Chapter 7: Momentum and Impulse

What is momentum?

Measure of the quantity of motion

- Symbol: $\vec{p}$
  ~~Work~~ Momentum is a vector.
- Formula for momentum: $\boxed{\vec{p} = m\vec{v}}$

m : mass (kg)
v : velocity (m/s)
↳ direction matters

- Units of momentum: $kg \cdot \frac{m}{s}$ or $N \cdot s$

$kg\frac{m}{s^{\cancel{2}}} \cdot \cancel{s}$

Example: What is the momentum of a 10 kg dog running at a speed of 4.0 m/s?

$\vec{p} = m\vec{v}$
$= (10kg)(4.0m/s)$
$= 40 kg\frac{m}{s}$

What is impulse?

Impulse is the change in momentum.

Impulse is provided when a force (net) is applied to an object for a certain amount of time.

↳ changes the velocity

Formulas relating impulse and momentum:

Impulse $= \Delta\vec{p}$

Impulse $= \vec{F} \cdot \Delta t$

$\vec{F} \cdot \Delta t =$

$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$

$\vec{F} \cdot \Delta t = m\vec{v}_f - m\vec{v}_i$

$F\Delta t = mv_f - mv_i$

$F\Delta t = m(v_f - v_i)$

$F = m\frac{(v_f - v_i)}{\Delta t}$

$F = ma$

Examples:

1. In which of the situations below is the greatest impulse provided?

   a. $\xrightarrow{} +$

   2 m/s →
   1.0 kg Clay — Stops at wall

   $$\text{Impulse} = m\cancel{v_f}^{\,0} - m v_i = -(1.0\,\text{kg})(2\,\text{m/s}) = -2.0\ \text{kg}\,\frac{\text{m}}{\text{s}}$$

   b. $\xrightarrow{} +$

   2 m/s
   1.0 kg Tennis ball — bounce back at 0.75 m/s

   $$\text{Impulse} = m v_f - m v_i = (1\,\text{kg})(-0.75\,\tfrac{\text{m}}{\text{s}}) - (1\,\text{kg})(2\,\tfrac{\text{m}}{\text{s}}) = -2.75\ \text{kg}\,\frac{\text{m}}{\text{s}}$$

2. A pitcher throws a 300 g baseball at a horizontal speed of 40 m/s. The batter hits the ball, exerting a force of 2700 N. The ball leaves the bat at a horizontal speed of 50 m/s in the opposite direction. For how long was the bat in contact with the ball?

   $\xrightarrow{} +$
   $v_i = 40\,\text{m/s}$ →
   ← $v_f = 50\,\text{m/s}$
   ← F

   $$F\Delta t = m v_f - m v_i$$

   $$\Delta t = \frac{m v_f - m v_i}{F}$$

   $$\Delta t = \frac{(0.3\,\text{kg})(-50\,\text{m/s}) - (0.3\,\text{kg})(40\,\text{m/s})}{-2700\,\text{N}} = \underline{\underline{0.01\,\text{s}}}$$

3. What impulse is provided to

   a. A 2.0 kg cart accelerated from rest to 5.0 m/s?

   $$\text{Impulse} = m v_f - \cancel{m v_i}^{\,0} = (2.0\,\text{kg})(5.0\,\text{m/s}) = 10\ \text{kg}\,\frac{\text{m}}{\text{s}}$$

   b. A box pushed for 8.0 s with a force of 100 N?

   $$\text{Impulse} = F\cdot\Delta t = (100\,\text{N})(8.0\,\text{s}) = 800\ \text{kg}\,\frac{\text{m}}{\text{s}}$$

Collisions and Explosions

Collision: two objects that come together

Explosion: two objects that start off together and then come apart

Momentum is always conserved in a collision. ↳at rest ↳and explosions

Energy is not always conserved.

- When energy is conserved in a collision, we say that the collision is elastic.
- When energy is not conserved in a collision, we say that the collision is inelastic. This occurs when two objects get stuck together or when energy is lost in deformation.

ex: Car crash is always inelastic

For two objects (A and B) are involved in a collision momentum is conserved.

total momentum before collision = total momentum after collision

$$\vec{p}_{A_i} + \vec{p}_{B_i} = \vec{p}_{A_f} + \vec{p}_{B_f}$$

$$\vec{p}_A + \vec{p}_B = \vec{p}_A\,' + \vec{p}_B\,'$$

$$\boxed{m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A\,' + m_B\vec{v}_B\,'}$$

* After the collision, A and B are stuck together. $v_A' = v_B' = v'$

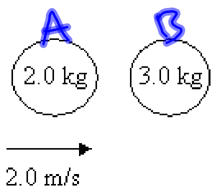
$$\boxed{m_A\vec{v}_A + m_B\vec{v}_B = (m_A+m_B)v'}$$

* If A and B started together and then seperate, $v_A = v_B = v$

$$\boxed{(m_A+m_B)v = m_A v_A' + m_B v_B'}$$

Examples:

1.



Two marbles are involved in a collision. The 3.0 kg Marble starts at rest. After the collision, both marbles travel together. What is the final velocity of both marbles?

$$m_A v_A + \cancel{m_B v_B}^{\,0} = (m_A + m_B) v'$$

$$v' = \frac{m_A v_A}{(m_A + m_B)}$$

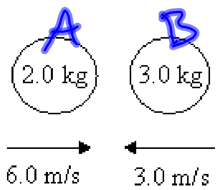
$$= \frac{(2.0\,kg)(2.0\,m/s)}{(2.0\,kg + 3.0\,kg)}$$

$$\frac{kg\,\frac{m}{s}}{kg}$$

$$= 0.80\,m/s$$

$\longrightarrow +$

2.



Two marbles are involved in a head on collision. After the collision, the 2.0 kg marble moves to the left at a speed of 1.0 m/s. What is the final velocity of the 3.0 kg?

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$m_A v_A + m_B v_B - m_A v_A' = m_B v_B'$$

$$\frac{m_A v_A + m_B v_B - m_A v_A'}{m_B} = v_B'$$

$$v_B' = \frac{(2.0\,kg)(6.0\,m/s) + (3.0\,kg)(-3.0\,m/s) - (2.0\,kg)(-1.0\,m/s)}{3.0\,kg}$$

$$= 1.7\,m/s$$

(1.7 m/s to Right)

Word problems

Examples:

$\xrightarrow{} +$

1. A 3.5 kg toy truck (A) is moving to the right at 2.7 m/s when a 0.5 kg piece of clay (B) is thrown to the right at 4.5 m/s. If the clay sticks to the truck, what is the final velocity of the system after the collision?

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

$$v' = \frac{m_A v_A + m_B v_B}{(m_A + m_B)}$$

$$= \frac{(3.5\,kg)(2.7\,m/s) + (0.5\,kg)(4.5\,m/s)}{(3.5\,kg + 0.5\,kg)}$$

$$v' = 2.9\ m/s$$

$\xrightarrow{} +$

2. ~~Zorba~~ A Clifford is sitting at rest in the boat (B) ~~when he~~ decides to jump out. The 155 kg dog jumps out of the 65 kg boat with a velocity of 4.5 m/s to the right. What is the velocity of the boat after the jump?

$$m_A \cancel{v_A}^{0} + m_B \cancel{v_B}^{0} = m_A v_A' + m_B v_B'$$

$$-m_A v_A' = m_B v_B'$$

$$v_B' = \frac{-m_A v_A'}{m_B}$$

$$= \frac{-(155\,kg)(4.5\,m/s)}{65\,kg}$$

$$v_B' = -10.7\ m/s$$

(10.7 m/s to left)

$\xrightarrow{+}$

3. A loaded 300 kg cannon (A) at rest fires a 10 kg projectile (B) at a velocity of 200 m/s to the right. What is the velocity of the cannon as it recoils backwards?

$$\cancel{m_A v_A}^{0} + \cancel{m_B v_B}^{0} = m_A v_A' + m_B v_B'$$

$$v_A' = -\frac{m_B v_B'}{m_A}$$

$$= -\frac{(10\,kg)(200\,m/s)}{300\,kg}$$

$$= -6.7\,m/s$$

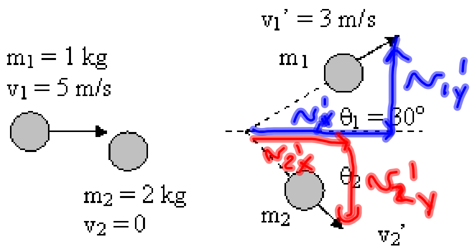
(6.7 m/s to left)

Momentum in 2 Dimensions

Remember that momentum is a vector. This allows us to solve collisions in 2 dimensions.

momentum in x-direction is conserved
" " y-direction " "

Examples: Consider the collision below. What is the speed and direction of m_2 after the collision?



$v_{1x}' = 3\frac{m}{s}\cos 30° = 2.60\frac{m}{s}$

$v_{1y}' = 3\frac{m}{s}\sin 30° = 1.50\frac{m}{s}$

x-direction

$$m_1 v_{1x} + \cancel{m_2 v_{2x}}^{\,0} = m_1 v_{1x}' + m_2 v_{2x}'$$

$$v_{2x}' = \frac{m_1 v_{1x} - m_1 v_{1x}'}{m_2}$$

$$= \frac{(1kg)(5m/s) - (1kg)(2.60m/s)}{2kg}$$

$$\boxed{v_{2x}' = 1.2 m/s}$$

Ans: $1.4\frac{m}{s}$, $-32°$

y-direction

$$\cancel{m_1 v_{1y}}^{\,0} + \cancel{m_2 v_{2y}}^{\,0} = m_1 v_{1y}' + m_2 v_{2y}'$$

$$v_{2y}' = \frac{-m_1 v_{1y}'}{m_2}$$

$$= \frac{-(1kg)(1.5m/s)}{2kg}$$

$$\boxed{v_{2y}' = -0.75 m/s}$$

1.2 m/s

0.75 m/s

$mag = \sqrt{(1.2m/s)^2 + (0.75m/s)^2}$
$= 1.4 m/s$

$\theta = \tan^{-1}\left(\frac{0.75}{1.2}\right) = 32°$

→ or inelastic

Elastic Collisions

In an elastic collision, both momentum and kinetic energy are conserved.

Examples: A 3.0 kg mass travels to the right at a speed of 24 m/s. A 1.0 kg mass travels to the left at a speed of 12 m/s. The two masses collide in a perfectly elastic collision. What is the velocity of each mass after the collision?

A B

$v_A' = ?$ $v_B' = ?$

Momentum

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \text{eq } 1$$

Energy

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \quad \text{eq } 2$$

$$v_B' = \frac{m_A v_A + m_B v_B - m_A v_A'}{m_B}$$

$$v_B' = \frac{(3kg)(24 m/s) + (1kg)(-12 m/s) - (3kg) v_A'}{1kg}$$

$$\boxed{v_B' = 60 \frac{m}{s} - 3 v_A'}$$

$$(60 - 3x)^2$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B \left(60 \frac{m}{s} - 3 v_A'\right)^2$$

$$(3kg)(24 m/s)^2 + (1kg)(-12 m/s)^2 = (3kg) v_A'^2 + (1kg)\left(3600 \frac{m^2}{s^2} - 360 \frac{m}{s} v_A' + 9 v_A'^2\right)$$

$$1872 J = 3kg\, v_A'^2 + 3600 J - 360 kg \frac{m}{s} v_A' + 9 kg\, v_A'^2$$

$$0 = 12 kg\, v_A'^2 - 360 kg \frac{m}{s} v_A' + 1728 J$$

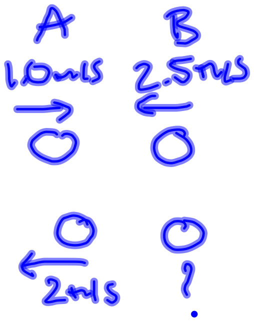
$$0 = v_A'^2 - 30 \frac{m}{s} v_A' + 144 \frac{m^2}{s^2}$$

$$0 = (v_A' - 6 m/s)(v_A' - 24 m/s)$$

$\boxed{v_A' = 6 m/s}$ or $v_A' = \cancel{24} m/s$ → implies no collision $v_A = v_A'$

$$v_B' = 60 \frac{m}{s} - 3(6 m/s)$$

$$\boxed{v_B' = 42 m/s}$$



2. Two identical billiard balls (each having a mass of 0.15 kg) are involved in a head on collision. One ball traveled to the left at a speed of 2.5 m/s while the other traveled at a speed of 1.0 m/s. After the collision, one of the balls travels at a speed of 2.0 m/s to the left. Is this collision elastic or inelastic?

Momentum

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$v_B' = \frac{m_A v_A + m_B v_B - m_A v_A'}{m_B}$$

$$= \frac{(0.15\,kg)(1.0\,m/s) + (0.15\,kg)(-2.5\,m/s) - (0.15\,kg)(-2.0\,m/s)}{0.15\,kg}$$

$$v_B' = 0.5\,m/s$$

Before collision

$$KE = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$= \frac{1}{2}(0.15\,kg)(1.0\,m/s)^2 + \frac{1}{2}(0.15\,kg)(-2.5\,m/s)^2$$

$$= 0.54\,J$$

After collision

$$KE = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$= \frac{1}{2}(0.15\,kg)(-2.0\,m/s)^2 + \frac{1}{2}(0.15\,kg)(0.5\,m/s)^2$$

$$= 0.32\,J$$

Inelastic (because $KE_i > KE_f$)

$\neq$

A little comment on elastic and inelastic collisions

If the problem does not say that the collision is elastic (or that no energy is lost during the collision) do not assume it is!