can sell no more than 500 articles. The number of caps sold must be greater than or equal to the number of sweaters sold. The manufacturer requires an order of a minimum of 150 sweaters.

The team will make a profit of \$3 on each cap and \$7 on each sweater.

How many caps and how many sweaters must the team sell to maximize its profit?

Show all your work.



Answer: The team must sell ___________ caps and ___________ sweaters to maximize its profit.

4

Matthew grows hay and corn on no more than 350 hectares of his farm. He uses at most twice as many hectares to grow hay as he does to grow corn. He uses at most 50 more hectares to grow corn as he does hay. He must use at least 100 hectares to grow hay. It costs \$200 to seed each hectare of hay and \$300 to seed each hectare of corn.

Let x represents the number of hectares used to grow hay

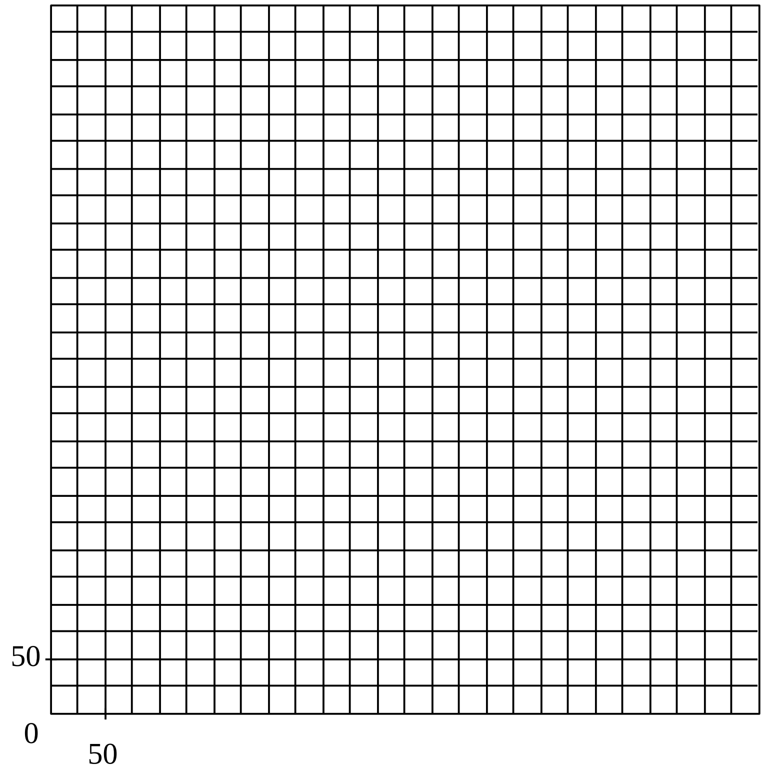
y represents the number of hectares used to grow corn

How many hectares of hay and of corn should Matthew seed to minimize his costs?

Show all your work.

Let x represents the number of hectares used to grow hay

y represents the number of hectares used to grow corn



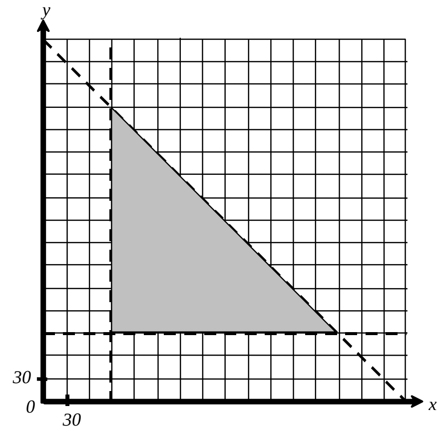
Answer: Matthew must seed ___________ hectares of hay and ___________ hectares of corn.

5

Secondary 5 students are organizing a fund-raising activity in order to lower the cost of their grad dance.

During a volleyball tournament, they plan to sell bottles of water and juice at a stand. The following constraints are to be respected:

- At least 90 bottles of water and at least 90 bottles of juice must be sold.
- They cannot store more than 480 bottles at the stand.



Letting *x* be the number of bottles of water to be sold and *y*, the number of bottles of juice to be sold, the students transformed the constraints into the following inequalities:

$$x \geq 90$$
$$y \geq 90$$
$$x + y \leq 480$$

The profit on each bottle of water is $0.90 and on each bottle of juice it is $1.40.

To determine their maximum profit, the students drew the graph of the polygon of constraints, shown above.

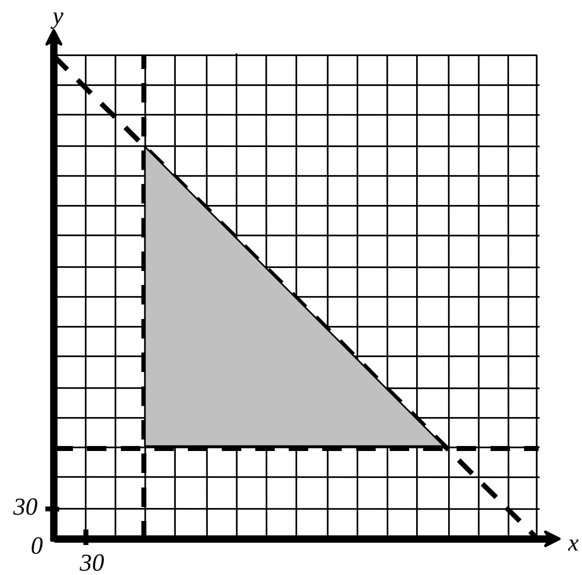
Before delivery, the supplier imposed a new condition: The students had to order a maximum of twice as many bottles of juice as bottles of water. This new constraint will lower their maximum profit.

By how much would the profit decrease because of the supplier’s new condition?

Show all your work.

Show all your work.

$x \geq 90$
$y \geq 90$
$x + y \leq 480$



Answer: The profit would decrease by $__________________.

6

The city council of a town wants to minimize the cost of staffing its recreation centres in the summer months. The council has determined that supervisors will be paid $3500 for the summer and staff workers will be paid $1500 for the summer.

The council wants to hire its employees using the following constraints:

- The maximum number of employees for its centres is 30 and the minimum is 18.
- The council also wants to hire at least 6 supervisors but no more than 14 supervisors.
- It wants to hire at least 8 staff workers.
- The number of staff workers must be at most twice the number of supervisors.

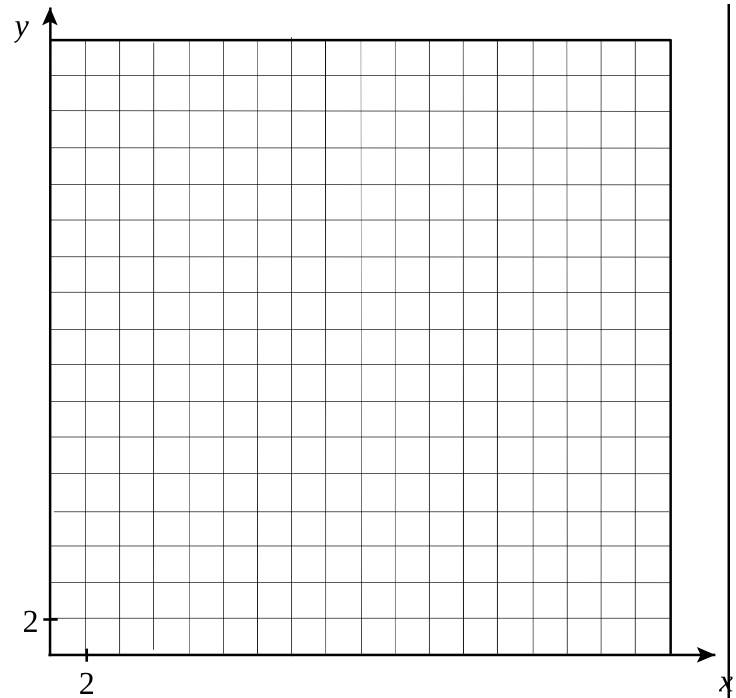
How many staff workers and how many supervisors can the town council hire and minimize its costs?

Show all your work.

Show all your work.

x: number of supervisors

y: number of staff workers



Answer: The town should hire _______ supervisors and _______ staff workers in order to minimize its costs.

7 Wheeler is a producer of mountain bikes and road bikes. Because of its small size, it can build no more than 80 bikes each week. To meet certain conditions in its workshop, it must build at least 45 mountain bikes, and at least 10 road bikes weekly. To meet consumer demand, it must manufacture at least 3 times as many mountain bikes as road bikes.

The following is the system of constraints for Wheeler's weekly bike production:

x = the number of road bikes produced weekly

y = the number of mountain bikes produced weekly

$x \geq 0$

$y \geq 0$

$x \geq 10$

$y \geq 45$

$x + y \leq 80$

$y \geq 3x$

For each road bike and mountain bike produced, Wheeler earns a profit of $250 and $175, respectively.

What is the maximum weekly profit that can be earned?

Show your work.

Show all your work.

x = number of road bikes

y = number of mountain bikes

$x \geq 0$

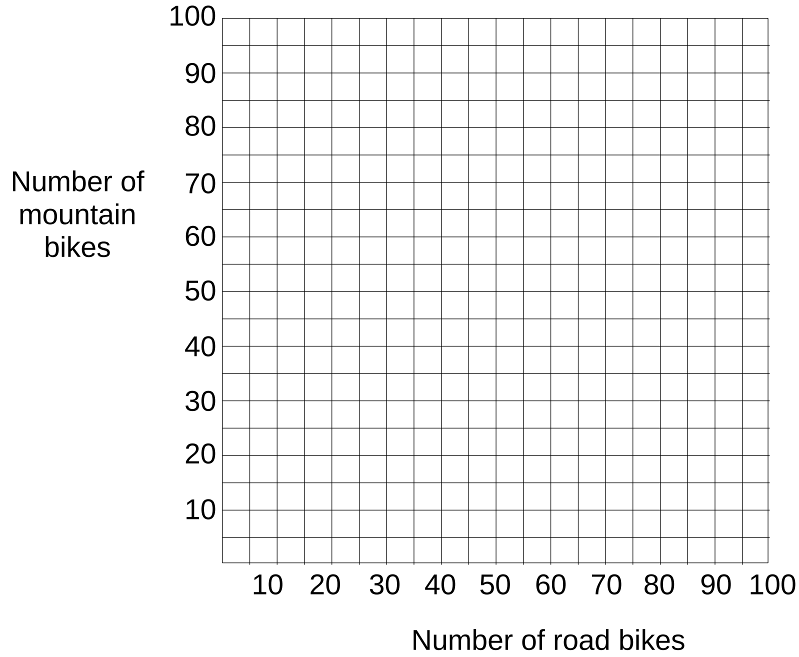
$y \geq 0$

$x \geq 10$

$y \geq 45$

$x + y \leq 80$

$y \geq 3x$



Answer: The maximum weekly profit is $_______________.

8

A fisherman has to separate his daily catch of shellfish into two categories before he can sell them. Lobsters are sold for $8.70 each and crabs are sold for $9.60 each.

On an average day, the fisherman can expect to catch a minimum of 35 crabs and a maximum of 60. By experience, there are at most twice as many lobsters as crabs in a daily catch and never has the fisherman caught more than 140 shellfish in a single day.

Using a polygon of constraints, determine the maximum revenue that this fisherman can expect to make.

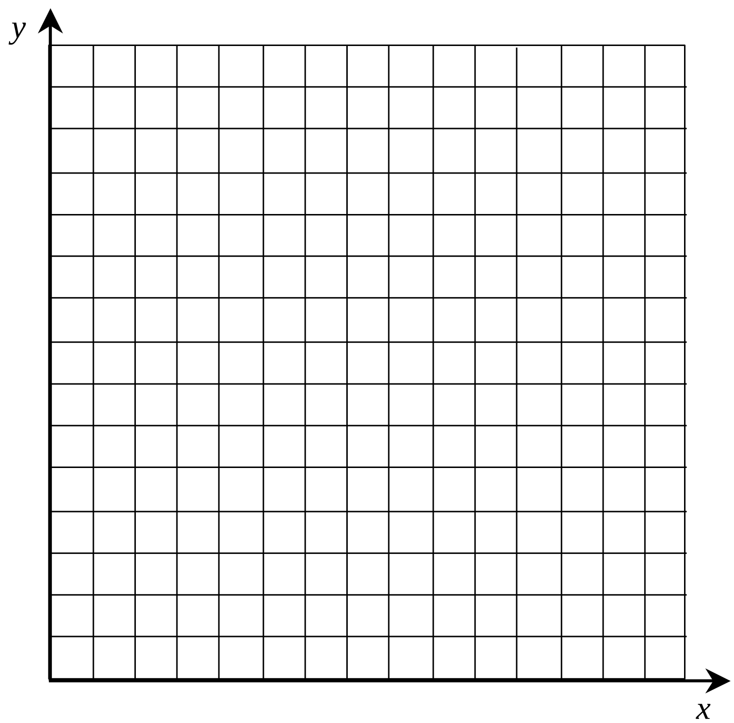
Show all your work.

Show all your work.

Let x: number of lobsters

y: number of crabs

Graph:



Answer: The maximum revenue this fisherman can expect to make is __________.

9 Instruments Quebecois makes two types of graphing calculators, the Gold Edition and the Bronze Edition. In order to meet daily demands, it must make at least 200 Gold Editions and at least 100 Bronze Editions.

The factory produces at least twice as many Gold Editions as Bronze Editions, but can make no more than 600 calculators a day.

Given x: number of Gold Editions

y: number of Bronze Editions

Constraints:

$x \geq 0$

$y \geq 0$

$x \geq 200$

$y \geq 100$

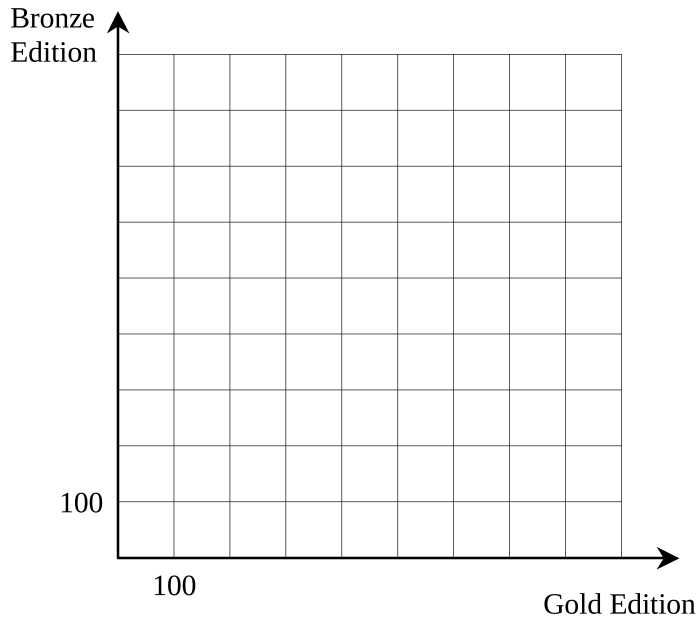
$x \geq 2y$

$x + y \leq 600$

The profit on a Gold Edition is $20.00 and $15.00 on a Bronze Edition.

In the answer booklet, graph the polygon of constraints and determine the number of calculators that the company should make to maximize its profit.

Show all your work.



Answer: Instruments Quebecois should produce ___________ Gold Editions and ___________ Bronze Editions to maximize profit.

10 The Grad Committee plans to sell chocolate bars to raise money for its upcoming dance. This year the committee members have decided to sell two types, one with roasted almonds and the other with caramel. They have a maximum of 500 bars to sell. They expect to sell a minimum of 120 almond chocolate bars. From past experience, almond chocolate bars sell at most 4 times as well as caramel ones. They make a profit of $0.80 for each almond chocolate bar and $1 for each caramel chocolate bar.

Let x: number of almond chocolate bars

y: number of caramel chocolate bars

What is the difference in the maximum profit if they had expected to sell a minimum of 160 almond chocolate bars rather than 120?

Show all your work.

Show all your work.

Show all your work.

x: number of almond chocolate bars

y: number of caramel chocolate bars

Answer: The difference in the maximum profit is $___________.

11 Murray plans a trip to New York in July. In order to save money, he works at two different part-time jobs on weekends. At the first job, he works a minimum of 10 hours per month and at the second, a maximum of 40 hours per month. Murray must work at least 30 hours per month but no more than 60 hours per month. He must work at least as many hours at the second job as he does at the first. He makes \$6.30 an hour at the first job and \$8 an hour at the second job.

Let x: number of hours per month at first job

y: number of hours per month at second job

The initial constraints for this situation are:

$$\begin{aligned} x &\geq 10 \\ y &\leq 40 \\ y &\geq 0 \\ x + y &\geq 30 \\ x + y &\leq 60 \\ y &\geq x \end{aligned}$$

Because of a shortage of employees, Murray was later advised that he could increase the number of hours he worked at the second job.

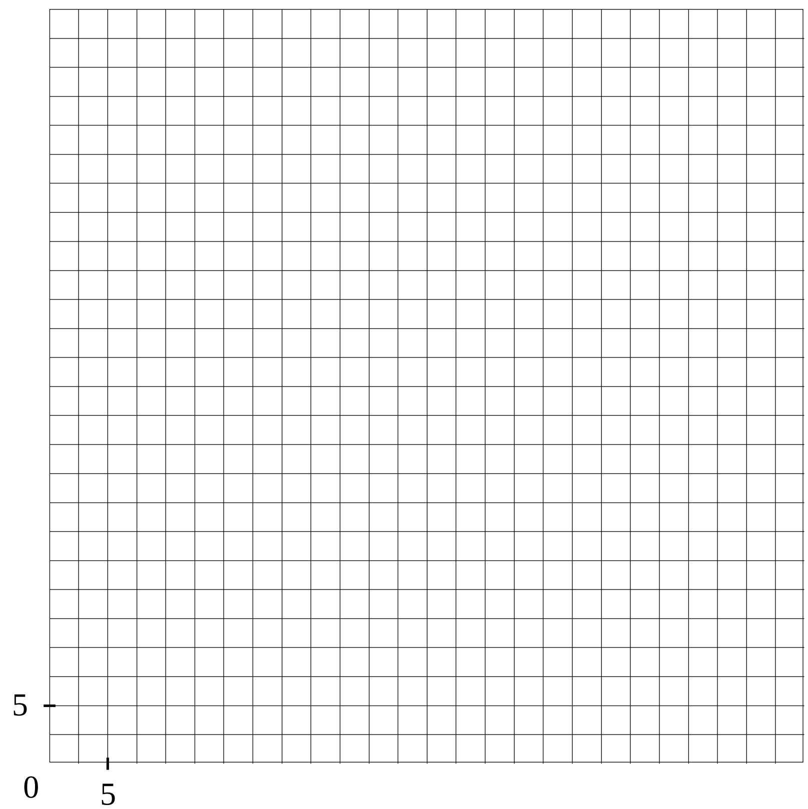
By how much did Murray's maximum possible salary increase because of the employee shortage?

Show all your work.

Show all your work.

x: number of hours per month at first job

y: number of hours per month at second job



Answer: Murray’s maximum possible salary increased by $__________.

12 Kim is organizing a fundraiser for her soccer team. She will sell hot dogs and hamburgers outside a popular grocery store. She needs to purchase enough supplies to be able to make the following:

– at most 800 hot dogs and hamburgers

– at least 150 hot dogs

– a minimum of 100 but not more than 400 hamburgers

– at most twice as many hamburgers as hot dogs

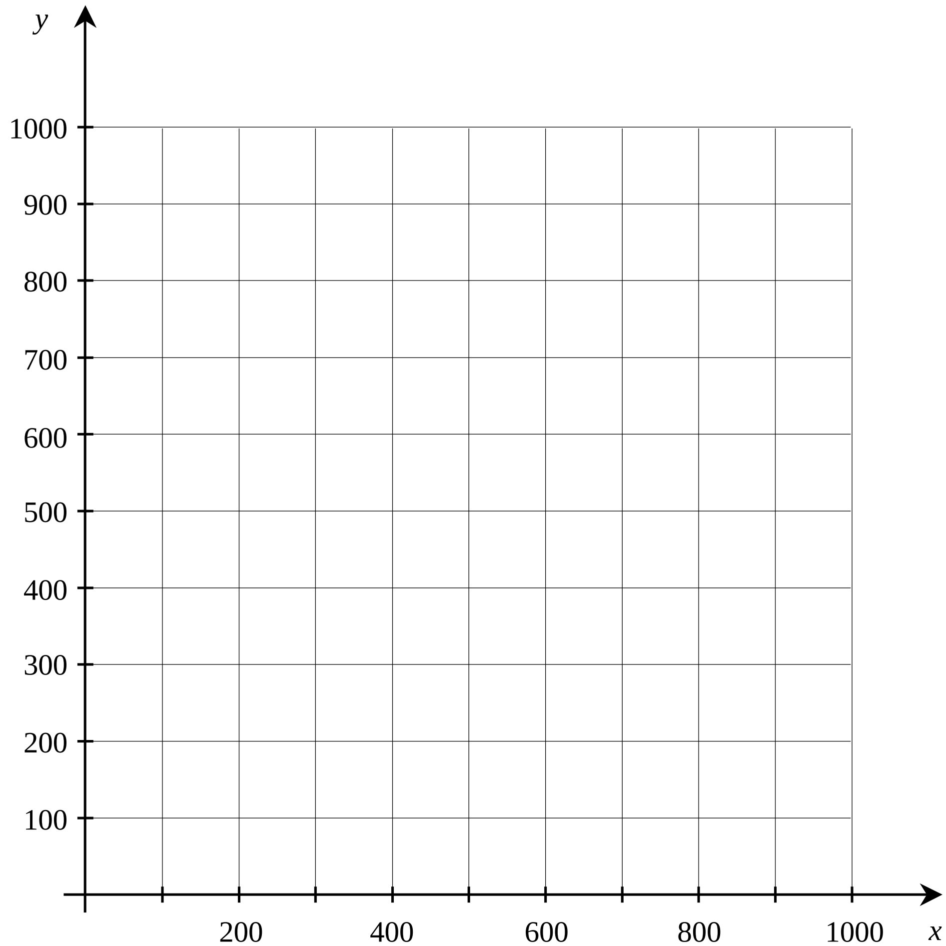
Her cost is \$0.45 per hot dog and \$0.75 per hamburger. She will sell the hot dogs at \$1 each and hamburgers at \$1.50 each.

Let x: the number of hot dogs

y: the number of hamburgers

Given her constraints, how many hot dogs and hamburgers does Kim need to sell to make the greatest profit possible?

Show all your work.



Answer: Kim needs to sell __________ hot dogs and __________ hamburgers to make the greatest profit.

13 A company produces different games. The two most popular are the Memory game and the Construction game. The number of games that the company stocks is based on past sales, which indicate that it sells at most twice as many Construction games as Memory games. The company cannot have more than 1500 of these games in stock. It costs \$5 to produce the Memory game and \$10 for the Construction game. The company expects to spend a minimum of \$10 000 to produce these games. The Memory game sells for \$35 while the Construction game sells for \$50.

Let x represent the number of Memory games in stock

y represent the number of Construction games in stock

What is the maximum profit the company can make selling these games?

Show all your work.

Answer: The maximum profit the company can make is $_______________.

14 CB Manufacturing Company makes both wireless and wired headsets for cellular phones.

- The assembly lines can produce no more than 25 wireless headsets.
- No more than 32 headsets in all can be produced per day.
- Workers can spend at most 68 person-hours per day testing headsets. It takes 1 person-hour to test a wireless headset and 4 person-hours to test a wired headset.
- The number of wireless headsets produced per day must be at least half the number of wired headsets.
- The company makes a profit of $20 on each wireless headset and $10 on each wired headset.

Let *x* represent the number of wireless headsets produced per day and

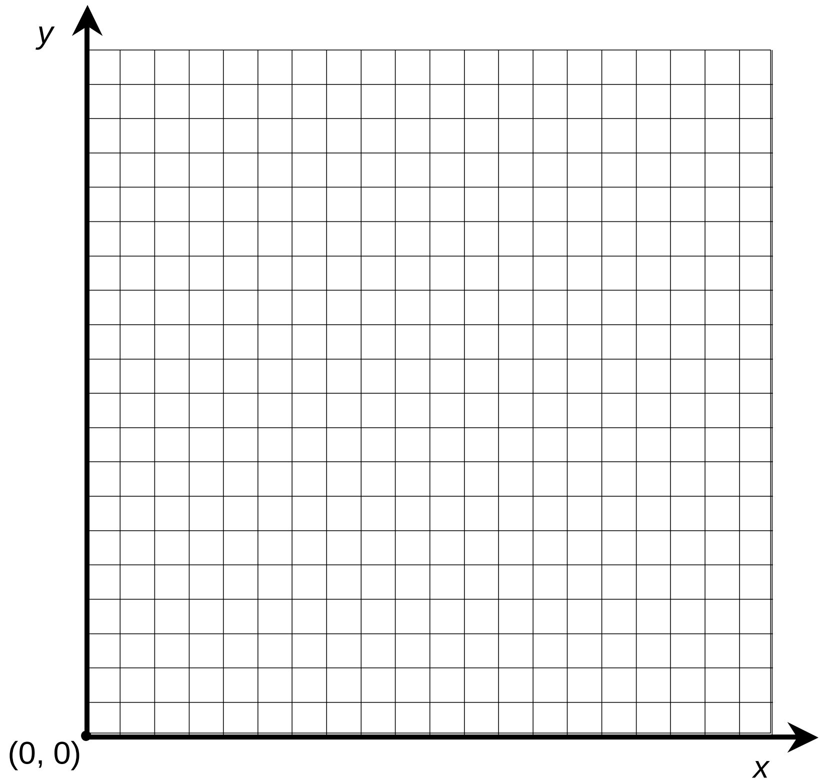
y represent the number of wired headsets produced per day

The budget director set up the following system of constraints:

$$\begin{aligned} x &\geq 0;\ y \geq 0 \\ x &\leq 25 \\ x + y &\leq 32 \\ x + 4y &\leq 68 \\ x &\geq \frac{1}{2}y \end{aligned}$$

Determine the maximum profit the company can make on the sale of the two types of headsets.

Show all your work.



Answer: The maximum profit that the company can make is $_________.

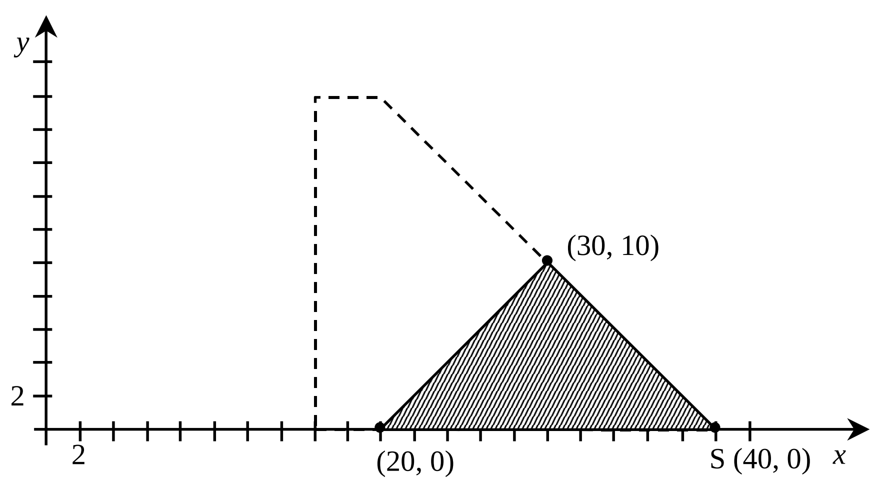
2- Correction key

1 Example of an appropriate method

Maximum possible income before the new constraint

Vertex	Income: 8x + 9.50 y
P (16, 0)	8 (16) + 9.50 (0) = $128
Q (16, 20)	8 (16) + 9.50 (20) = $318
R (20, 20)	8 (20) + 9.50 (20) = $350 ← maximum income
S (40, 0)	8 (40) + 9.50 (0) = $320

Vertices of the new polygon of constraints



Maximum possible income with the new constraint

Vertex	Income: 8x + 9.50 y
(20, 0)	8 (20) + 9.50 (0) = $160

(30, 10)	8 (30) + 9.50 (10) = \$335 ← maximum income
S (40, 0)	8 (40) + 9.50 (0) = \$320

Difference between the two maximum possible incomes

\$350 – \$335 = \$15

Answer: With this new constraint, Jonathan's maximum possible income will decrease by \$**15**.

Note: Students who used an appropriate method in order to determine the maximum possible income before **or** with the new constraint have shown that they have a partial understanding of the problem.

2

Example of an appropriate solution

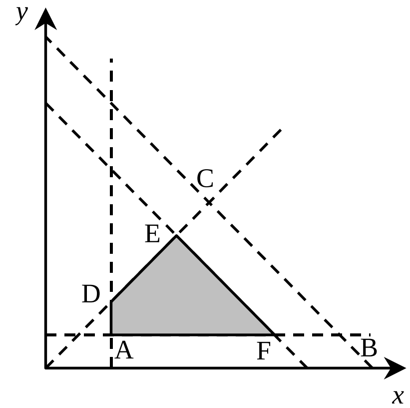
Calculation of revenue before the new constraint is considered

Vertices	R(*x*, *y*) = 1.00*x* + 1.50*y*
A(100, 50)	100 + 75 = \$175
B(450, 50)	450 + 75 = \$525
C(250, 250)	250 + 375 = \$625
D(100, 100)	100 + 150 = \$250

Maximum Revenue ⇒ \$625 250 cases of oranges and 250 cases of grapefruit

Calculation of revenue after consideration of the constraint $x + y \leq 400$

Vertices	$R(x, y) = 1.00x + 1.50y$
A(100, 50)	\$175
D(100, 100)	\$250
E(200, 200)	\$500
F(350, 50)	\$425



Maximum Revenue $\Rightarrow$ 500 \$ 200 cases of oranges

200 cases of grapefruit

Decrease of revenue because of the flood

625 – 500 = 125

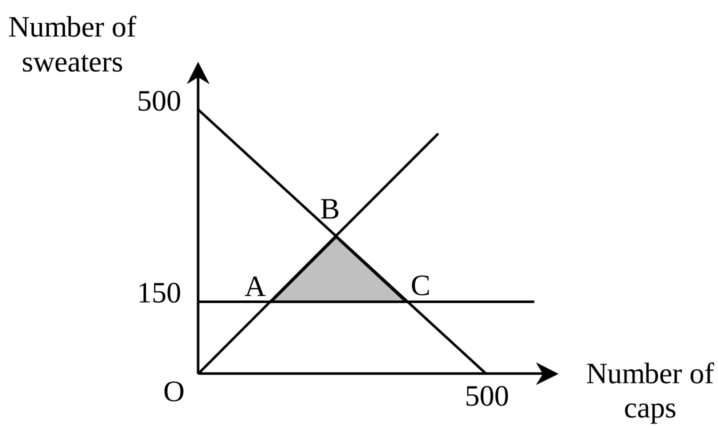
Answer: The decrease in revenue caused by the flood is \$**125**.

3 Example of an appropriate solution

Inequalities representing the constraints

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 500$$
$$x \geq y$$
$$y \geq 150$$

Polygon of constraints



Coordinates of vertices A, B, and C

A(150, 150), B(250, 250) and C(350, 150)

Profit according to the function $P = 3x + 7y$ and vertices A, B and C

A P = \$1500

B P = \$2500

C P = \$2100

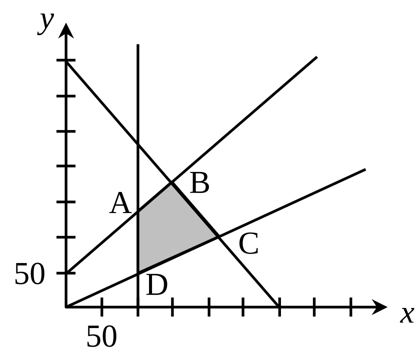
Answer: The team must sell **250** caps and **250** sweaters to maximize its profit.

4 Example of an appropriate solution

Constraints

$$x \geq 0$$
$$y \geq 0$$
$$x \leq 2y$$
$$y \leq x + 50$$
$$x + y \leq 350$$
$$x \geq 100$$

The polygon of constraints:



Cost function

C = 200*x* + 300*y*

Coordinates of the vertices

A(100, 150), B(150, 200) C(233.33, 116.67) and D(100, 50)

Costs

C_A = \$65 000

C_B = \$90 000

C_C = \$81 667

C_D = \$35 000

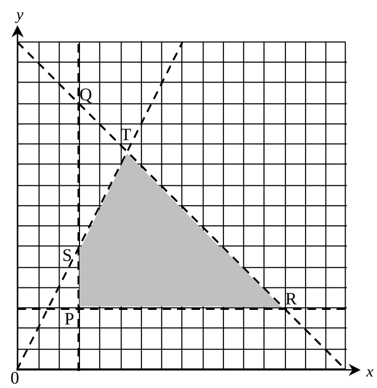
Answer: Matthew must seed **100** hectares of hay and **50** hectares of corn.

5 Example of an appropriate solution

The supplier's new constraint is

$2x \geq y$ or $y \leq 2x$

Polygon of constraints



Objective function

$Z = 0.90x + 1.40y$

Vertices of the polygon of constraints

System of equations	Vertices	Value of objective function	Profit
$x = 90$ $x + y = 480$	Q (90, 390)	0.90(90) + 1.40(390) = 627	\$627 (maximum profit before additional constraint)
$x = 90$ $2x = y$	S (90, 180)	0.90(90) + 1.40(180) = 333	\$333

$x + y = 480$ $2x = y$	T(160, 320)	0.90(160) + 1.40(320) = 592	\$592 (maximum profit with additional constraint)
$x + y = 480$ $y = 90$	R(390,90)	0.90(390)+1.40(90) = 477	\$477
$x = 90$ $y = 90$	P(90,90)	0.90(90) + 1.40(90) = 207	\$207

Difference in profit

\$627 - \$592 = \$35

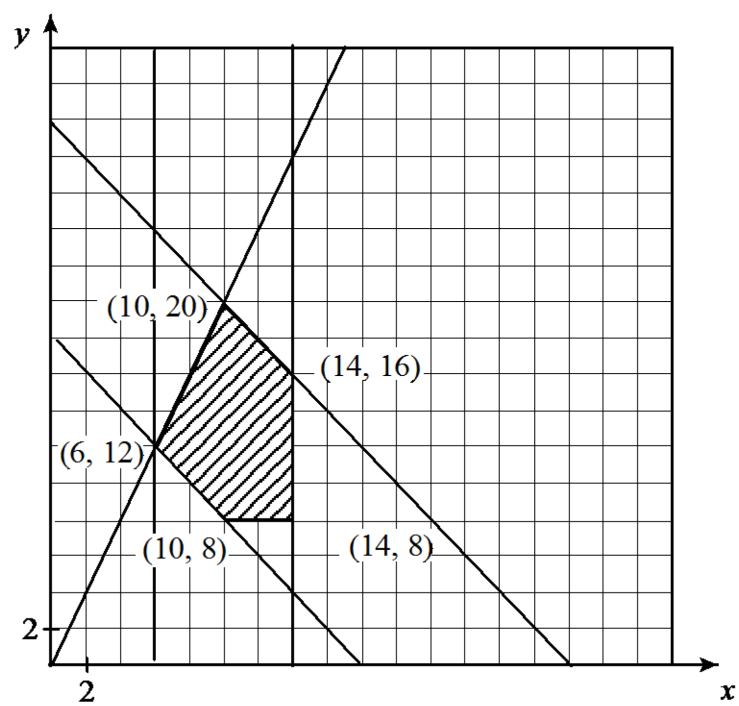
Answer: The profit would decrease by \$**35**.

6

Example of an appropriate solution

x: number of supervisors

y: number of staff workers



Constraints : $x \geq 0$, $y \geq 0$

$x + y \leq 30$

$x + y \geq 18$

$x \geq 6$

$x \leq 14$

$y \geq 8$

$y \leq 2x$

Vertices of polygon of constraints:	(10, 8)	$\Rightarrow$	10(3500) + 8(1500) =	\$47 000
	(6, 12)	$\Rightarrow$	6(3500) + 12(1500) =	\$39 000
	(10, 20)	$\Rightarrow$	10(3500) + 20(1500) =	\$65 000
	(14, 16)	$\Rightarrow$	14(3500) + 16(1500) =	\$73 000
	(14, 8)	$\Rightarrow$	14(3500) + 8(1500) =	\$61 000

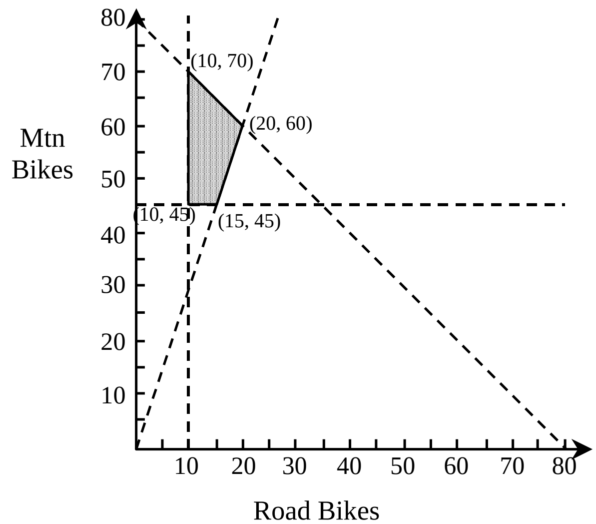
The minimum cost is \$39 000.

Answer The town should hire 6 supervisors and 12 staff workers in order to minimize its costs.

7 Example of an appropriate method

x = number of road bikes

y = number of mountain bikes



$x \geq 0$

$y \geq 0$

$x \geq 10$

$y \geq 45$

$x + y \leq 80$

$y \geq 3x$

Objective Function

Max. Profit = $250x + 175y$

Points (x, y)		Calculation	Profit
1.	(10, 45)	250(10) + 175(45)	\$10 375
2.	(15, 45)	250(15) + 175(45)	\$11 625
3.	(20, 60)	250(20) + 175(60)	\$15 500
4.	(10, 70)	250(10) + 175(70)	\$14 750

Answer The maximum weekly profit is \$15 500.

8

Example of an appropriate method

Let x: number of lobsters

y: number of crabs

Constraints:

$x \geq 0 \quad y \geq 0$

$y \geq 35$

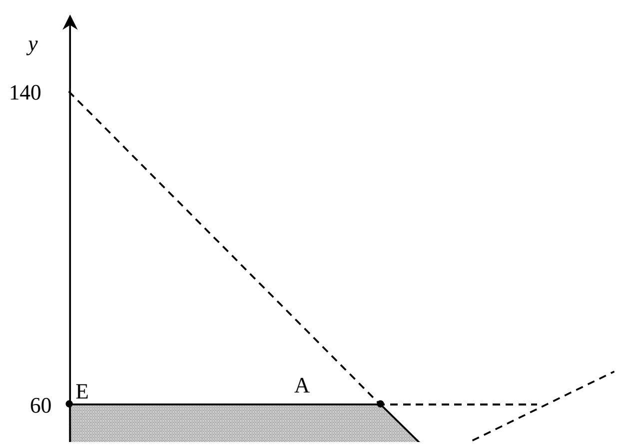
$y \leq 60$

$x \leq 2y$

$x + y \leq 140$

Objective Function: $R = 8.70x + 9.60y$

Graph:



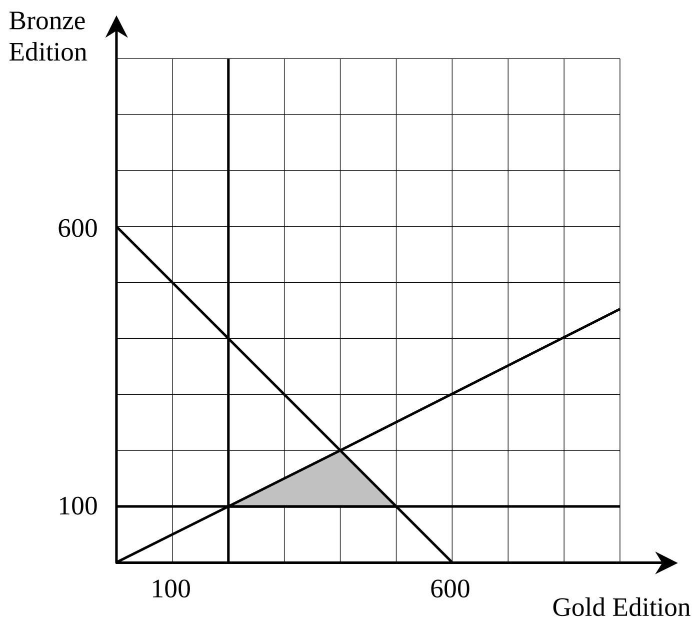
Vertex	$R = 8.70x + 9.60y$
A(80, 60)	1272 ← max
B($93.\bar{3}$, $46.\bar{6}$)	1259
C(70, 35)	945
D(0, 35)	336
E(0, 60)	576

Answer: The maximum revenue this fisherman can expect to make is **$1272**.

Note: Do not penalize students who did not include the non-negative constraints.

Students who determined the constraints and graphed the polygon have shown a partial understanding of the problem.

9 Example of an appropriate solution



Maximum possible profit

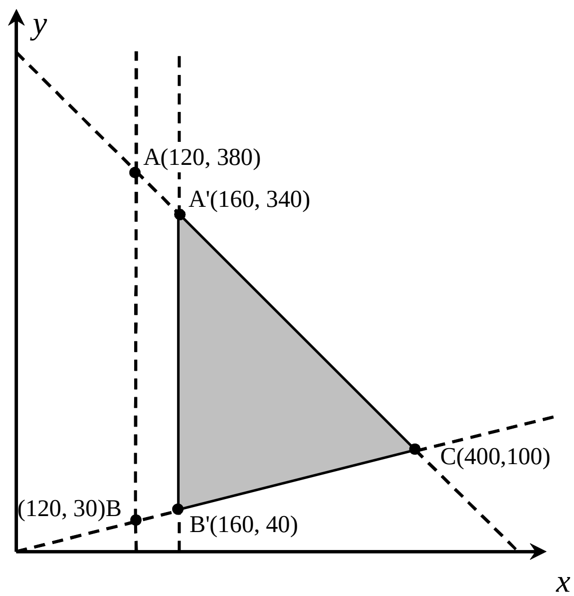
Vertex	Profit = 20x + 15y
A(200, 100)	20(200) + 15(100) = \$5500
B(400, 200)	20(400) + 15(200) = \$11 000
C(500, 100)	20(500) + 15(100) = \$11 500

Answer: Instruments Quebecois should produce **500** Gold Editions and **100** Bronze Editions to maximize profit.

10 Example of an appropriate solution

x: number of almond chocolate bars

y: number of caramel chocolate bars



Constraints

$$\begin{aligned} x + y &\leq 500 \\ x &\leq 4y \\ x &\geq 120 \end{aligned}$$

Profit

P = 0.8x + y

Point	Profit
A(120, 380)	$476
B(120, 30)	$126
C(400, 100)	$420

Point	Profit
A'(160, 340)	$468
B'(160, 40)	$168
C'(400, 100)	$420

Difference in profit

$476 – $468 = $8

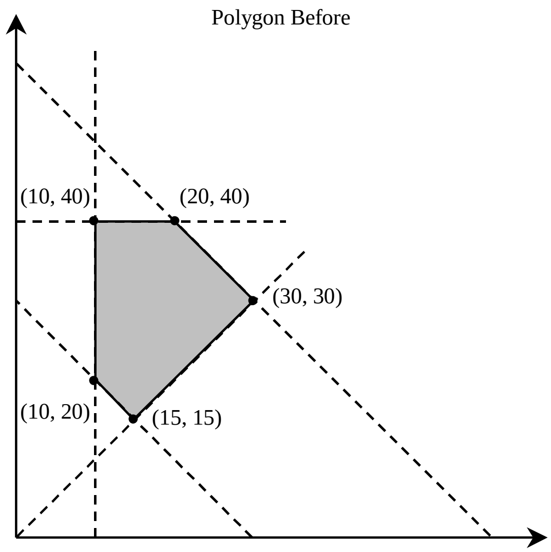
Answer: The difference in the maximum profit is $**8**.

Note: Students who use an appropriate method in order to determine the constraints, graph the polygon and find the original corner points have shown they have a partial understanding of the problem.

11 Example of an appropriate solution

x: number of hours at first job per month

y: number of hours at second job per month

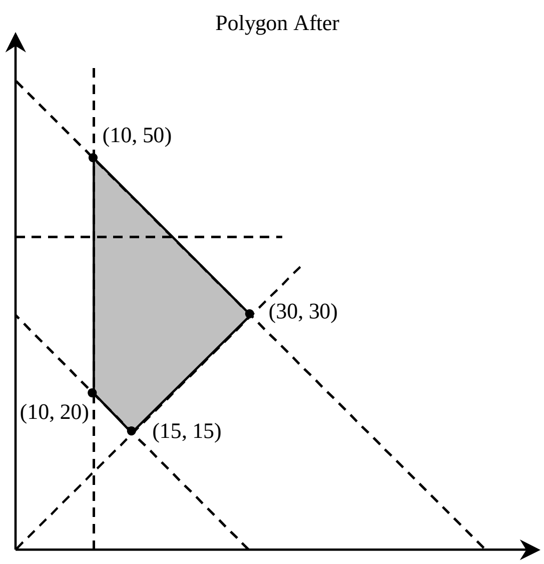


Constraints before

$$x \geq 10$$
$$y \leq 40$$
$$y \geq 0$$
$$x + y \geq 30$$
$$x + y \leq 60$$
$$y \geq x$$

Constraints after

$$x \geq 10$$
$$y \geq 0$$
$$x + y \geq 30$$
$$x + y \leq 60$$
$$y \geq x$$



Maximum Before

Vertices	S = 6.3x + 8y ($)
A(10, 40)	383
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
E(20, 40)	446

Maximum After

Vertices	S = 6.3x + 8y (\$)
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
F(10, 50)	463

Difference in maximum salary

$463 – $446 = $17

Answer: Murray's maximum possible salary increased by $17

Note: Students who use an appropriate method in order to determine **the constraints, graph the original polygon, and find its vertices** have shown they have a partial understanding of the problem.

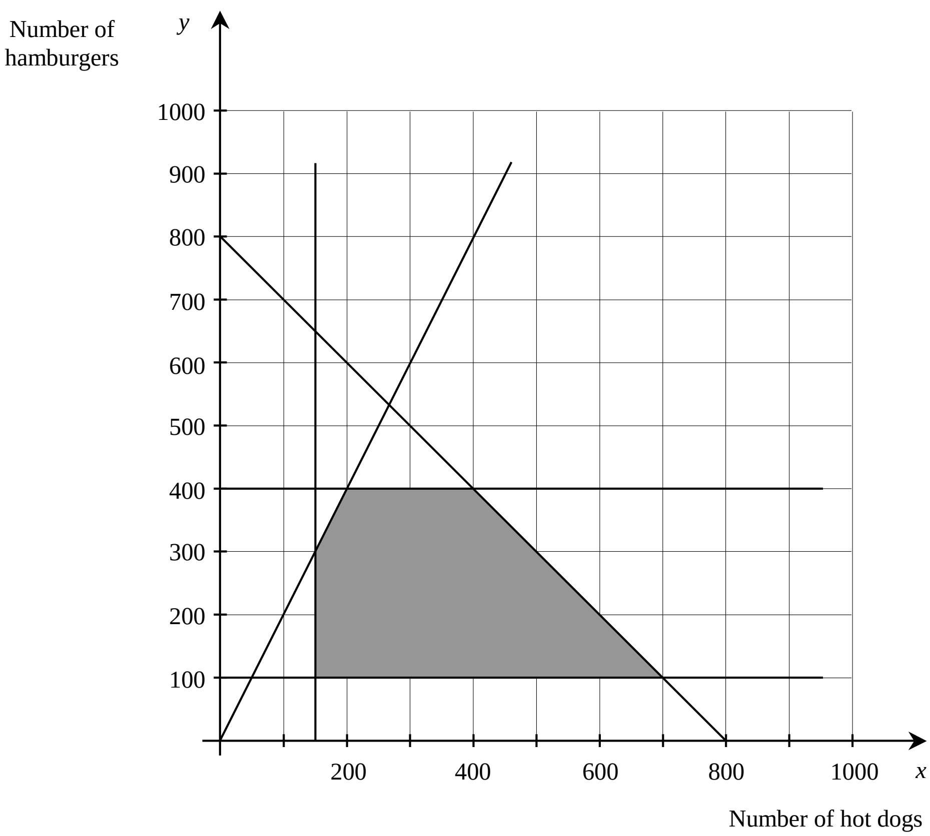
12 Example of an appropriate solution

x: number of hot dogs

y: number of hamburgers

Constraints

$$\begin{aligned} x + y &\leq 800 \\ x &\geq 150 \\ y &\geq 100 \\ y &\leq 400 \\ y &\leq 2x \end{aligned}$$



Points	P = 0.55x + 0.75y
(150, 100)	157.5

(150, 300)	307.5
(200, 400)	410
(400, 400)	520
(700, 100)	460

Answer: Kim needs to sell **400** hot dogs and **400** hamburgers to make the greatest profit.

Note: Students who have found **the constraints** have shown they have a partial understanding of the problem.

13 Example of an appropriate solution

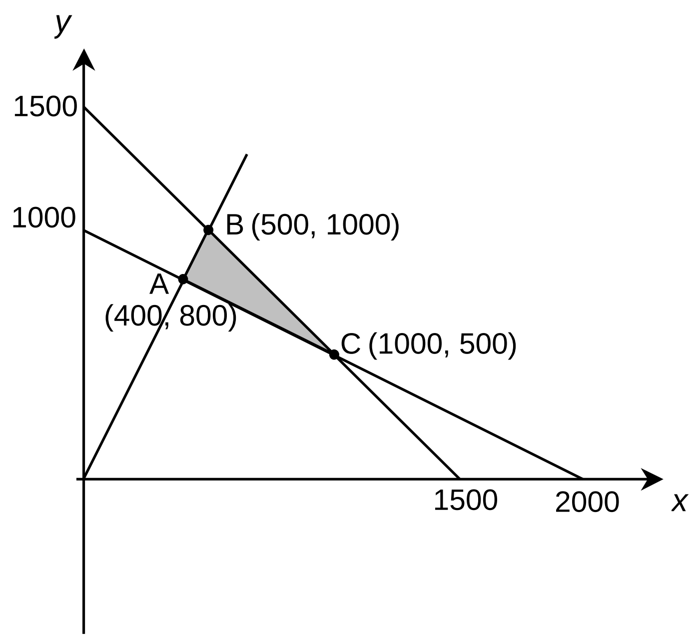
Constraints:

$$y \le 2x$$
$$x + y \le 1500$$
$$5x + 10y \ge 10\,000$$
$$x \ge 0$$
$$y \ge 0$$

Objective function: maximize profit

P = $30x + 40y$

Vertices	P = $30x + 40y$
A (400, 800)	$ 44 000
B (500, 1000)	$ 55 000
C (1000, 500)	$ 50 000



Answer: The maximum profit the company can make is **$55 000**.

Note: Deduct 1 mark if students use P = $35x + 50y$

Students who have the inequalities and the correct graph have shown they have a partial understanding of the problem.

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Example of an appropriate solution:

Boundary lines for the polygon of constraints:

①	②	③	④
$x = 25$	$x + y = 32$	$x + 4y = 68$	$x = \frac{1}{2} y$

②

x	y
32	0
0	32

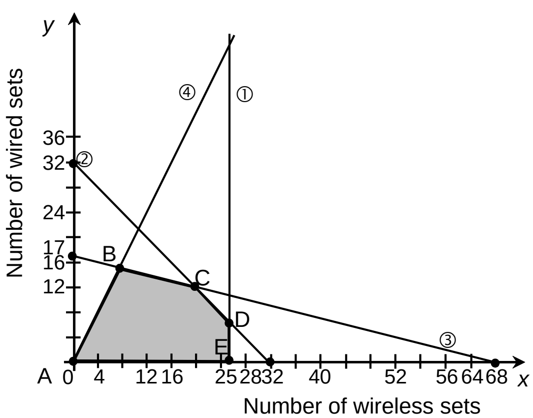
③

x	y
68	0
0	17

④

x	y
0	0
8	16

Polygon of constraints:



The vertices of the polygon of constraints:

A: origin B: ③ and ④ C: ② and ③ D: ① and ② E: on x-axis

$x = 25$

A (0, 0) $x + y = 32$ E (25, 0)

$$x = \frac{1}{2}y$$
$$x + 4y = 68$$

$$\frac{1}{2}y + 4y = 68$$
$$\frac{9}{2}y = 68$$
$$y = \frac{136}{9}$$
$$y \approx 15.1$$

$$x = \frac{1}{2}\left(\frac{136}{9}\right)$$
$$x = \frac{68}{9}$$
$$x \approx 7.6$$

B (7.6, 15.1)

$$x + y = 32$$
$$x + 4y = 68$$

$$3y = 36$$
$$y = 12$$
$$x + 12 = 32$$
$$x = 20$$

C (20, 12)

25 + y = 32

y = 7

D (25, 7)

Projected profit at each vertex:

Vertices	20x + 10y = Profit	
A (0, 0)	20 (0) + 10 (0) =	0
B (7.6, 15.1)	20 (7.6) + 10 (15.1) =	503
C (20, 12)	20 (20) + 10 (12) =	520
D (25, 7)	20 (25) + 10 (7) =	570
E (25, 0)	20 (25) + 10 (0) =	500

Answer: The maximum profit the company can earn is **$570**.