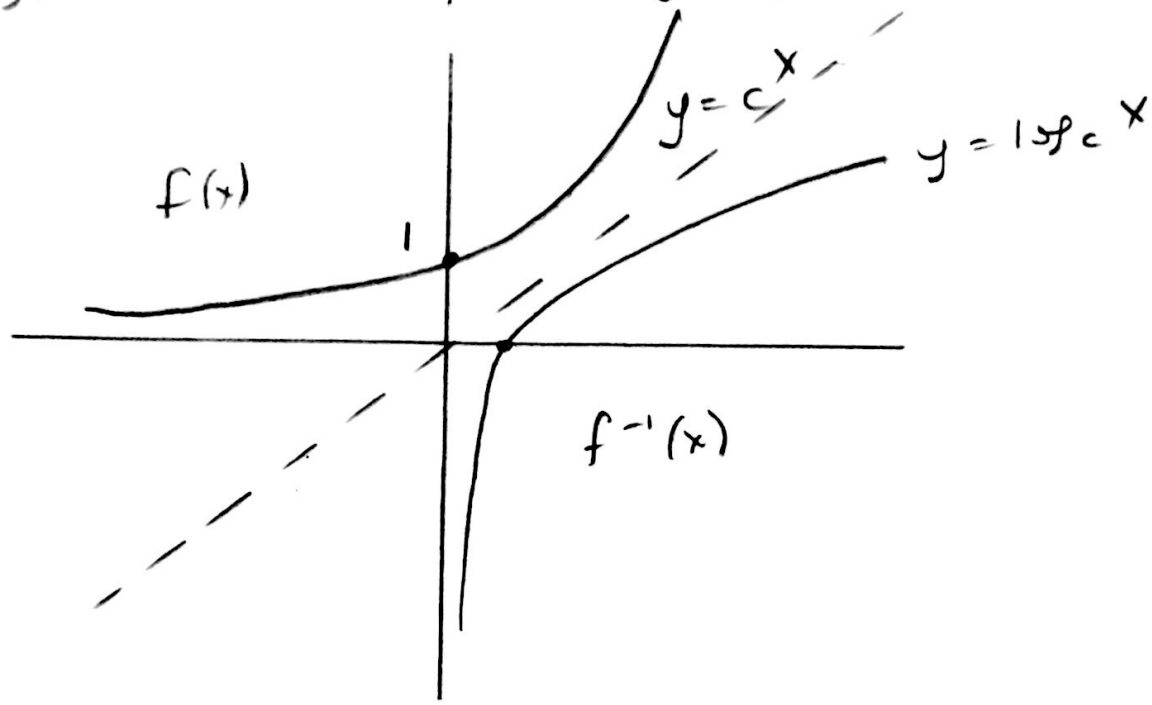
LOGARITHMIC FUNCTIONS

A. Basic Logarithmic Function

- the inverse of an exponential function, $y = c^x$, is called a logarithmic function, written $y = \log_c x$.



B. Definition:

there is an equivalent relationship between

$$\boxed{y = a^x \iff \log_a y = x}$$

Other rules.

$\log_c 1 = 0$

$\log_c c = 1$

$\log x \in \mathbb{R}$, if $x > 0$ (domain)

$\log x = \log_{10} x$

$\ln x = \log_e x$ $\longrightarrow$ precalculus course

$\log_b a = \dfrac{\log a}{\log b}$ (change of base rule)

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C. Examples

- Rewrite in logarithmic form

$2^4 = 16$

$3^{-2} = \frac{1}{9}$

$7^3 = 343$

- Rewrite in exponential form

$\log_5 25 = 2$

$\log_3 \left(\frac{1}{27}\right) = -3$

$\log 1000 = 3$

- Evaluate

$\log_4 16$

$\log_{25} 5$

$\log 0.001$

$\log_3 3$

$\log_7 1$

$\log_{\frac{2}{3}} (4/9)$

$\log (-3)$

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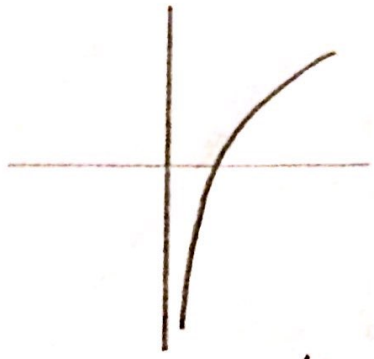
D. <u>Basic Log Function</u>

$y = \log_c x$

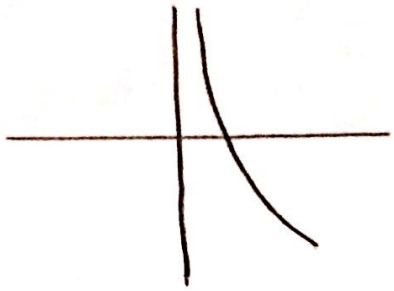
$\boxed{c > 1}$



reflection about x-axis

$y = \log_c x$

$\boxed{0 < c < 1}$

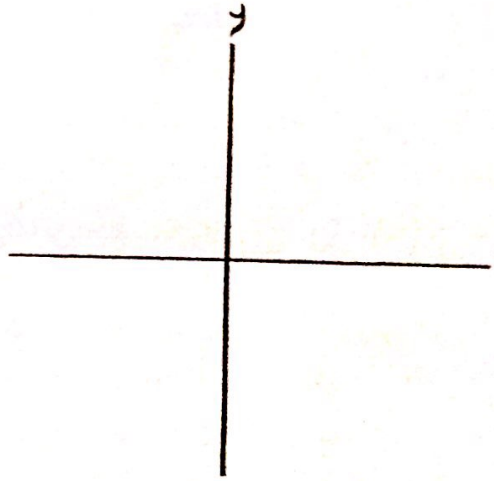


note: $\log_2 x = -\log_{\frac{1}{2}} x$

<u>examples</u>:

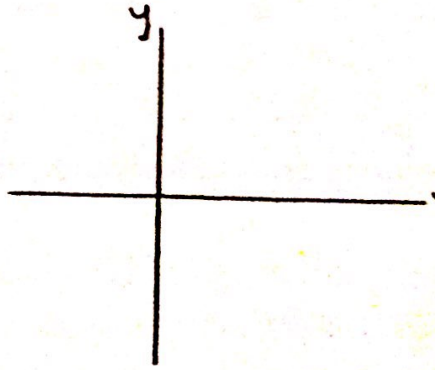
1) <u>Sketch</u> $y = \log_3 x$

x	1	3
y		



2) <u>Sketch</u> $y = \log_{\frac{1}{3}} x$

x	1/3	1
y		



3) <u>Determine the rule</u> for a) $f(x) = \log_c x$ passing through $(9, 2)$

b) $g(x) = \log_c x$ through $(100, 2)$

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E. Exponential Equations, using logarithms

example

$$2(3)^{2x} - 10 = 0$$

$$2(3)^{2x} = 10$$

$$(3)^{2x} = 5$$

$$\log_3 5 = 2x$$

$$\frac{1}{2} \cdot \log_3 5 = x$$

can be estimated using change of base

$$x = \frac{1}{2} \cdot \frac{\log 5}{\log 3}$$

$$\approx \frac{1}{2}\left(\frac{0.6989}{0.4771}\right) \approx 0.7324$$

example

$$3^x = 7$$

$$2^x + 1 = 6$$

$$2(5^{3x}) = 22$$

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F. Transformed Logarithmic Function

$$f(x) = a \log_c b(x-h) + k$$

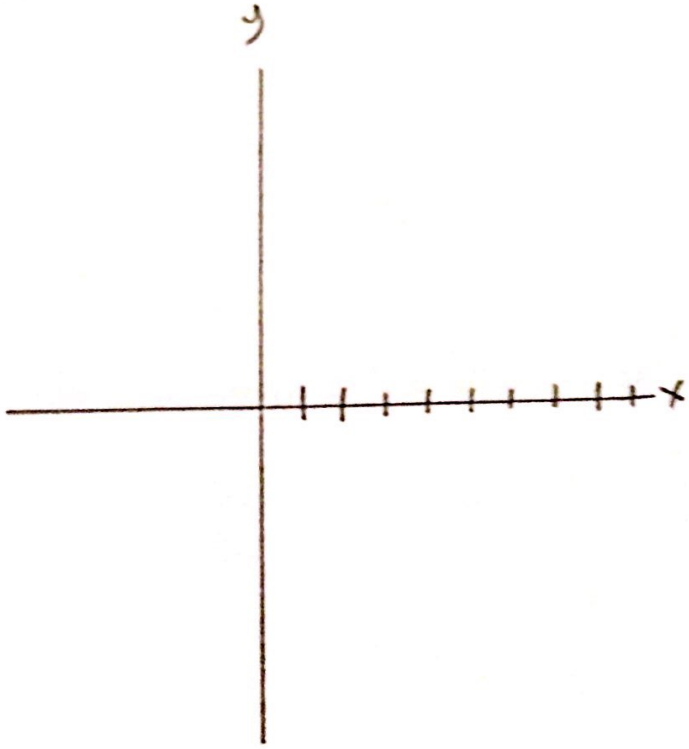
$h =$ vertical asymptote

$a =$ S_x (reflection across x axis)

$b =$ S_y (reflection across y axis)

Example:

sketch $y = -\log_2(x-3) + 2$



Study

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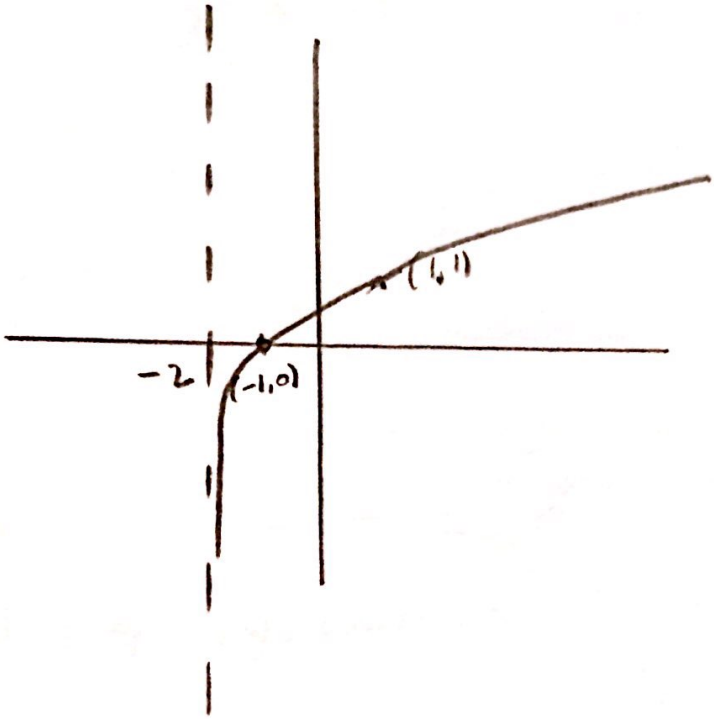
G. Finding the rule $y = \log_c(x-h)$

example



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H. Finding the Inverse

example

$$y = 3^x - 6$$

example $y = \log_3(x-1) + 2$

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I.

Properties of Logarithms

Rules

$\log_a xy = \log_a x + \log_a y$		product rule
$\log_a \frac{x}{y} = \log_a x - \log_a y$		quotient rule
$\log_a x^n = n \log_a x$		power rule

Expand the following

$$\log_3 \frac{x^2 y^3}{\sqrt{w}}$$

$$\log_5 \left(\frac{x^5 y^2}{t}\right)^3$$

$$\log x^{2/3} \sqrt{y^3}$$

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Condense the following

$3\log_4 x + 4\log_4 y - \frac{1}{2}\log_4 w$

$\log_2(x+3) + \log_2(x-3)$

$\log_3 7 - \log_3 2$

Calculate

→ $\log_{\frac{1}{2}} 8 + 3\log_2 4^2 - 10\log_3 \sqrt{3} + \log_7 1$

→ if $\log 2 = x$ and $\log 5 = y$
calculate $\log 400$, in terms of x and y

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J. Logarithmic Equations

Type 1 $\log_c p = q \longleftrightarrow c^q = p$

example

$\log_6 (x-5) = 2 - \log_6 x$

* Remember to verify for extraneous solutions

$\log (x+1) = 1 - \log (x-2)$

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Type 2 $\log_c u = \log_c v \longleftrightarrow u = v$

example

$\log(x+2) = \log(-x+6) + \log(x-1)$

$\log(x-2) = \log 6 - \log x$

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Type 3 $a^u = b^v$

example

$$3^{x-1} = 2^{x+1}$$

K. composition

$f(x) = x+2$

$g(x) = \log_2(x-4)$

$f(g(x)) =$

$g(f(x)) =$

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K. Word problems

1) • In a laboratory experiment, there are initially 12 insects. We notice that the number of insects doubles every 3 days.

a) What is the rule which gives the resulting number y of insects as a function of the number t of days since the beginning of the experiment? _____

b) After how many days will there be at least 6144 insects? _____

2) 1. The formula $c(t) = c_0\left(1+\frac{i}{n}\right)^{nt}$ gives the accumulated capital after t years of an initial capital c_0 invested at an annual interest rate i compounded n times per year.

a) A capital of \$2500 is invested at an annual interest rate of 6% compounded monthly. What will the accumulated capital be after 5 years? _____

b) How long will it take for a capital invested at an annual interest rate of 8% compounded twice per year to double its value? _____

3) 1. The population of a village decreased by approximately 2% per year since 1990. In the year 2000, the village had a population of 2500.

a) What was the population of this village in

1. 1995? _____ 2. 2007? _____

b) If the rate of decay is maintained, in what year will the population of this village be equal to half the population recorded in the year 2000? _____

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