Mathematics 5 SN

LOGARITHMIC FUNCTIONS

1 Computers have changed a great deal since 1950 because of the miniaturization of the circuits.

Year	Number of circuits on a chip $f(x)$
1950	35
1960	3 500
1970	350 000
1980	35 000 000
1990	....

The function representing this situation is given by

$$f(x) = 35(10^{wx})$$

where x represents the number of years since 1950 and w, a parameter.

What is the value of the parameter?

Work

Result : ________________

2

A basket of groceries today costs $200. If the rate of inflation remains at 4 % for the next few years, how much will the same grocery basket cost in 5 years?

Note : Express your answer to the nearest hundredth.

Show all the steps in your solution.

Work

Result : $________________

3

A radioactive substance disintegrates at a rate such that after 2 years it has $\frac{4}{9}$ of its initial mass. If you have 60 grams of this substance, how much of it will remain after 12 years?

Note : Express your answer in grams to the nearest hundredth.

Show all the steps in your solution.

Work

Result : ________________ grams

4

The value of a certain amount of capital C_o invested at a given rate of interest i for *t* years will be as follows:

$$C(t) = C_o(1 + i)^t$$

Emily invested the $2000 she received for winning a design contest. This money will earn the same rate of interest throughout the term of the investment.

The following table of values shows the value of Emily's investment as a function of time in years.

Term of the Investment (years)	Value of the Investment ($)
1	2200
3	2662

How much will Emily's investment be worth after 10 years?

Show all your work.

Show all your work

Answer Emily's investment will be worth $_______________ after 10 years.

5 Reproduction of a certain type of insect is the focus of a laboratory experiment. There were 25 insects at the beginning of the experiment. It was noted that the number of insects increases by 3% every 7 days.

After how many days will there be 20 425 insects?

Show all your work.

Show all your work.

Answer: There will be 20 425 insects after _____________ days.

6

The number of people living in Kilwat, Germany, varies according to the rule of an exponential function. On January 1^{st} 1975, the city's population was 130 000. On January 1^{st} 1985, it was 260 000.

What was the population of this German city on January 1^{st} 2000, given that the growth rate remained constant?

Show all your work.

Show all your work.

Answer On January 1^{st}, 2000, there were ______________ people in Kilwat.

7

When interest is paid n times a year, the value of a certain amount of capital C_0 invested at an annual interest rate i for t years will be as follows:

$$C(t) = C_0\left(1+\frac{i}{n}\right)^{nt}$$

Gerry wants to invest $2 000 for 2 years. He has two investment options.

Investment option A	Investment option B
Annual interest rate of 5%	Annual interest rate of 4.2%
Interest paid once a year	Interest paid 12 times a year

Gerry chose investment option A because he was told it would provide the best return.

Rounded to the nearest whole month, how many months would Gerry have had to invest his money under investment option B in order to earn the same amount he will earn under investment option A?

Show all your work

Answer: Gerry would have had to invest his money for __________ months, to the nearest whole month, under investment option B in order to earn the same amount he will earn under investment option A.

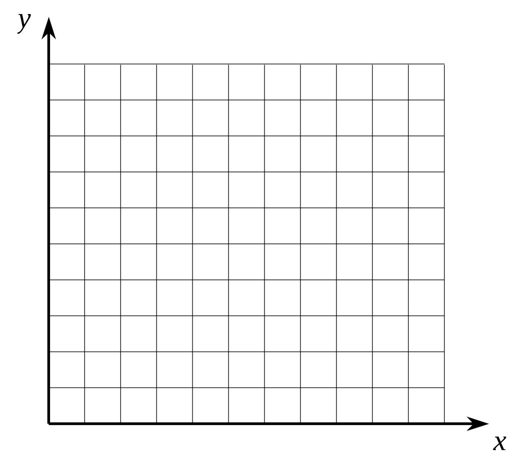
8

An airplane is flying at an altitude of 10 000 m. At 21:00, the pilot begins the descent towards Pierre Elliott Trudeau Airport. The descent follows an exponential model ending with the plane's landing. At 21:04, the airplane is at an altitude of 5222 m.

At what time will the airplane be at an altitude of 280 m?

Show all your work.

Show all your work.



Answer: The airplane will be at an altitude of 280 m at ______________.

9 Jeremy inherited $80 000 from an aunt. He wants to invest this money for his retirement needs. One bank offers him an annual rate of 8% paid out twice a year.

How long will it take Jeremy to quadruple his original investment?

Show all your work.

Show all your work.

Answer: It will take ____________ years for Jeremy to quadruple his original investment.

10

When Jennifer bought a new car in 1995, she paid $17 500. In 1998 the value of her car had fallen to $10 000. She decided that she would sell her car when the value fell below $5000.

Assuming the decline in the price of a car is modelled by an exponential function, how old will Jennifer's car be when its value falls below $5000? Round your answer to the nearest month.

Show all your work.

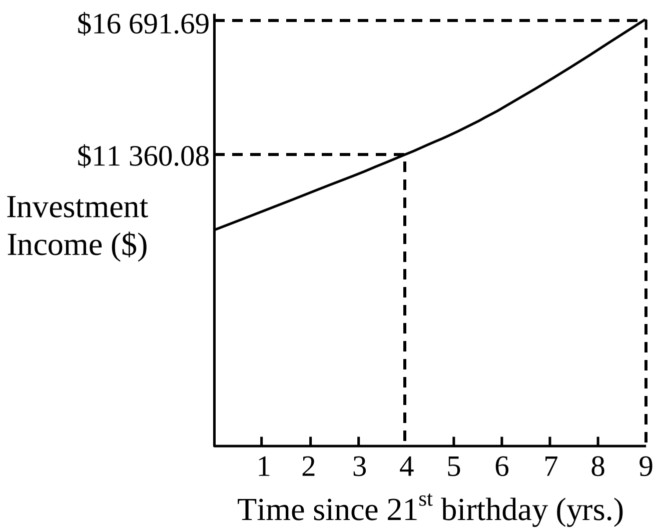
Show all your work.

Answer: The value of the car falls below $5000 when it is ____ years _____ months old.

11 On her 21st birthday, Marie received a lump sum of money. At that time, she decided to invest it at a fixed annual interest rate, compounded yearly, until her 30th birthday.

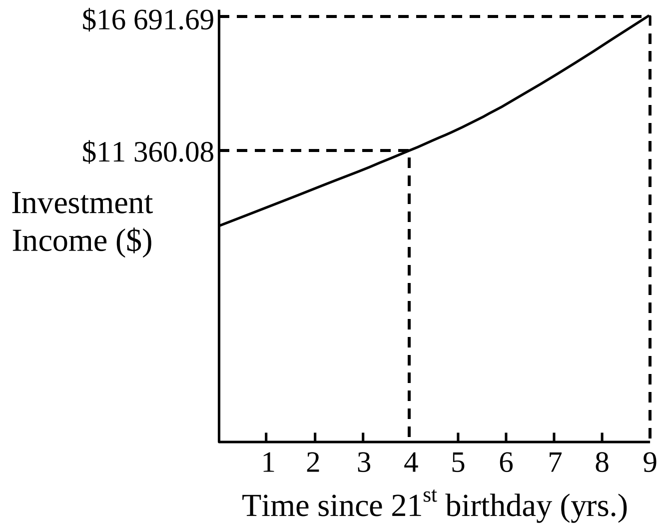
On her 25th birthday, her investment had grown exponentially to \$11 360.08. On her 30th birthday, it had further grown to \$16 691.69.

This situation is represented in the graph below.



Rounded to the nearest tenth of a percent, what was the fixed annual interest rate over this nine-year period?

Show all your work.



Answer: The annual fixed interest rate is approximately ___________ %.

12 A virus appeared in South America in the middle of the last decade. Scientists knew that the number of people infected with this virus would increase according to a specific exponential function.

At the beginning of 1996, authorities found 110 infected people. Five years later, the number had grown to 835. Wide-scale inoculation began once 2000 people had been infected with the virus.

In what year did these inoculations begin?

Show all your work.

Show all your work.

Answer: The inoculations began in the year ___________.

13 In January 1990, there were 5.5 billion people living on this planet. The population has been growing at a rate of 1.9% per year.

In which year will the population reach 9 billion?

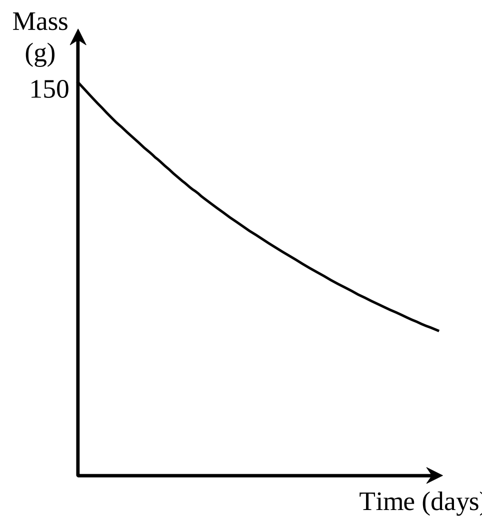
Show all your work.

Show all your work.

Answer: The population will reach 9 billion in the year ___________.

14

A chemist is working with a dangerous compound she has just created. She began with 150 g of the compound, but noticed that it decays exponentially. After observing for 10 days, 123 g remained. She needs to know how long it will take until only half of the compound will be left.



Rounded to the nearest day, how many days after the experiment started will only half of the compound remains?

Show all your work.

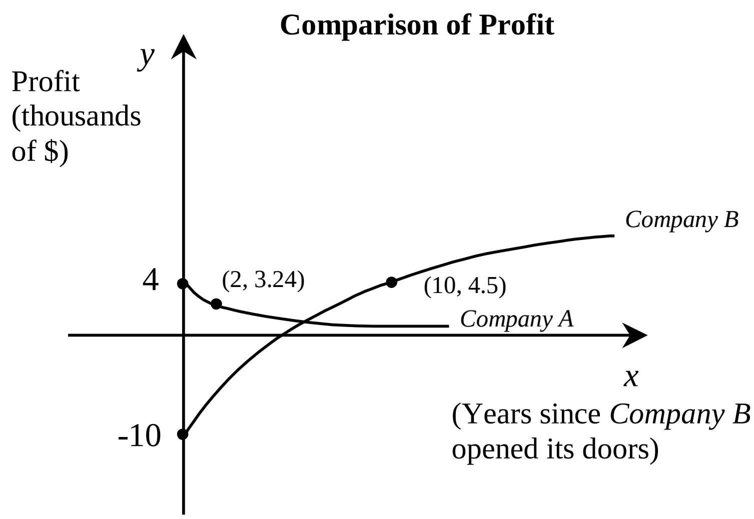
Show all your work.

Show all your work.

Answer: To the nearest day, half of the compound will remain after ________ days.

15 *Company A* has seen a decrease in profit since its competitor, *Company B*, opened its doors. The decrease can be estimated using an exponential function in the form of $g(x) = ac^x$.

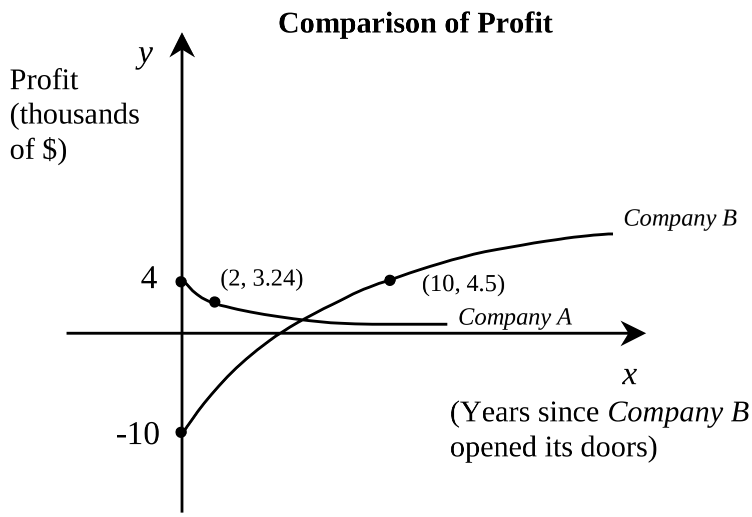
The profit of *Company B* can be estimated according to an exponential function in the form of $f(x) = ac^x + 15$.



Based on these estimates, how much more profit would *Company B* make than *Company A*, 11 years after it opened its doors?

Round your answer to the nearest dollar.

Show all your work.



Answer: *Company B* would make $ _____________ more than *Company A*.

16 When rabbits were first brought to Australia, they had no natural enemies. From January 1865 to January 1867, the rabbit population increased exponentially from 60 000 members to 2 400 000 members.

According to this exponential model, in which year were the first pair of rabbits brought to Australia?

Show all your work.

Answer: The first pair of rabbits was brought to Australia in the year ___________.

17 Given the equation : $\log_4(x^2 + 15x) = 2$, where $x \in \Re$.

What value(s) of x satisfies(satisfy) this equation?

The value(s) that satisfy this equation is (are) : ______________________________

18 Solve the following logarithmic equation :

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

Show your work.

Work

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

Result : ______________________________

19

Solve the following logarithmic equation :

$$\log_2(x - 3) + \log_2(2x) = 3$$

Show your work.

Work

$$\log_2(x - 3) + \log_2(2x) = 3$$

Answer : x equals ________________.

20 Find the value of x in

$$\frac{1}{9} = \exp_x(-2)$$

____

21 To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6 % per year starting in 1994.

Mr. Blais paid $1225 in taxes for the year 1993. He wants to find a function t that can be used to calculate the amount of annual taxes as a function of the number of years n elapsed since 1993.

What rule corresponds to function t?

The rule that corresponds to function t is ____.

22 During an experiment in a science laboratory, a population of 100 mosquitoes triples every 12 hours.

What rule can be used to calculate the number of mosquitoes Q(t) as a function of the number of days t?

The rule is Q(t) = ________________.

23 Solve the following equation :

$$3 \log_a 2 + \log_a(x - 2) = \log_a 12(x + 3) - \log_a 3$$

In this equation, x equals _______________.

24 Two rival companies A and B decided to make the same product using two different processes.

The following functions represent the gross profits of the two companies:

$a(t) = 1000 \log_4 t$ for company A

$b(t) = 1000 \log_5 t$ for company B

where t is time in months.

After the twelfth month, the expenses of company A were \$800 and company B, \$500.

Which company had the greatest net profit after the twelfth month?

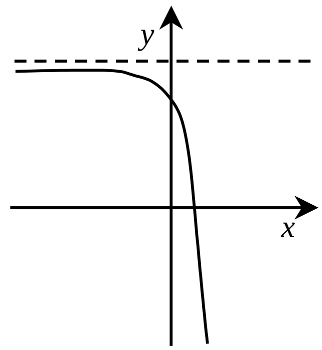
N.B. net profit = gross profit ! expenses

Work

Result : ________________

25 The rule of a function g is $g(x) = \mathrm{p}(\mathrm{c})^x + \mathrm{q}$. The equation of its asymptote is $y = 8$. This function is represented by the table of values and the graph given below.

x	y
0	6
1	2
2	-10



What is the rule of function g?

The rule of function g is $g(x)$ = ____________________.

26 Half-life is the time required for a radioactive element to decay to half its original mass.

Radium (Ra) is a radioactive element that decays naturally according to the following equation:

$$N = N_O\left(\frac{1}{2}\right)^{\frac{t}{d}}$$

where N is the mass in grams left after *t* year(s),

N_O is the original mass in grams, and *d* is the half-life of the element.

In 3000 years, a 100-g sample of radium decays to a mass of 27.04 g.

In 1000 years, how much of a 250-g sample of radium will be left?

Show all your work.

Show your work.

Answer In 1000 years, there will be ____________ g of radium left.

27 Electronic mail makes it easy to contact many people rapidly. Justin received a message on Monday. The next day he sent it to 12 people. The following day, each of those 12 recipients sent the message to 12 people, and so on.

On what day will the message have been sent exactly one million times?

The message will have been sent to exactly one million people on _________.

28 Given the following equation:

$$2\log_a 8 + \log_a \left(\frac{1}{a}\right)^5 = 19$$

What is the numerical value of base *a*, to the nearest hundredth?

Show all your work.

Answer: The numerical value of base *a* is __________.

29

Use the properties of logarithms to express $\log_2\left(\frac{\sqrt{x}(x-5)}{x^3}\right)$ as a sum, a difference or a product of logarithms.

$\log_2\left(\frac{\sqrt{x}(x-5)}{x^3}\right) =$ ____

30

What is the solution of the following equation?

$$\log_5 (x-1) + \log_5 (x+3) - 1 = 0$$

Show all your work.

Answer: The solution of the equation is ____________________.

31 Solve the following equation:

$$2^x = 3^{2x-1}$$

Round your answer to the nearest hundredth.

Rounded to the nearest hundredth, the value of x is _________.

32 Solve for x in the following equation:

$$\log_3(x + 4) = \log_7 7 - \log_3(x + 2)$$

The value of x: ____________________.

33 Solve the following logarithmic equation:

$$\log (x + 3) + \log (2x - 7) = \log (2x - 1)$$

The solution is ________________________________.

34 Solve the following logarithmic equation.

$$\log_4 (x + 3) + \log_4 (x - 3) = 2$$

Show all your work.

$$\log_4 (x + 3) + \log_4 (x - 3) = 2$$

Answer: ____

35 Solve the following logarithmic equation:

$$\log_2(x+3) = 3 - \log_2(x-4)$$

Answer : ____

36 Given log (x + 2)(x – 1) = 1

What is/are the solution(s) to the equation?

The solution(s) to the equation is/are __________________.

37 In 1991, Albert invested $4000. In 1999, his investment had grown to $5474.28. He eventually was able to triple his initial investment.

Jocelyn, Albert's brother, also invested a sum of money at the same interest rate. In the number of years it took Albert to triple his initial investment, Jocelyn's investment grew to $15 000.

What was the difference between Albert's initial investment and Jocelyn's initial investment? (Round your final answer to the nearest dollar.)

Show all your work.

Show all your work.

Answer: The difference between Albert’s initial investment and Jocelyn’s initial investment is $___________.

38

Express the following as a logarithm of a single simplified expression.

$$\log 5x^2 - 3\log y^4 - \frac{1}{2}\log x + \log y^7$$

Show all your work.

$$\log 5x^2 - 3\log y^4 - \frac{1}{2}\log x + \log y^7$$

Answer: ____

2- Correction key

1 Work : (example)

Calculation of w

In 1960, x was 10 and $f(x) = 3500$ so that

$$3500 = 35(10^{10w})$$

$$35 \times 100 = 35(10^{10w})$$

$$100 = 10^{10w}$$

$$10^2 = 10^{10w}$$

thus $2 = 10w$ i.e. $w = 0.2$

Result : The parameter is 0.2.

2 Work : (example)

$f(x) = 200 \times (1.04)^5$

$f(x) = \$243.33$

Result : $243.33

3 Work : (example)

$$f(x) = \mathrm{M}\left(\frac{4}{9}\right)^{\frac{t}{2}}$$

$$f(x) = 60 \times \left(\frac{4}{9}\right)^{\frac{12}{2}}$$

$f(x) = 60 \times 0.0077$

$f(x) = 0.46$

Result : 0.46 grams

4 Example of an appropriate method

Value of parameter i

$C_o = 2000$

$C(t) = C_o(1 + i)^t$

$2662 = 2000(1 + i)^3$

$1.331 = (1 + i)^3$

$1.1 = 1 + i$

$0.1 = i$

$C(t) = 2000(1.1)^t$

Value after 10 years

$C(10) = 2000(1.1)^{10} \approx 5187.4849$

Answer Emily's investment will be worth $5187.48 after 10 years.

5 Example of an appropriate method

Given x, the number of days gone by

$f(x)$, the number of insects

Rule of the exponential function

$$f(x) = 25(1.03)^{\frac{x}{7}}$$

Value of x when $f(x)$ = 20 425

$$20\ 425 = 25(1.03)^{\frac{x}{7}}$$

$$817 = 1.03^{\frac{x}{7}}$$

$$\frac{x}{7} = \log_{1.03} 817$$

$$x = \frac{7 \log 817}{\log 1.03}$$

x = 1588.003...

Answer 20 425 insects will be present after 1588 days.

6 Example of an appropriate method

Equation of the function

The population doubles every 10 years

$$\mathrm{P}(x) = 130\,000(2)^{\frac{x}{10}}$$

Calculation of the population in the year 2000

P(x) = $130\,000(2)^{\frac{x}{10}}$ if $x = 25$

P(x) = $130\,000(2)^{2.5}$

P(x) $\approx$ 735 391.05

Answer On January 1st 2000, there were 735 391 people in Kilwat.

7 Example of an appropriate method

Value of investment under option A after 2 years

$C(t) = 2000\left(1+\frac{0.05}{1}\right)^{1t}$

$C(t) = 2000\,(1.05)^{t}$

$C(t) = 2000\,(1.05)^{2}$

$= \$2205$

Rule associated with investment option B

$C(t) = 2000\left(1+\frac{0.042}{12}\right)^{12t}$

$C(t) = 2000\,(1.0035)^{12t}$

Number of months required under investment option B to earn $2205

$2205 = 2000(1.0035)^{12t}$

$\frac{2205}{2000} = (1.0035)^{12t}$

$1.1025 = (1.0035)^{12t}$

$12t = \log_{1.0035} 1.1025$

$12t = 27.9288\ldots$

Answer: Gerry would have had to invest his money for **28** months, to the nearest whole month, under investment option B in order to earn the same amount he will earn under investment option A.

Note: Do not penalize students who did not round off their final answer or who made a mistake in rounding it off.

Students who used an appropriate method in order to determine the value of investment A after 2 years have shown that they have a partial understanding of the problem.

8

Example of an appropriate solution

Let t: time, in minutes, which has past since 21:00

$A(t)$: altitude of the airplane after t minutes

The rule of correspondence

$A(t) = a \times c^t$

$$\begin{aligned}
10\ 000 &= a \times c^0 && P_1(0,\ 10\ 000) \\
10\ 000 &= A && P_2(4,\ 5222) \\
A(t) &= 10\ 000c^t \\
5222 &= 10\ 000c^4 \\
0.5222 &= c^4 \\
(0.5222)^{\frac{1}{4}} &= c \\
0.85 &\approx c
\end{aligned}$$

$$A(t) = 10\ 000(0.85)^t$$

t when $A(t) = 280$

$$\begin{aligned}
280 &= 10\ 000(0.85)^t \\
0.028 &= 0.85^t \\
t &= \log_{0.85}\ 0.028 \\
t &= \frac{\log\ 0.028}{\log\ 0.85} \\
t &\approx 22
\end{aligned}$$

Answer: The airplane will be at an altitude of 280 m at **21:22**.

9

Example of an appropriate solution

Let C_0, be the original investment

i, the rate of interest

n, the number of interest payments per year

t, the length of the investment period

$$C(t) = C_0\left(1 + \frac{i}{n}\right)^{nt}$$

$$320\ 000 = 80\ 000\left(1 + \frac{0.08}{2}\right)^{2t}$$

$$4 = \left(1 + \frac{0.08}{2}\right)^{2t}$$

$$4 = (1.04)^{2t}$$

$$2t = \log_{1.04} 4$$

$$2t = 35.35$$

$$t = 17.67$$

Answer: It will take Jeremy **17.67** years or **17 years and 8 months** to quadruple his investment.

Accept all answers between 17.67 and 18 years.

10 Example of an appropriate solution

Let t: time after 1995 (years)

$V(t)$: value of the car ($)

$V(t) = 17\,500(r)^t$ where r is the rate at which the value declines and

$10\,000 = 17\,500(r)^3$

$0.5714285 = (r)^3$

$r \approx \sqrt[3]{0.571428}$

$r \approx 0.83$

When $V(t) = 5000$

$5000 \approx 17\,500(0.83)^t$

$\dfrac{5000}{17\,500} \approx (0.83)^t$

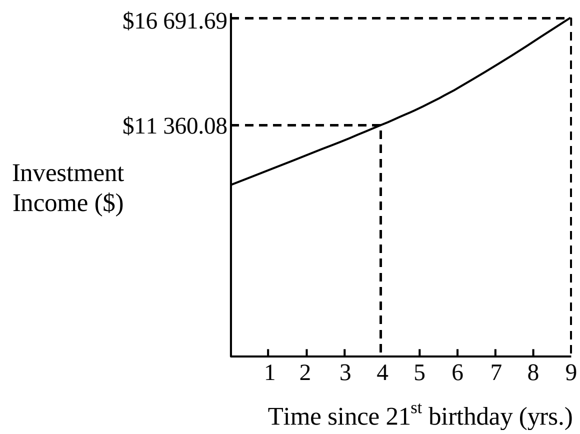
$$t \approx \frac{\ln\left(\dfrac{5000}{17\,500}\right)}{\ln(0.83)}$$

$t \approx 6.72$

Answer The value of the car falls below $5000 when it is 6.72 years $\approx$ 6 years 9 months.

11 Example of an appropriate method

Use exponential model:



$y = 11\ 360.08(1 + i)^{9-4}$

where i = annual interest rate

x = time, in years, since 21st birthday

y = accumulated investment ($)

Substituting (9, 16 691.69), we get

$16\ 691.69 = 11\ 360.08(1 + i)^{9-4}$

$\Rightarrow \dfrac{16\ 691.69}{11\ 360.08} = (1+i)^5 \qquad \Rightarrow \sqrt[5]{\dfrac{16\ 691.69}{11\ 360.08}} = 1 + i \qquad \Rightarrow \sqrt[5]{\dfrac{16\ 691.69}{11\ 360.08}} - 1 = i$

$\Rightarrow 1.08 - 1 \approx i \qquad \Rightarrow i \approx 0.08$ or 8%

Alternate solution let $y = 11\ 360.08(1 + i)^x$

where x = number of years since 25th birthday

Substituting (5, 16 691.69), we get

$16\ 691.69 = 11\ 360.08(1 + i)^x$

$$\Rightarrow i = \sqrt[5]{\frac{16\ 691.69}{11\ 360.08}} - 1 \approx 0.08 \quad \text{or} \quad 8\%$$

Answer The annual fixed interest rate is approximately 8%.

(Accept 8.0% as well as 8%)

12 Example of an appropriate method

Find the rate

$$y = a \bullet b^x$$

$$y = 110 \bullet b^x$$

$$835 = 110 \bullet b^x$$

$$7.59 = b^5$$

$$1.5 = b$$

Find time that elapsed when 2000 victims have been infected

$$110 \times 1.5^t = 2000$$

$$1.5^t = 18.\overline{18}$$

$$t = \frac{\log 18.\overline{18}}{\log 1.5} \approx 7.15$$

Find the year the vaccine will be offered

$$1996 + 7.15 = 2003.15$$

Answer: The population will be offered the vaccine in the year **2003**.

Note: Students who have determined the rate have shown a partial understanding of the problem.

13 Example of an appropriate solution

Let t be the number of years after 1990

$$5.5(1 + 0.019)^t = 9$$

$$(1 + 0.019)^t = 1.\overline{63}$$

$$t \log (1.019) = \log 1.\overline{63}$$

$$t = \frac{\log 1.\overline{63}}{\log 1.019} \approx 26.165\ldots$$

Answer: The population will reach 9 billion in the year **2016**.

Note: Do not penalize students who round the answer to 2017.

Students who have entered the correct values in the formula have shown they have a partial understanding of the problem.

14 Example of an appropriate solution

Let t: number of days

$f(t)$: amount of the compound remaining (g)

$$
\begin{aligned}
f(t) &= 150c^t \\
123 &= 150c^{10} \\
0.82 &= c^{10} \\
c &= \sqrt[10]{0.82} \\
c &\approx 0.98
\end{aligned}
$$

Time for 75 g to remain:

$$
\begin{aligned}
f(t) &= 150(0.98)^t \\
75 &= 150(0.98)^t \\
0.5 &= (0.98)^t \\
t &= \log_{0.98}(0.5) \\
t &= \frac{\log(0.5)}{\log(0.98)} \\
t &\approx 34.3
\end{aligned}
$$

Answer: To the nearest day, half of the compound will remain after **34** days.

Note: Students who do not round $\sqrt[10]{0.82}$ will obtain a rounded answer of 35 days. Accept answer of 34 or 35 days if appropriate work is shown.

Students who use an appropriate method in order to correctly determine the value $c \approx 0.98$ have shown they have a partial understanding of the problem.

Students who use the half-life formula $N = N_o(0.5)^{\frac{t}{H}}$ where H is the half-life, and who obtain $\frac{10}{H} = \log_{0.5}(0.82))$ have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

15 Example of an appropriate solution

Equation representing profit decrease for *Company A*

$$
\begin{array}{ll}
 & g(x) = ac^{x} \\
\text{Substituting } (0, 4) & 4 = ac^{0} \\
 & 4 = a \\
(2, 3.24) & 3.24 = 4c^{2} \\
 & 0.81 = c^{2} \\
 & 0.9 = c \\
 & g(x) = 4(0.9)^{x}
\end{array}
$$

Equation representing growth for *Company B*

$f(x) = ac^x + 15$

Substituting (0, -10)

$$
\begin{aligned}
-10 &= ac^{0} + 15 \\
-25 &= a \\
f(x) &= -25c^{x} + 15 \\
4.5 &= -25c^{10} + 15 \quad \text{Point } (10, 4.5) \\
-10.5 &= -25c^{10} \\
0.42 &= c^{10} \\
0.92 &\cong c
\end{aligned}
$$

$\therefore f(x) = -25(0.92)^x + 15$

Solving for $f(11)$ and $g(11)$

$$f(11) = -25(0.92)^{11} + 15$$
$$= 5.009 \text{ thousands} \quad \text{or } \$5009$$
$$g(11) = 4(0.9)^{11}$$
$$= 1.255 \quad \text{thousands} \quad \text{or } \$1255$$

Difference between the two companies

$5009 – $1255 = $3754

Answer: *Company B* would make **$3754** more than *Company A*.

Accept answers in the **interval** $[\mathbf{3754,\ 4200}]$

Accept also $[3.7 \text{ thousands} \quad \text{and} \quad 4.0 \text{ thousands}\ ]$

Note: Students who find the correct rule for either one of the two companies, have shown they have a partial understanding of the problem.

16 Example of an appropriate solution

Mathematical model: $y = ac^x$

Determine base, c using ordered pairs (1865, 60 000) and (1867, 2 400 000)

$$
\begin{aligned}
2\,400\,000 &= 60\,000\,(c)^{1867-1865} \\
\frac{2\,400\,000}{60\,000} &= (c)^2 \\
40 &= (c)^2 \\
\sqrt{40} &= c
\end{aligned}
$$

Let x represent the number of years from the introduction of rabbits to 1865:

$$
\begin{aligned}
60\,000 &= 2\left(\sqrt{40}\right)^x \\
\frac{60\,000}{2} &= \left(\sqrt{40}\right)^x \\
x &= \log_{\sqrt{40}}(30\,000) \\
x &= \frac{\log 30\,000}{\log \sqrt{40}} \\
x &= \frac{4.477\ldots}{0.801\ldots} \\
x &= 5.589\ldots
\end{aligned}
$$

Year when first pair of rabbits was brought to Australia.

1865 – 5.589... = 1859.410...

Answer: The first pair of rabbits was brought to Australia in the year **1859**.

Note: Students who used an appropriate method to determine a base have shown they have a partial understanding of the problem.

17 The values that satisfy this equation is (are) : $x = -16$ and $x = 1$.

18 Work : (example)

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

$$\log_2 \frac{(x^2 + 5)}{5} = \log_2 6$$

$$\frac{x^2 + 5}{5} = 6$$

$$x^2 + 5 = 30$$

$$x^2 = 25$$

$$x = -5 \quad \text{ou} \quad x = 5$$

Result : $x = -5$ or $x = 5$

19 Example of an appropriate solution

$\log_2(x - 3)(2x) = 3$

$2^3 = (x - 3)(2x)$

$8 = 2x^2 - 6x$

$0 = 2x^2 - 6x - 8$

$0 = (2x - 8)(x + 1)$

$0 = 2x - 8$ or $x + 1 = 0$

$x_1 = 4$ $x_2 = -1$

Value to reject

Answer : x equals 4.

20 3

21 The rule that corresponds to function t is $t(n) = 1225 \times 1.06^n$ or any equivalent rule.

22 The rule is $Q(t) = 100 \times 3^{2t}$

or any equivalent rule.

23 In this equation, x equals 7.

24 Work : (example)

Gross profit of company A after the 12th month

$$a(t) = 1000\log_4 12 = 1000\frac{\log 12}{\log 4} \approx 1000 \times 1.792\,481 \approx 1792.48$$

Gross profit of company B after the 12th month

$$b(t) = 1000\log_5 12 = 1000 = \frac{\log 12}{\log 5} \approx 1000 \times 1.543\,959 \approx 1543.96$$

Net profit of company A after the 12th month.

$1792.48 – $800.00 = $992.48

Net profit of company B after the 12th month.

$1543.96 – $500.00 = $1043.96

Result : Company B

25 The rule of function *g* is *g*(*x*) = **-2(3)^{x} + 8**.

26

Example of an appropriate solution

$N = 27.04$ g $\quad N_O = 100$ g $\quad t = 3000$ years

$$N = N_O \left(\frac{1}{2}\right)^{\frac{t}{d}}$$

$$27.04 = 100\left(\frac{1}{2}\right)^{\frac{3000}{d}}$$

$$\frac{27.04}{100} = \left(\frac{1}{2}\right)^{\frac{3000}{d}}$$

$$\log_{\frac{1}{2}}\left(\frac{27.04}{100}\right) = \frac{3000}{d}$$

$$d = \frac{3000}{\log_{\frac{1}{2}}(0.2708)}$$

$$d \approx 1590 \text{ years}$$

$$N = 250\left(\frac{1}{2}\right)^{\frac{1000}{1590}}$$

$$N = 161.66 \text{ g}$$

Answer: In 1000 years, there will be **161.66** g of radium left.

27 The message will have been sent to exactly one million people on **Sunday**.

28 Example of an appropriate solution

$$2\log_a 8 + \log_a \left(\frac{1}{a}\right)^5 = 19$$

$$2\log_a 2^3 + \log_a a^{-5} = 19$$

$$6\log_a 2 - 5\log_a a = 19$$

$$6\log_a 2 - 5 = 19$$

$$\log_a 2 = \frac{19+5}{6}$$

$$\frac{\log 2}{\log a} = 4$$

$$\log a = \frac{\log 2}{4}$$

$$a = 10^{\left(\frac{\log 2}{4}\right)}$$

$$a \approx 1.19$$

Answer: The numerical value of base a is $\approx$**1.19**.

29 $\log_2(x-5) - \dfrac{5}{2}\log_2 x$

Note Accept $\dfrac{1}{2}\log_2 x + \log_2 (x-5) - 3\log_2 x$

30 Example of an appropriate method

$\log_5 (x - 1) + \log_5 (x + 3) - 1 = 0$

$\log_5 (x - 1)(x + 3) = 1$ Sum of logs = log of the product

$(x - 1)(x + 3) = 5^1$

$x^2 + 2x - 3 = 5$

$x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$x - 2 = 0$ or $x + 4 = 0$

$x = 2$ or -4

-4 is an extraneous root

Answer The solution of the equation is 2.

31 Rounded to the nearest hundredth, the value of x is **0.73**.

Note: Do not penalize students who did not round their answer to the nearest hundredth.

32 The value of x is -1.

33 The solution set is $x = 4$

34 Example of an appropriate solution

$$\begin{aligned}
\log_4 (x+3) + \log_4 (x-3) &= 2 \\
\log_4 (x+3)(x-3) &= 2 \\
(x+3)(x-3) &= 4^2 \\
x^2 - 9 &= 16 \\
x^2 &= 25 \\
x &= 5 \quad \text{or} \quad x = -5 \text{ (reject)}
\end{aligned}$$

Answer: $\boldsymbol{x = 5}$

Deduct 1 mark if –5 is **not** rejected.

35 Answer: **5**

Deduct marks if student did not reject -4.

36 The solutions to the equation are **–4** and **3**.

37 Examples of appropriate solutions

Example 1

Let $v(t)$ be the value of Albert's investment t years after 1991

$$v(t) = 4000(\text{base})^t \quad \text{therefore in 1991,}$$
$$\begin{aligned} 4000(c)^8 &= 5474.28 \\ c^8 &= 1.368\ 57 \\ c &= \sqrt[8]{1.368\ 57} \\ &\approx 1.04 \end{aligned}$$

Time to triple investment

$$4000(1.04)^t = 12\ 000$$
$$\begin{aligned} (1.04)^t &= 3 \\ t &= \frac{\log 3}{\log 1.04} \\ &\approx 28 \text{ years} \end{aligned}$$

Let v_o be the value of Jocelyn's initial investment

$$\begin{aligned} 15\ 000 &= v_o\ (1.04)^{28} \\ v_o &= \left(\frac{15\ 000}{(1.04)^{28}}\right) \\ &\approx 5002.16 \end{aligned}$$

Difference between both initial investments

\$5002 – \$4000 = \$1002

Example 2

$$y = ab^x$$

$\dfrac{y}{a} = b^x$ since the length of time and the rate are both the same.

$$\therefore \frac{y}{a} = \text{constant}$$

∴ Albert's investment triples in the same length of time that Jocelyn's investment does.

$$\frac{y_1}{a_1} = \frac{y_2}{a_2}$$

$$\begin{aligned} \frac{12\ 000}{4000} &= \frac{15\ 000}{a_2} \\ a_2 &= \$5000 \end{aligned}$$

Difference between both initial investments

\$5000 – \$4000 = \$1000

Answer: The difference between Albert's and Jocelyn's initial investments is $**1002**.

Note: Accept answers in the range of $1000 to $1002, as a result of rounding differences.

Students who use an appropriate method in order to correctly determine **the value *c* ≈ 1.04 (example 1)** have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

38

Example of an appropriate solution

$$\log 5x^2 - 3\log y^4 - \frac{1}{2}\log x + \log y^7$$

$$= \log 5x^2 - \log y^{12} - \log x^{\frac{1}{2}} + \log y^7$$

$$= \log\left(\frac{5x^2y^7}{x^{\frac{1}{2}}y^{12}}\right)$$

$$= \log\left(5x^{\frac{3}{2}}y^{-5}\right) \text{ or } \log\left(\frac{5x^{\frac{3}{2}}}{y^5}\right)$$