Chapter 6: Work, Power, Energy

What is work (or work done)?

Energy transfered to an object by a Force applied over a distance.
* Work is energy.

- Symbol: W
  Work is a scalar. (no direction)

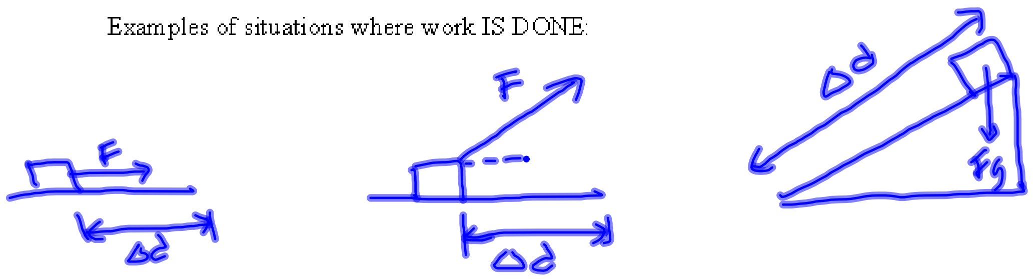
- Units of work: Joules (J)

  Note: $1J = 1\frac{kg \cdot m^2}{s^2}$

Mechanical work is done when the force and the displacement are parallel. both horizontal " vertical " diagonal

When there is an angle (other than 90 °) between the force and the displacement, some work is done.

Examples of situations where work IS DONE:



NO mechanical work is done when the force and the displacement are perpendicular.

Examples of situations where NO WORK IS DONE:

Formula for calculating work:

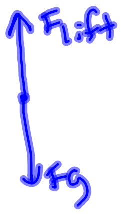
$$\boxed{W = F \cdot \Delta d}$$

F can be the component of the force parallel to Δd.

If there is an angle between the force and the displacement, we must find the component of the force that is parallel to the displacement. (Or we can find the component of the displacement parallel to the force.)

Examples:

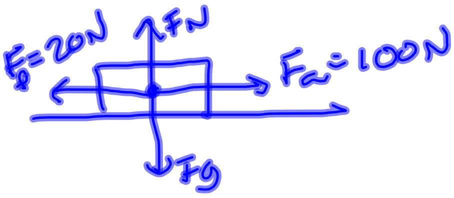
1. A worker lifts a 20 kg box from the floor and places it on a shelf 1.5 m above the ground. How much work does the worker do to accomplish this task?



$F_g = mg$
$= (20\,kg)(9.8\,\frac{m}{s^2})$
$= 196\,N \rightarrow F_{lift}$

$W = F \cdot \Delta d$
$= 196\,N \times 1.5\,m$
$= 294\,J$

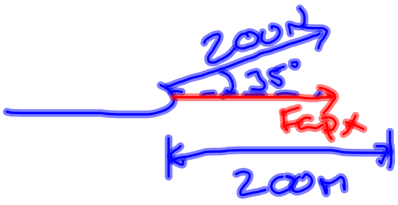
2. A boy applies a horizontal force of 100 N (to the right) to a 40 kg box that is on the ground. Friction exerts a force of 20 N. How much work does the boy do to push the box over a distance of 5.0 m?



* We do not take friction into account to calculate work.

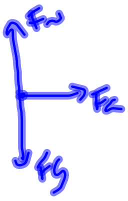
$W = F \cdot \Delta d$
$= 100\,N \cdot 5.0\,m$
$= 500\,J$

3. A girl pulls a sled with a force of 200 N at an angle of 35 ° above the horizontal. She pulls the sled over a distance of 200 m. How much work does the girl do?



$F_{apx} = 200N \cos 35°$
$= 163.8 N$

$W = F_{apx} \times \Delta d$
$= 163.8N \times 200N$
$= 3.28 \times 10^4 J$
$= 32.8 \text{ kJ}$



4. A boy applies a horizontal force of 10 N to a 5.0 kg toy car moving at a speed of 1.0 m/s for 2.0 s. How much work is done by this boy? (Frictionless)

① $F_{net} = F_a$
$= 10N$

② Find a
$F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{10N}{5.0kg}$
$= 2.0 m/s^2$

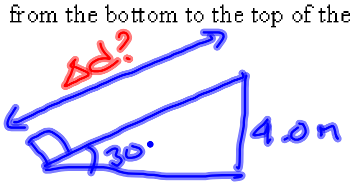
③ Find Δd
$v_i = 1.0 m/s$
$\Delta t = 2.0 s$
$a = 2.0 m/s^2$
$\Delta d = ?$

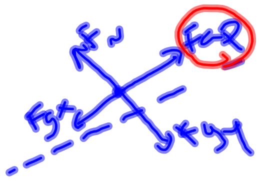
$\Delta d = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$
$= (1.0 \frac{m}{s})(2.0s) + \frac{1}{2}(2.0 \frac{m}{s^2})(2.0s)^2$
$= 6.0 m$

④ $W = F \cdot \Delta d$
$= 10N \times 6.0m$
$= 60J$

490N

5. A 50 kg box is to be brought to the top of a frictionless incline plane. The incline is set at 30° and is 4.0 m high. Find how much work is needed to get this box from the bottom to the top of the incline.





Diagonal

① $F_{ap} = F_{gx}$

$= 490N \sin 30°$

$= 245N$

② Find Δd

$\sin 30° = \dfrac{4.0m}{\Delta d}$

$\Delta d = \dfrac{4.0m}{\sin 30°}$

$= 8.0m$

③ $W = F \cdot \Delta d$

$= 245N \times 8.0m$

$= 1960J$

For a frictionless incline, we can use the vertical F and Δd.

$W = F \cdot \Delta d$

$= F_g \times (\text{height})$

$= 490N \times 4.0m$

$= 1960J$

Same because frictionless

Mechanical Power

What is power?

rate at which energy is transfered
" " " work is done

- Symbol: P
  Power is a scalar.
- Units of Power: Watts (W)
  Note: $1W = 1\frac{J}{s}$

Formula

$$\boxed{P = \frac{W}{\Delta t}}$$ → Work (J) → time (s)

Examples:

1. A winch is used to raise a 50 kg box to a height of 10 m above the ground in 20 seconds at a constant velocity. What is the power of the winch?

F_{ap} ↑ ↓ F_g

① $F_g = 50kg \times 9.8\frac{m}{s^2}$
$= 490N$

② $W = F \cdot \Delta d$
$= 490N \times 10m$
$= 4900J$

③ $P = \frac{W}{\Delta t}$
$= \frac{4900J}{20s}$
$= 245W$

2. A girl pushes a box on the floor at a constant velocity of 1.5 m/s. She exerts a horizontal force of 100 N over a distance of 12 m. What is the power generated by the girl?

F=100N →

① Find Δt
$v = \frac{\Delta d}{\Delta t}$
$\Delta t = \frac{\Delta d}{v}$
$= \frac{12m}{1.5m/s}$
$= 8.0s$

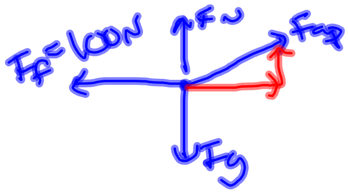
② $W = F \cdot \Delta d$
$= (100N)(12m)$
$= 1200J$

③ $P = \frac{W}{\Delta t}$
$= \frac{1200J}{8.0s}$
$= 150W$

$* P = \frac{F \cdot \Delta d}{\Delta t}$

$\boxed{P = F \times v}$ for constant velocity

3. A boy pulls a 20 kg sled using a force of 200 N at an angle of 30° above the horizontal. Friction provides a force of 100 N. The sled starts from rest, and the covers a distance of 8.0 m. What is the power generated by the boy?



$F_{apx} = 200\,N \cos 30°$
$= 173\,N$

① $W = F \Delta d$
$= 173\,N \times 8.0\,m$
$= 1384\,J$

② F_{net}?
$F_{net} = F_{apx} - F_f$
$= 173\,N - 100\,N$
$= 73\,N$

③ $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{73\,N}{20\,kg}$
$= 3.65\,m/s^2$

④ $\Delta d = 8.0\,m$
$a = 3.65\,m/s^2$
$v_i = 0$
$\Delta t = ?$

$\Delta d = \cancel{v_i \Delta t}^{\,0} + \frac{1}{2} a (\Delta t)^2$

$\Delta t = \sqrt{\frac{2 \Delta d}{a}}$
$= \sqrt{\frac{2(8.0\,m)}{3.65\,m/s^2}}$
$= 2.1\,s$

⑤ $P = \frac{W}{\Delta t}$
$= \frac{1384\,J}{2.1\,s}$
$= \underline{\underline{659\,W}}$

Mechanical Energy

Mechanical energy is composed of kinetic energy and gravitational potential energy.

Gravitational Potential Energy

Potential energy is the amount of energy associated with the position (height) of an object with respect to a reference point.

↳ height that corresponds to 0 potential energy (choose it)

- Symbol: PE or E_p
  Energy is a scalar.

- Units of Energy: Joules (J)

  Note: $1J = 1\frac{kgm^2}{s^2}$

- Formula: $\boxed{PE = mgh}$

m : mass (kg)
g : 9.8 m/s²
h : height above "zero" (m)

Kinetic Energy

Kinetic energy is the amount of energy associated with the motion of an object.

- Symbol: KE or E_k
  Energy is a scalar.

- Units of Energy: Joules (J)

  Note: $1J = 1\frac{kgm^2}{s^2}$

- Formula: $\boxed{KE = \frac{1}{2}mv^2}$

m : mass (kg)
v : velocity or speed (m/s)
*instantaneous

Examples:

1. A 2500 kg car travels a speed of 60 km/h. How much kinetic energy does this car have?

$$\frac{60\,km}{h} \times \frac{1000\,m}{1\,km} \times \frac{1\,h}{3600\,s} = 16.67\,m/s$$

$$KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(2500\,kg)(16.67\,m/s)^2$$
$$= 3.47\times10^5\,J$$

2. A 500 g apple is in a tree. The apple has 12.25 J of potential energy relative to the ground. How high above the ground is the apple located?

$PE = mgh$

$h = \frac{PE}{mg}$

$= \frac{12.25\,J}{(0.5\,kg)(9.8\,\frac{m}{s^2})}$

$= 2.5\,m$

$\frac{\cancel{kg}\,m^{\cancel{2}}}{\cancel{s^2}} \times \frac{1}{\cancel{kg}} \times \frac{\cancel{s^2}}{\cancel{m}}$

Conservation of Mechanical Energy

→ no external forces

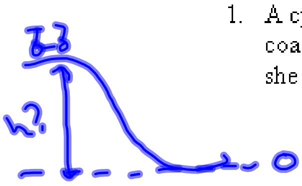
In a close (isolated) system, the total mechanical energy of system is constant.

This mean: $ME_i = ME_f$

$$\boxed{PE_i + KE_i = PE_f + KE_f}$$

Examples:

1. A cyclist and her bicycle have a combined mass of 65 kg. She starts from rest and coasts down the hill without pedaling. When she reaches the bottom of the hill, she has a speed of 12 m/s. What is the height of the hill?



$PE_i + \cancel{KE_i}^{\,0} = \cancel{PE_f}^{\,0} + KE_f$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$h = \frac{1}{2}\frac{v^2}{g}$

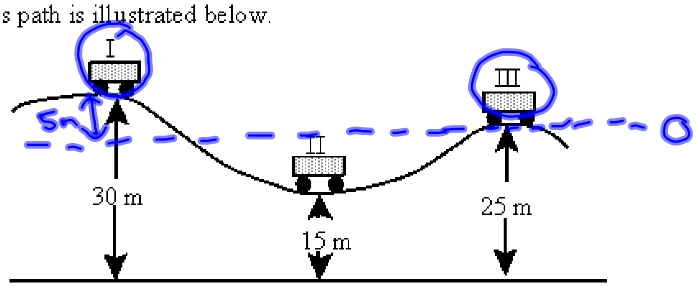
$= \frac{1}{2}\frac{(12\,m/s)^2}{9.8\,m/s^2}$

$= 7.3\,m$

$\frac{m^{\cancel{2}}}{\cancel{s^2}} \times \frac{\cancel{s^2}}{\cancel{m}}$

2. A roller coaster car passes through point I at a speed of 12 m/s and then keeps going, passing through points II and III.

The car's path is illustrated below.



Frictional forces are negligible. What is the speed of the cart at point III?

$$PE_i + KE_i = \cancel{PE_f}^{\,0} + KE_f$$

$$KE_f = PE_i + KE_i$$

$$\frac{1}{2}\cancel{m}v_f^2 = \cancel{m}gh + \frac{1}{2}\cancel{m}v_i^2$$

$$\frac{1}{2}v_f^2 = gh + \frac{1}{2}v_i^2$$

$$v_f^2 = 2\left(gh + \frac{1}{2}v_i^2\right)$$

$$= 2\left(9.8\frac{m}{s^2}(5m) + \frac{1}{2}(12m/s)^2\right)$$

$$v_f^2 = 242\,m^2/s^2$$

$$v_f = 16\,m/s$$

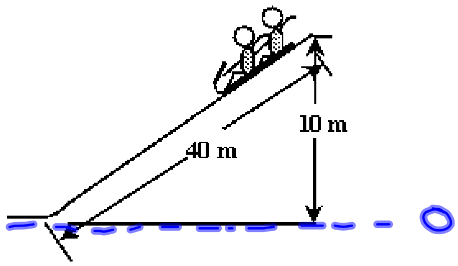
When there is friction on the system, total energy is still conserved. Friction does work to remove energy from the system.

So this means: $PE_i + KE_i - W_f = PE_f + KE_f$

$\uparrow W_f = F_f \cdot \Delta d$

Example:

1. Two persons slide down a snow- covered hill in a sled from a height of 10 m. The force of friction on the sledders is 200 N. The total mass of the persons and sled is 100 kg. The slope of the hill is 40 m long. Calculate the speed of the sled and its occupants at the bottom of the hill.



① $W_f = F_f \cdot \Delta d$
$= (200N) \cdot 40m$
$= 8000J$

② $PE_i + \cancel{KE_i}^{0} - W_f = \cancel{PE_f}^{0} + KE_f$

$KE_f = PE_i - W_f$
$= mgh - W_f$
$= (100kg)(9.8\frac{m}{s^2})(10m) - 8000J$ ← not 200 N
$= 1800J$

③ $KE_f = \frac{1}{2}mv^2$

$v^2 = \frac{2KE}{m}$
$= \frac{2(1800J)}{100kg}$

$v^2 = 36m^2/s^2$

$v = 6.0m/s$

Work done by external forces

We saw that when a force of friction does work, it removes energy from the system.

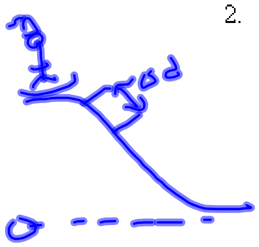
When a force is applied to a system, it can ADD energy to the system.

So the equation would look something like:

$PE_i + KE_i + W_{Fapp} - W_f = PE_f + KE_f$

Example:

2. A 65.0 kg skier is at rest at the top of a hill 10.0 m high. She pushes with her poles with a force of 50.0 N, to give herself an initial speed. She then coasts down the rest of the hill. When she reaches the bottom of the hill, she has a speed of 14.1 m/s. Over what distance did the skier push with her poles? Disregard the effects of friction.



① $PE_i + \cancel{KE_i}^{0} + W_{ap} = \cancel{PE_f}^{0} + KE_f$

$W_{ap} = KE_f - PE_i$

$= \frac{1}{2} m v_f^2 - mgh_i$

$= \frac{1}{2}(65kg)(14.1\frac{m}{s})^2 - (65kg)(9.80\frac{m}{s^2})(10m)$

$= 91.33J$

② $W_{ap} = F_{ap} \cdot \Delta d$

$\Delta d = \frac{W_{ap}}{F_{ap}}$

$= \frac{91.33J}{50.0N}$

$= 1,83m$

Work Energy Theorem

When work is done on an object, it changes the kinetic energy of the object.

So the total work done on an object corresponds to the change in kinetic energy of that object.

This means: $W_{total} = \Delta KE$

$W_{total} = KE_f - KE_i$

$$\cancel{PE_i}^{0} + KE_i + W_{ap} - W_f = \cancel{PE_f}^{0} + KE_f$$

$$W_{ap} - W_f = KE_f - KE_i$$

Examples:

3. A 2000 kg car going at speed of 25 m/s comes to a stop over a distance of 70 m. What is the force applied to the car by the brakes?

① $W = \cancel{KE_f}^{0} - KE_i$

$= -\frac{1}{2}mv^2$

$= -\frac{1}{2}(2000kg)(25\,m/s)^2$

$= -625\,000J$

② $W = F \cdot \Delta d$

$F = \frac{W}{\Delta d}$

$= \frac{-625\,000J}{70m}$

$F = -8929\,N$

the force is opposite to motion

4. A wagon starting from rest is pulled with a force of 25 N over a distance of 5.0 m. Friction exerts a force of 10 N. What is the final kinetic energy of the wagon?

$W_{ap} - W_f = KE_f - \cancel{KE_i}^{0}$

$KE_f = W_{ap} - W_f$

$= F_{ap}\Delta d - F_f\Delta d$

$= (25N)(5m) - (10N)(5m)$

$= 75J$

Energy in a Spring

When a spring is stretched or compressed, it stores elastic potential energy. When the spring is released, the extremity of the spring moves, transforming the elastic potential energy into kinetic energy.

- Symbol: E_s, E_e
  Energy stored in a spring is a scalar.
- Units of Energy: Joules (J)
  Note: $1J = 1\frac{kgm^2}{s^2}$
- Formula: $\boxed{E_s = \frac{1}{2}kx^2}$

k: spring constant (N/m)
x: deformation (m)

Examples:

1. In order to compress a spring by 40 cm, 48 J of work must be done. What is the spring constant of this spring?

$$E_s = \frac{1}{2}kx^2$$

$$k = \frac{2E_s}{x^2}$$

$$= \frac{2(48J)}{(0.40m)^2} = 600 N/m$$

2. A dart of mass 0.100 kg is pressed against a spring of a toy dart gun. The spring has a spring constant k = 250 N/m, and it is compressed by a distance of 0.06m. If the dart detaches from the spring once the spring reaches it's equilibrium position, what is the speed of the dart as it leaves the spring?

$$\cancel{PE_i}^{0} + \cancel{KE_i}^{0} + \cancel{W_p}^{0} - \cancel{W_f}^{0} + E_s = \cancel{PE_f}^{0} + KE_f$$

$$E_s = KE_f$$

$$\frac{1}{\cancel{2}}kx^2 = \frac{1}{\cancel{2}}mv^2$$

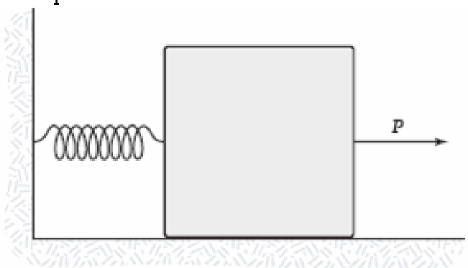
$$v^2 = \frac{kx^2}{m}$$

$$= \frac{(250N/m)(0.06m)^2}{0.100kg}$$

$$= 9 m^2/s^2$$

$$v = 3.0 m/s^2$$

3. A 10-kg block on a horizontal frictionless surface is attached to a light spring of constant 800 N/m. The block is initially at rest at the spring's equilibrium position when a force (magnitude $P = 80$ N) acting parallel to the surface is applied to the block, as shown. What is the speed of the block when it is 13 cm from its equilibrium position?



$$\cancel{KE_i}^{0} + W_{ap} - \cancel{W_f}^{0} = KE_f + E_s$$

$$W_{ap} = KE_f + E_s$$

$$KE_f = W_{ap} - E_s$$

$$= F \cdot \Delta d - \frac{1}{2}kx^2$$

$$= (80N)(0.13m) - \frac{1}{2}(800 N/m)(0.13m)^2$$

$$= 3.64J$$

$$KE = \frac{1}{2}mv^2$$

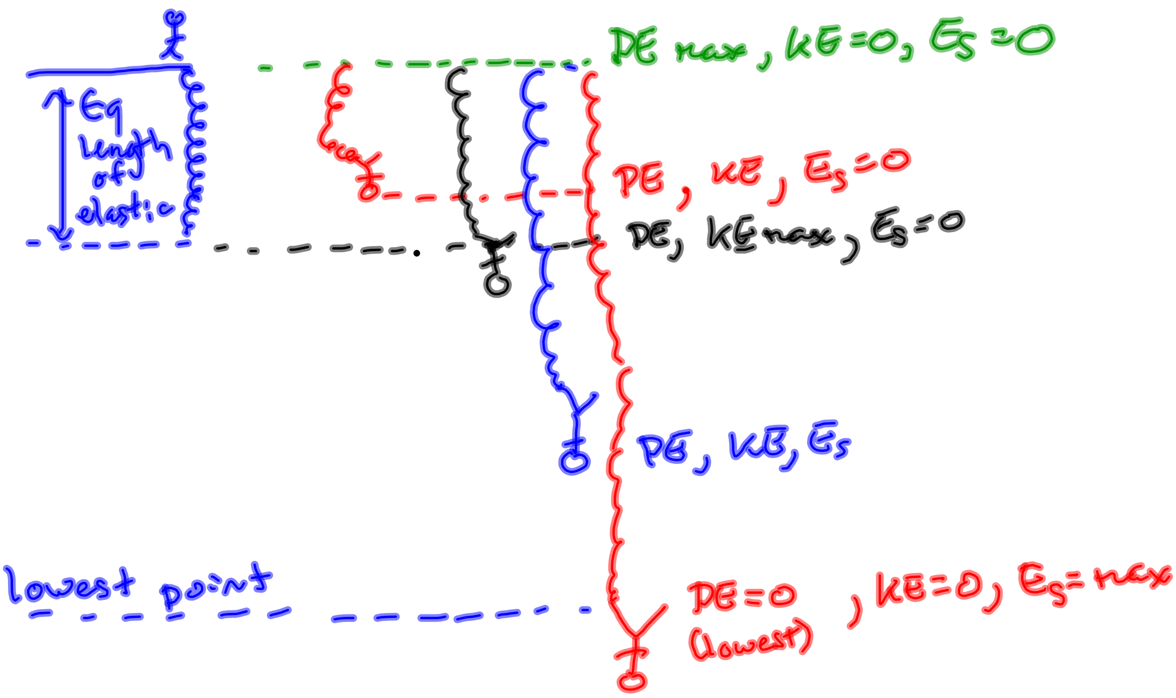
$$v^2 = \frac{2KE}{m}$$

$$= \frac{2(3.64J)}{10kg}$$

$$v^2 = 0.728 m^2/s^2$$

$$v = 0.85 m/s$$

A little something about the bungee…



When the person comes to rest:

Equilibrium of force

$F_s = F_g$