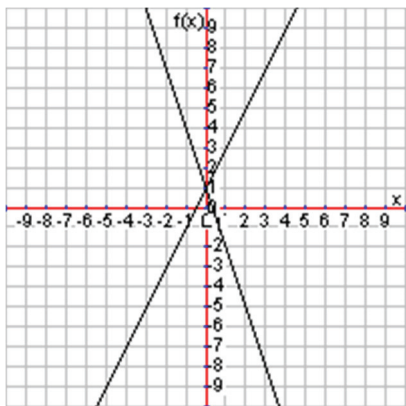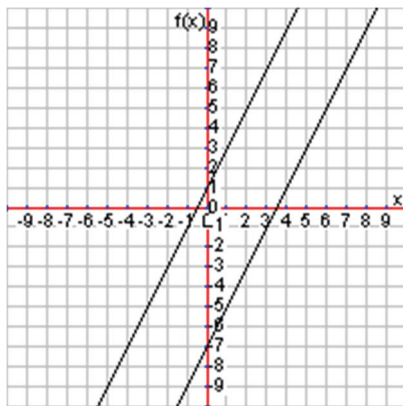# 6.5 Solving Systems by Inspection

What makes a system of linear equations have a single solution, no solutions, or infinite solutions? One of the first representations we looked at for systems was the graphical representation. What is true about the following systems of linear equations that have either infinite or no solution?
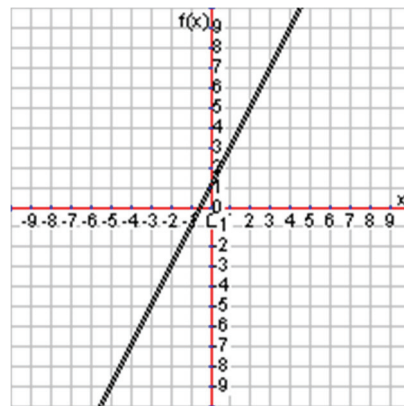

One solution
$$y = 2x + 1$$
$$y = -3x + 1$$

No solutions
$$y = 2x + 1$$
$$y = 2x - 7$$

Infinite solutions
$$y = 2x + 1$$
$$y = 2x + 1$$

Notice that the systems with no solutions and infinite solutions both have the same slope. In other words, the lines are parallel. If those parallel lines have different y-intercepts, then there are no solutions to the system. If those parallel lines are in fact the same exact line (same slope and same y-intercept), then there are infinite solutions. The lines are sitting right on top of each other. Therefore if we could quickly determine whether two lines have the same slope, we could know if it will have infinite or no solutions.

Standard Form

If the two equations are given in slope-intercept form, then we can readily see the slope and y-intercept. Same slope and different intercept would mean no solution. Same slope and same intercept would mean infinite solutions. However, not all equations are given in slope-intercept form. Another common form of a linear equation is called **standard form**, which is: $Ax + By = C$.

Consider the following system of equations given in standard form. We can't readily see the slope or y-intercept since they are both in standard form.

$$2x + y = 5$$

$$4x + 2y = 10$$

So how can we find the slope? We could solve each equation for y, but this method is called inspection. We're looking for a quicker way. Let's get the second equation in slope-intercept form and see if we can find any patterns of where the slope comes from.

$$4x + 2y = 10$$

$$4x + 2y - 4x = 10 - 4x$$

$$2y = -4x + 10$$

$$\frac{2y}{2} = \frac{-4x + 10}{2}$$

$$y = -2x + 5$$

Notice that we got the slope from dividing the coefficients of the variables. Specifically, if we started with the standard form equation $Ax + By = C$, we took $-A$ divided by B. In other words, we can simply look at the ratio of the coefficients in each equation. If they are the same, then the lines will have the same slope meaning it will definitely either have no solutions or infinite solutions. Look at the original system again:

$$2x + y = 5$$

$$4x + 2y = 10$$

Notice that the ratio of the coefficients, $\frac{-A}{B}$, for both equations is equal: $\frac{-2}{1} = \frac{-4}{2} = -2$. That means there are either no solutions or infinite solutions. The y-intercept will tell us which one, but remember that if they two equations are the exact same, there will be infinite solutions. Otherwise it will be no solutions.

If we divided the second equation by 2 on both sides we would get the first equation. Since the two equations would be the same, any point on the line represented by the first equation would be on the line of the second equation. That means we know there are infinite solutions and didn't have to do any work at all.

Now consider the following system. How many solutions are there?

$$3x - 2y = 5$$

$$2x - 3y = 5$$

Check the ratios of the coefficients. Notice that $\frac{-3}{-2} \neq \frac{-2}{-3}$ which means that the lines are not parallel. That tells us there is one solution, and we should use graphing, substitution, or elimination to find the solution.

Solution Steps

In essence, we follow these steps if the equations are not in slope-intercept form:

1) Make sure both equations are in standard form and check if the ratio of the coefficients are equal

    a. If the ratios are not equal, there is a single solution, and you need to solve.

    b. If the ratios are equal, then check if you can make the equations exactly the same.

        i. If the equations can be made the same, there are infinite solutions.

        ii. If the equations cannot be made the same, there are no solutions.

Decide if the following systems of equations have a single solution, no solutions, or infinite solutions. If it has a solution, solve the system.

1. $x + y = 1$
   $x + y = 5$

2. $2x + 3y = 7$
   $4x + 5y = 13$

3. $\frac{1}{2}x + 3y = 1$
   $x + 6y = 2$

4. $x + \frac{1}{3}y = -10$
   $3x + y = 30$

5. $2y = 6$
   $3(x + y) = 12$

6. $x + y = 2$
   $3x + 3y = 6$

7. $x + 5y = 9$
   $x + 5y = 6$

8. $2y = 5$
   $4y = 15$

9. $x + \frac{3}{5}y = 2$
   $y = -2x + 3$

10. $3x + y = 10$
    $y - 10 = -3x$

11. $3x + y = 5$
    $y = -3x + 5$

12. $6x + 4y = 10$
    $3y - 10 = -7x$

13. $2x + y = 4$
    $y - 5 = -2x$

14. $5x - 4y = 3$
    $5x = 4y - 3$

15. $7x + 5y = 3$
    $5y - 3 = -7x$

16. $\frac{2}{3}x - y = 0$
    $2x = 3y$

17. $4x = 4$
    $2x + 2y = 4$

18. $x = 2$
    $2(x + y) = 4$

19. $x + 4y = 2$
    $2(x + 4y) = 10$

20. $10x = 10 - 2y$
    $5x + y = 5$

Write a system of equations for each situation and solve using inspection.

21. The sum of two numbers is 100. Twice the first number plus twice the second number is 200. What are the numbers?

22. The perimeter of a rectangle is 40 in. Twice the length of the rectangle is 20 minus twice the width. What are the length and width?

23. Coffee worth $2.95 a pound was mixed with coffee worth $3.50 a pound to produce a blend worth $3.30 a pound. How much of each kind of coffee was used to produce 44 pounds of blended coffee?

24. Jeri has a total of 40 pets with a total of 160 legs. If she owns only cats and dogs, how many of each does she have?

25. Pam's age plus Tom's age is 65. Twice Pam's age is equal to 130 minus twice Tom's age. How old are they?

26. The sum of two numbers is 50. Three times the first number minus three times the second number is 30. What are the numbers?

27. The perimeter of a rectangle is 30 cm. Four times the length of the rectangle is equal to 120 minus four times the width. What are the length and width?

28. A customer bought six cups of coffee and four bagels and paid $10. Another customer bought three cups of coffee and two bagels and paid $15. How much are each cup of coffee and each bagel?

29. A family went to Six Flags and bought two adult tickets and five child tickets and paid $160. A second family bought two adult tickets and eight child tickets and paid $220. How much is each adult ticket and each child ticket?

30. Jorge bought two T-shirts and four hoodies for the CMS Student Council for $80. Xavier bought one T-shirt and two hoodies for $40. How much is each T-shirt and each hoodie?