Dynamics

Chapter 5: Forces and Newton's Laws (Part I)

What is a force?

push or pull
forces affect the motion of objects

- Symbol: $\vec{F}$
  Force is a vector, it has a magnitude and a direction.
- Units of Force: Newtons (N)
  Note: $1N = 1\frac{kg \cdot m}{s^2}$
- To represent a force: $\longrightarrow$

Types of forces

1) **Gravitational Force**

force of attraction between any two bodies

Symbol: F_g

Formula: $\boxed{F_g = \frac{G m_1 m_2}{r^2}}$

G: gravitational constant ($6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$)
m_1: one body's mass (kg)
m_2: other body's mass (kg)
r: distance between the 2 bodies (m)

2) **Simplified Force of gravity (aka Weight)**

force due to gravity on objects near the surface of the earth

Symbol: F_g

Formula:

$\boxed{F_g = mg}$

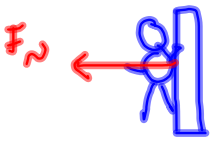
m : mass (kg)
g : $9.8 \frac{m}{s^2}$

→ perpendicular

3) **Normal force**

force exerted by a surface, perpendicular to that surface

Symbol: F_N (no formula) F_N is what it "needs" to be



leaning on the wall

4) **Force of friction**

force between 2 surfaces in contact (direction is opposite to motion)

Symbol: F_f

Formula:

$$\boxed{F_f = \mu F_N}$$

μ: coefficient of friction (no units)

$0 < \mu < 1$

There are two types of friction:

- Static Friction (not moving) μ_s
- Kinetic Friction (sliding) μ_k

$\mu_s > \mu_k$

5) **Tension**

force in a string/cable (as it is pulled)

Symbol: T or F_T (no formula)

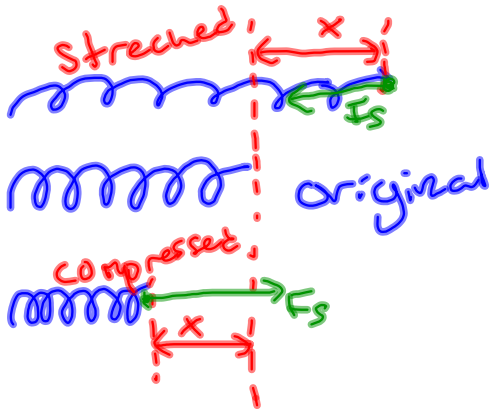
6) **Force in a spring**

force that tends to bring the spring back to its original length

Symbol: F_s

Formula:

$$\boxed{F_s = kx}$$



k: spring constant (N/m)

x: deformation (m)

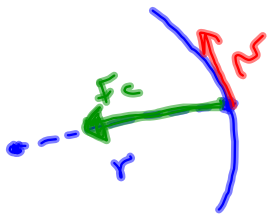
7) **Centripital Force**

force that allows an object to follow a circular path

Symbol: F_c

Formula:

$$\boxed{F_c = \frac{mv^2}{r}}$$



m: mass (kg)

v: linear speed (tangential)

r: radius of circular path

8) Applied force

Force applied by person/engine

Symbol: F_a

9) Net force

Vector sum of all forces on an object

Symbol: F_{net}

Free body diagrams

Free body diagrams (FBD) are used to show the forces acting on an object. You should always start problem solving by drawing a FBD.

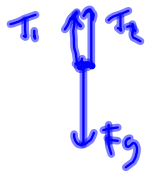
In a FDB, objects are represented by ·

Arrows starting from this dot are used to show the forces acting on the objects.

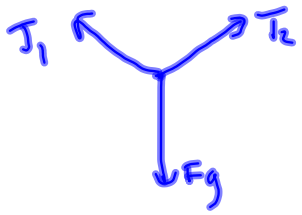
Examples: Draw the FBD for the situations below

1) A box rests on a table.

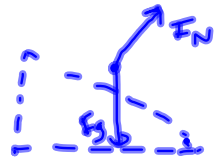
2) A sign is supported by 2 vertical cables.



3) A sign is supported by 2 angled cables.



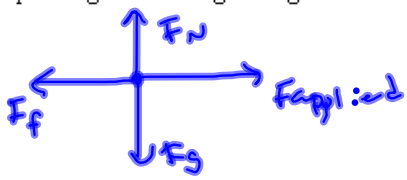
4) A box slides down a frictionless ramp.



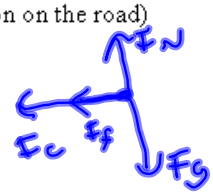
(F_N is $\perp$ to surface)

5) A box is resting on a spring.

6) A girl is pushing a box along a rough floor.



7) A car turning a sharp corner. (There is friction on the road)



Finding the net force

To find F_{net}, we find the vector sum of all the forces exerted on one object.

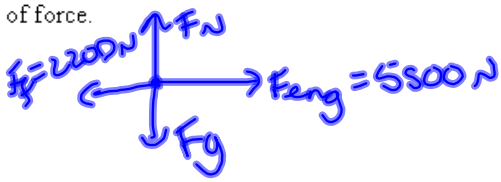
Examples: Find the net force for each situation below.

1) A 2.0 kg brick gets detached from the wall of a building. The brick falls towards the ground. (Ignore air resistance.)

$F_g = (2.0\,kg)(9.8\,N/kg)$
$= 19.6\,N$

$F_{net} = F_g$
$= 19.6\,N$
(down)

2) A car's engine provides 5500 N of force. Friction slows the car down with 2200 N of force.



$F_{net} = F_{eng} - F_f$
$= 5500\,N - 2200\,N$
$= 3300\,N$

$F_N = F_g$ (they cancel out)

↑ +

3) The engine of a 2000. kg rocket provides 85 700 N of force. Air resistance provides 12 400 N.

$F_g = mg = (2000\,kg)(9.8\,m/s^2) = 19\,600\,N$

$F_{net} = F_{eng} - F_{air} - F_g$
$= 85\,700\,N - 12\,400\,N - 19\,600\,N$
$= 53\,700\,N$

Equilibrium of forces

A system is said to be in equilibrium when $F_{net} = 0$.

When a system is in equilibrium,

- the object is not moving (at rest)
  OR
- the object is moving at a constant velocity

When $F_{net} = 0$, it means that

- all forces pushing the object "up" are equal to all forces pushing the object "down"
- all forces pushing the object "left" are equal to all forces pushing the object "right"

i.e. the vector sum of all forces is zero.

Knowing that a system is in equilibrium allows us to solve some problems.

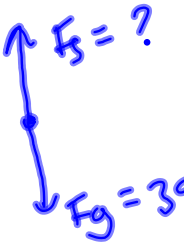
Examples:

1) A 50. kg box rests on a table. What is the normal force applied by the table?

$F_N = F_g$
$= 490\,N$

$F_g = 39.2\,N$

2) When you hang a 4.0 kg mass from a spring, the spring stretches by 10 cm. What is the spring constant of this spring?

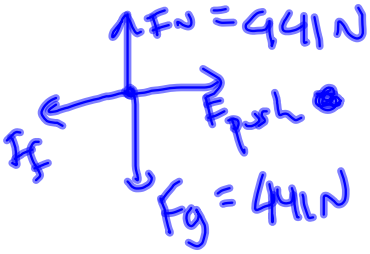


① $F_s = F_g$
$= 39.2\,N$

② $F_s = kx$
$k = \frac{F_s}{x} = \frac{39.2\,N}{0.10\,m} = 392\,N/m$

$F_g = 45\,kg \times 9.8\,\frac{m}{s^2} = 441\,N$

3) A 45 kg box is being pushed along a rough floor. The coefficient of friction between the box and the floor is 0.20. What is the force of friction?

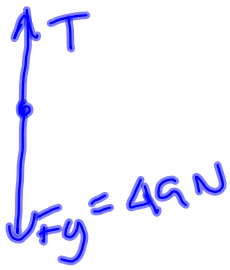


$F_f = \mu F_N$

$= (0.2)(441\,N)$

$= 88.2\,N$

4) A 5.0 kg light fixture is suspended from the ceiling using a cable. What is the tension in the cable?



$T = F_g$

$T = 49\,N$

Newton's Laws

1st Law: Law of inertia

When no net force acts on an object, the object remains at rest or continues moving at a constant velocity.

Examples: no net force → things continue doing what they were doing.

1) When you hit the brakes in a car, all objects that are not strapped down go flying forward. The objects continue moving at same velocity because no net force is acting on them.

2) Ketchup in a glass bottle!

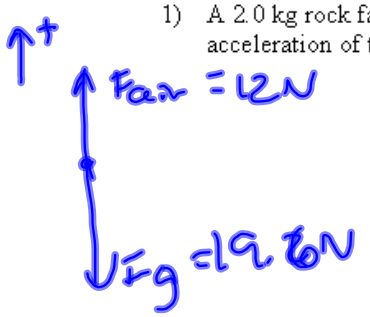
2nd Law: *Law of acceleration*

The acceleration of an object is directly proportional to the net force acting on the object, and inversely proportional to the mass of the object.

$$F_{net} = ma$$

m : mass (kg)
a : acceleration (m/s²)

1) A 2.0 kg rock falls from the edge of a cliff. Air resistance exerts 12 N. What is the acceleration of the rock?

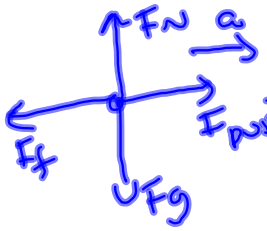


① $F_{net} = F_{air} - F_g$
$= 12N - 19.6N$
$= -7.6N$

$kg \frac{m}{s^2} \times \frac{1}{kg}$

② $F_{net} = ma$
$a = \frac{F_{net}}{m} = \frac{-7.6N}{2kg} = -3.8\ m/s^2$ down

2) A 70.0 kg box is pushed along a rough surface with a force of 452 N. The box accelerates at a rate of 0.500 m/s². What is the force of friction acting on the box?



① $F_{net} = ma$
$= (70kg)(0.500\ m/s^2)$
$= 35N$

② $F_{net} = F_{push} - F_f$
$F_f = F_{push} - F_{net}$
$= 452N - 35N$
$= 417N$

3rd Law: *Action -Reaction*

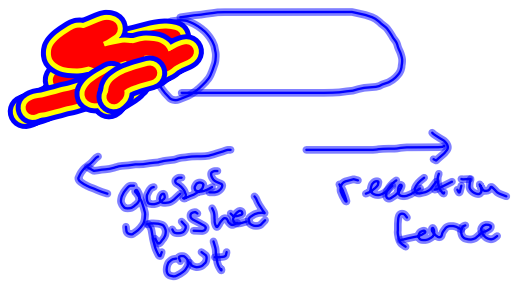
For every force applied to a object, the object applies a force of equal magnitude but opposite direction.

* For every action there's an equal and opposite reaction.

Examples:

1) You push the wall with 50 N, the wall pushed you back with 50 N

2) Jet engines.



Solving problems using Newton's Second Law.

Basic Idea:

There are 2 ways of finding the net force

- $F_{net} = ma$
- F_{net} = the vector sum of all forces

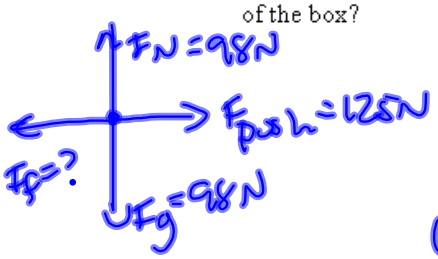
So to solve problems

1) Find F_{net} using both methods (you may get a "number" or an algebraic expression)
2) Make both F_{net}'s equal to each other
3) Solve for what you are looking for

Examples:

1) A boy pushed a 10 kg box along a rough surface. He applies a force of 125 N. The coefficient of friction between the floor and the box is 0.4. What is the acceleration of the box?

$F_g = 98N$

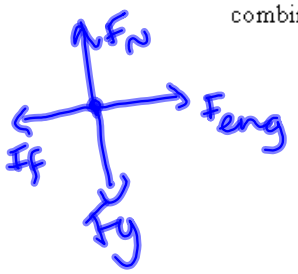


① $F_f = \mu F_N$
$= (0.4)(98N)$
$= 39.2N$

② $F_{net} = F_{push} - F_f$
$= 125N - 39.2N$
$= 85.8N$

③ $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{85.8N}{10kg}$
$= 8.58 \frac{m}{s^2}$

2) A 2500 kg car's engine provides a force of 8500 N. The car accelerates at a rate of 2.6 m/s^2. What is the magnitude of the force that slows down the car (air resistance combined with friction between parts)?



① $F_{net} = ma$
$= (2500kg)(2.6 m/s^2)$
$= 6500N$

② $F_{net} = F_{eng} - F_f$
$F_f = F_{eng} - F_{net}$
$= 8500N - 6500N$
$= 2000N$

Some classics

Rocket Problems:

- Don't forget that gravity is involved
- Acceleration and velocity don't always have to be in the same direction

1) A 4.00 kg toy rocket is being propelled by its engine using a force of 50.0 N. What is the acceleration of the rocket?

$F_{eng} = 50N$

$F_g = 39.2N$

① $F_{net} = F_{eng} - F_g$

$= 50N - 39.2N$

$= 10.8N$

② $F_{net} = ma$

$a = \frac{F_{net}}{m} = \frac{10.8N}{4kg} = 2.7m/s^2$

2) The engine of a rocket applies a force of 3.0×10^3 N. As a result, the rocket experiences an upward acceleration of $5.2\ m/s^2$. What is the mass of the rocket?

$F_{eng} = 3000N$

$F_g = 9.8\frac{m}{s^2} \times m$

① $F_{net} = ma$

$= m(5.2m/s^2)$

② $F_{net} = F_{eng} - F_g$

$\left(5.2\frac{m}{s^2}\right)m = 3000N - \left(9.8\frac{m}{s^2}\right)m$

$\left(5.2\frac{m}{s^2} + 9.8\frac{m}{s^2}\right)m = 3000N$

$15\frac{m}{s^2}m = 3000N$

$m = \frac{3000N}{15\ m/s^2}$ $\quad kg\frac{m}{s^2} \times \frac{s^2}{m}$

$= 200\ kg$

Elevator Problems (standing on a scale in an elevator, of course)

- When the elevator has an upward acceleration

  feel heavier $F_N > F_g$

- When the elevator has a downward acceleration

  feel lighter $F_N < F_g$

- We use F_N as the force shown by the scale (assuming it is in Newtons)

3) As she often does, Carrie is standing on a scale in an elevator. Carrie has a mass of 62 kg. The elevator has an upward acceleration of 1.5 m/s^2. What is Carrie's weight, according to the scale (which is in Newtons)?

$\uparrow F_N = ?$

$F_g = mg$
$= 62\,kg \times 9.8\,\frac{m}{s^2}$
$= 607.8\,N$

① $F_{net} = ma$
$= 62\,kg \times 1.5\,m/s^2$
$= 93\,N \uparrow$

② $F_{net} = F_N - F_g$
$F_N = F_{net} + F_g$
$= 93\,N + 607.8\,N$
$= 700.8\,N$

4) Tony, who has a mass of 75 kg, also likes to stand on scales in elevator. The elevator is moving, and he sees that scale indicated 585 N. What is the acceleration of the elevator? (Don't forget to specify the direction.)

$\uparrow +$

$\uparrow F_N = 585\,N$

$F_g = 735\,N$

① $F_{net} = F_N - F_g$
$= 585\,N - 735\,N$
$= -150\,N$

② $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{-150\,N}{75\,kg}$
$= -2.0\,m/s^2$
$\uparrow$ down

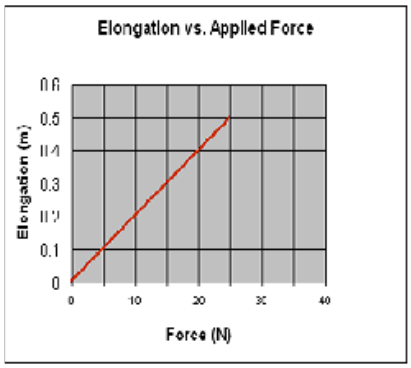
Using graph to determine a coefficient

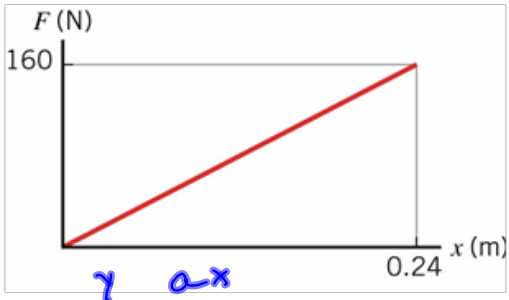
Sometimes we get graphs that allow us to determine either the coefficient of friction or the spring constant.

We find the rate of change (slope) or the inverse of the slope to determine the value of coefficient of friction or the spring constant.

Examples:

1. Find the spring constant for each spring below.





$y \quad a \quad x$

$F_s = kx$

$k = \frac{x_2 - x_1}{y_2 - y_1}$

$= \frac{25N - 0N}{0.5m - 0m}$

$= 50N/m$

$k = \text{slope}$

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{160N - 0}{0.24m - 0}$

$= 667N/m$

2. Find the coefficient for the surfaces represented below.