Name : ________________________________

Math 5 SN – June Review

1 For a science project, Louise and Bob recorded the temperature outside their school for a 24 hour period. At the beginning, the thermometer read -1 °C. The minimum temperature of -5 °C was reached 8 hours later.

The data recorded show that the temperature t(x) varied as a function of the number of hours x elapsed and that t is an absolute value function.

During how many hours was the temperature less than or equal to -3 °C?

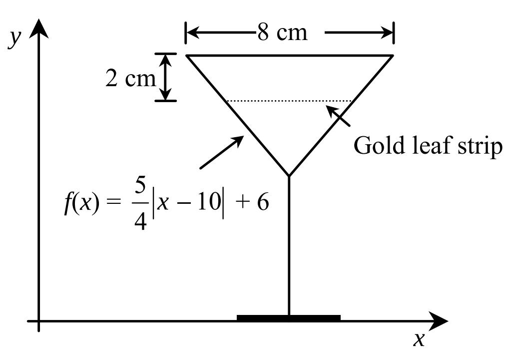
2 Jenny and Eric analyzed the changes in the value of Future Telecom's shares in 1999. On January 1st 1999, the initial value of a share was $25. On may 31st, the share reached its minimum value of $10. Since then, the value of shares has been on the increase.

They noticed that the relation between the elapsed time, in months, and the value of a share, in dollars, was an absolute value function.

What was the value of a share on December 31st, 1999?

3 A glassblower wants to apply a gold leaf strip around a cocktail glass. The gold leaf strip will be applied 2 cm from the rim of the glass.

The side view of this glass is represented in the following Cartesian plane. The scale of the graph is in centimetres.



The rule $f(x) = \frac{5}{4}|x-10|+6$ is associated with the top part of the glass.

The maximum diameter of the glass is 8 cm.
What is the diameter of the glass at the point where the gold leaf strip will be applied?

4 Starting 26 weeks before an election, a firm holds a weekly poll on voter intention.

During this polling period, the popularity of Party "A" varies according to an absolute value function.

In the first survey, Party "A" polled 28%. Ten weeks later, the party reached its maximum survey result of 43%.

When did Party "A" have 25% of the survey results?

Show all your work.

5 What are the exact values of A that satisfy the following trigonometric equation?

$$\sin A \cot A + 2\cos^2 A = 1, \quad A \in [0, 2\pi]$$

6 Each morning, the traffic on a certain highway increases, reaches a peak, and then decreases again.

This situation is represented mathematically by the function:

$$V(t) = -25\,|\,t - 8\,| + 65$$

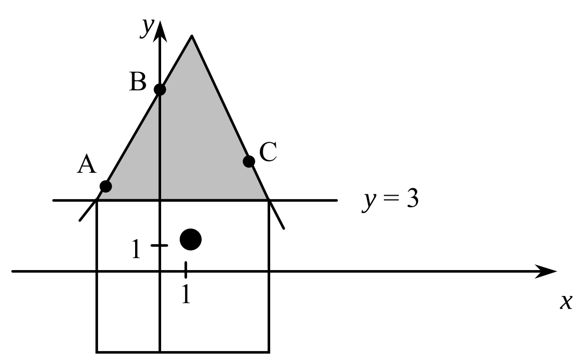
$V(t)$ = the number of vehicles passing per minute
t = the time, in hours, since midnight

To ensure a smooth and safe flow of traffic, a police officer has been assigned to monitor this section of highway when the volume of traffic is at least 35 vehicles per minute.

For how many hours should the police officer be on duty to ensure the safe flow of traffic?

7 Ethan's diagram, not drawn to scale, shows the front of a birdhouse. The base of the roof corresponds to the line $y = 3$.

The sides of the roof form an absolute value function that passes through the points A(-2, 4), B(0, 7) and C(4, 5).



What is the area of the shaded triangular section of the front of the birdhouse?

8 The centre of a circle coincides with the vertex of the parabola with equation $y^2 - x + 8y + 17 = 0$.

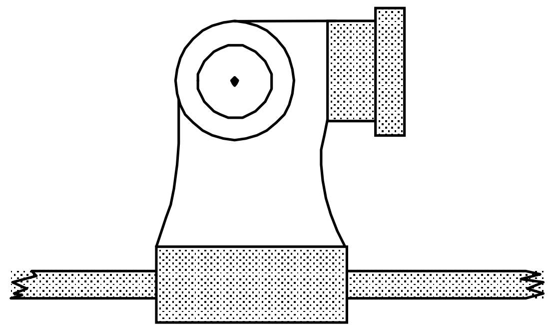
This circle passes through the points of intersection of the parabola and the line with equation $x = 5$.

What is the degree measure of the arc of the circle located inside the parabola?

9 The drawing at the right was prepared by an architect. It includes two concentric circles whose radii differ by 8 cm.

In a Cartesian coordinate system, the equation of the larger circle is :

$$x^2 + y^2 - 12x - 16y - 44 = 0$$

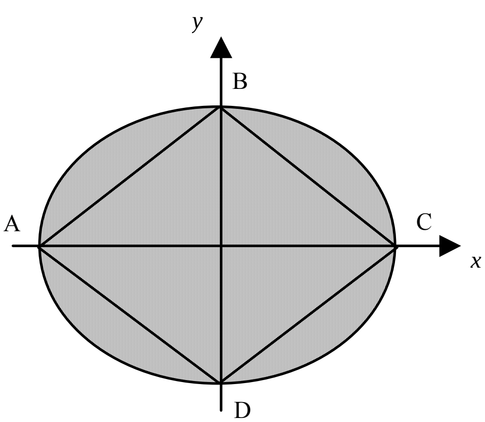


In the same system of axes, what is the equation of the smaller circle?

10 A municipal pool is in the shape of an ellipse. Its equation on a plane graduated in metres is :

$$\frac{x^2}{400} + \frac{y^2}{225} = 1$$

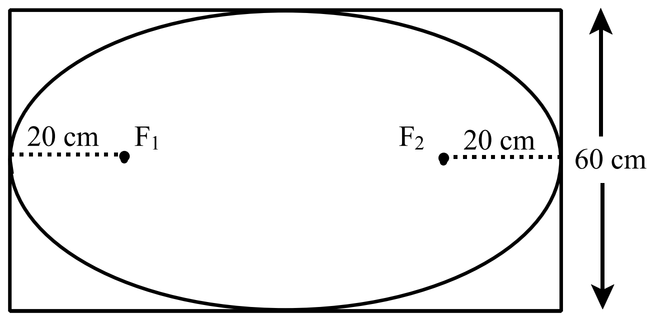
Lifeguards are placed at points A, B, C and D of the ellipse. A cable passing through these points has been installed for safety.



Rounded to the nearest degree, what are the measures of the angles of the rhombus thus formed?

11 A carpenter places two nails on a rectangular piece of plywood as illustrated in the diagram below. Each nail is placed 20 cm from each of the smaller edges. The smaller edges are 60 cm in length.

The carpenter uses these two nails as focal points to draw the largest ellipse possible on the rectangular plane.



Find the area of the rectangular piece of plywood.
Round your answer to the nearest cm^2.

12 A microbiologist is studying two bacteria populations.

Last Monday, the 1st population numbered 2000 and the 2nd numbered 2 048 000.

He noted that the 1st population doubled every day while the 2nd population was reduced by half each day.

After how many days would the two populations of bacteria be equal in number?

13 At last count, a certain species of bird numbered 200, raising fears that its extinction was imminent. Hunting this species is now forbidden. With this law in place, biologists claim that the population of this species will double every 6 months.

They believe that the species will be saved once its population reaches 18 500.

In how many years will this species no longer be threatened with extinction?

14 Three years ago Greg invested \$1000 at a fixed interest rate compounded every 6 months. His investment is currently valued at \$1400.

Given $C_n = C_0\left(1 + \frac{t}{k}\right)^{nk}$ where

C_n	is the capital after n years
C_0	is the capital invested
t	is the annual interest rate
k	is the number of times per year that interest is paid
n	is the number of years

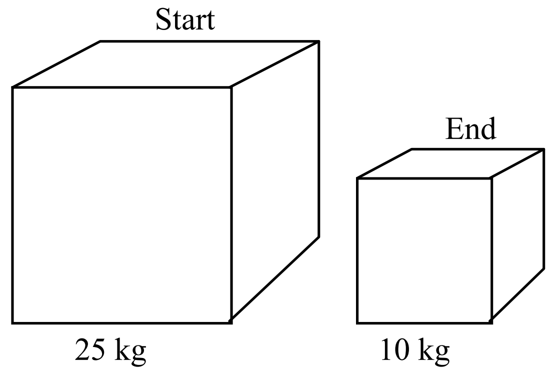
To the nearest centimetre, what is the annual rate of interest?

15 On their last fishing trip, Louis and Carmen took a block of dry ice with them.

When they started out, the block had a mass of 25 kg. During the trip, the block sublimated at a constant rate such that every 12 hours its mass decreased by 9 %.

At the end of the trip, the block had a mass of 10 kg.

To the nearest tenth of an hour, how long did their trip last?



16 In 1990, the deer population on Anticosti Island was estimated at 70 000. If this population doubles every 50 years, this situation can be translated by the following function:

$$P(t) = (70\,000)2^{\frac{t}{50}}$$

In what year will there be 100 000 deer?

17 A car loses 15 % of its value per year for the first 4 years and 10 % a year for the years that follow.

After how many years will a \$28 000 car be worth \$6 632.13?

18 When Jennifer bought a new car in 1995, she paid \$17 500. In 1998 the value of her car had fallen to \$10 000. She decided that she would sell her car when the value fell below \$5000.

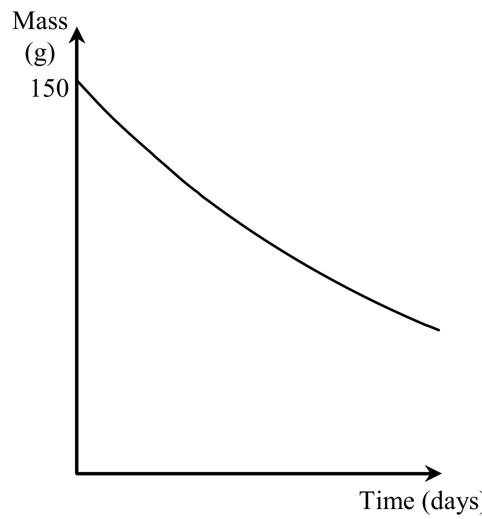
Assuming the decline in the price of a car is modelled by an exponential function, how old will Jennifer's car be when its value falls below \$5000? Round your answer to the nearest month.

19 In January 1990, there were 5.5 billion people living on this planet. The population has been growing at a rate of 1.9% per year.

In which year will the population reach 9 billion?

20 A chemist is working with a dangerous compound she has just created. She began with 150 g of the compound, but noticed that it decays exponentially. After observing for 10 days, 123 g remained. She needs to know how long it will take until only half of the compound will be left.

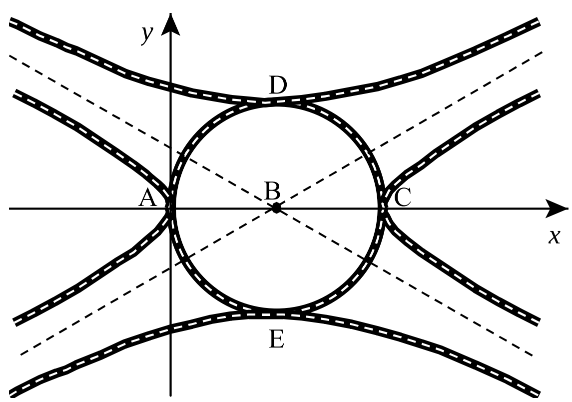
Rounded to the nearest day, how many days after the experiment started will only half of the compound remains?



21 A radioactive substance disintegrates at a rate such that after 2 years it has $\frac{4}{9}$ of its initial mass. If you have 60 grams of this substance, how much of it will remain after 12 years?

Note : Express your answer in grams to the nearest hundredth.

22 Highways in the shape of hyperbolas run alongside asymptotes and merge with a circular intersection.



In the diagram above:

- points D, B and E are collinear,
- the circle is 12 units in diameter,
- the foci of the hyperbola with vertices D and E are 20 units apart,
- vertex A is located at the origin of the Cartesian coordinate system,
- point B is the centre of the circle.

What is the equation of the hyperbola with vertices D and E?

23 The line whose equation is $4x + 3y - 43 = 0$ is tangent to a circle with centre C(3, 2).

What is the equation of this circle?

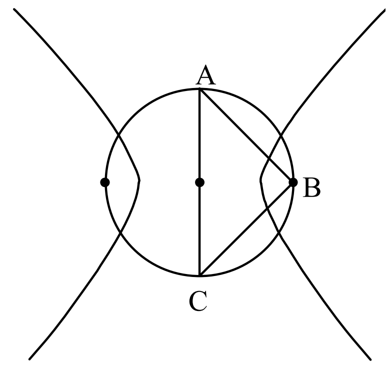
24 A figure is formed by two conic sections whose measures are in centimetres.

The equation of the first conic is $\dfrac{(x-3)^2}{64} - \dfrac{(y-4)^2}{36} = 1$.

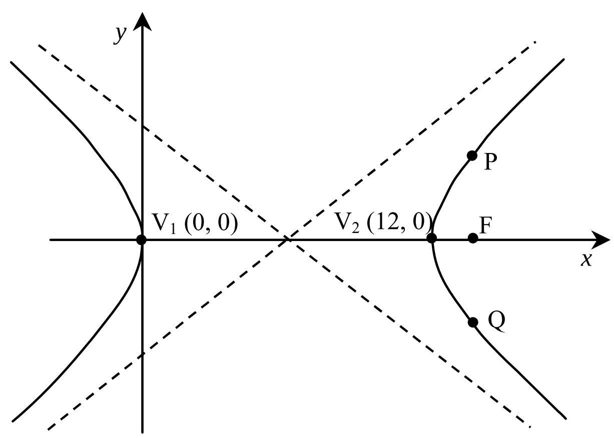
The second conic is a circle that passes through the foci of the first.

A right isosceles triangle ABC is inscribed in the semi-circle, as illustrated in the diagram on the right.

What is the perimeter of triangle ABC?



25 A park development plan is represented in the Cartesian plane below. The scale of the graph is in metres.



The branches of the hyperbola represent two hedges. Point V_1 and V_2 represent the vertices of the hyperbola.

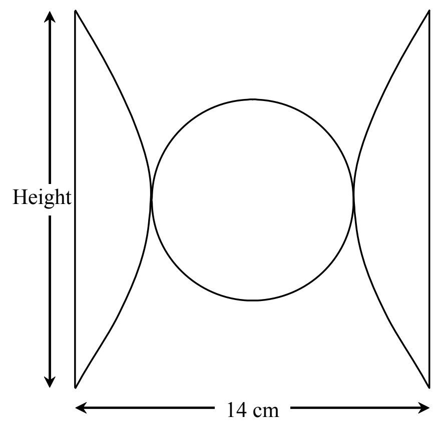
Two statues are located in the hedge on the right. Points P and Q represent the location of these statues. Line segment PQ is parallel to the y-axis. Focus F of the hyperbola is located on segment PQ.

The asymptotes of this hyperbola represent the sidewalks.

The equations of the asymptotes are $y = \dfrac{4}{3}x - 8$ and $y = \dfrac{-4}{3}x + 8$.

What is the distance between the two statues to the nearest metre?

26 A modern picture frame is in the shape of a circle between the two branches of a hyperbola, as shown in the diagram below.



The equation of the circle is $(x - 13)^2 + (y - 10)^2 = 16$. The centre of the circle and the centre of the hyperbola coincide. The vertices of the hyperbola are the endpoints of the horizontal diameter of the circle and the vertical edges of the picture frame pass through the foci of the hyperbola. The total length of the frame is 14 cm.

What is the height of the frame?

27 Solve the following logarithmic equation :

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

28 Two rival companies A and B decided to make the same product using two different processes.

The following functions represent the gross profits of the two companies:

$a(t) = 1000 \log_4 t$ for company A
$b(t) = 1000 \log_5 t$ for company B
where t is time in months.

After the twelfth month, the expenses of company A were \$800 and company B, \$500.

Which company had the greatest net profit after the twelfth month?

N.B. net profit = gross profit ! expenses

29 What is the solution of the following equation?

$$\log_5 (x - 1) + \log_5 (x + 3) - 1 = 0$$

30 Wheeler is a producer of mountain bikes and road bikes. Because of its small size, it can build no more than 80 bikes each week. To meet certain conditions in its workshop, it must build at least 45 mountain bikes, and at least 10 road bikes weekly. To meet consumer demand, it must manufacture at least 3 times as many mountain bikes as road bikes.

Use:
x = the number of road bikes produced weekly
y = the number of mountain bikes produced weekly

For each road bike and mountain bike produced, Wheeler earns a profit of $250 and $175, respectively.

What is the maximum weekly profit that can be earned?

31 A fisherman has to separate his daily catch of shellfish into two categories before he can sell them. Lobsters are sold for $8.70 each and crabs are sold for $9.60 each.

On an average day, the fisherman can expect to catch a minimum of 35 crabs and a maximum of 60. By experience, there are at most twice as many lobsters as crabs in a daily catch and never has the fisherman caught more than 140 shellfish in a single day.

Using a polygon of constraints, determine the maximum revenue that this fisherman can expect to make.

32 Murray plans a trip to New York in July. In order to save money, he works at two different part-time jobs on weekends. At the first job, he works a minimum of 10 hours per month and at the second, a maximum of 40 hours per month. Murray must work at least 30 hours per month but no more than 60 hours per month. He must work at least as many hours at the second job as he does at the first. He makes $6.30 an hour at the first job and $8 an hour at the second job.

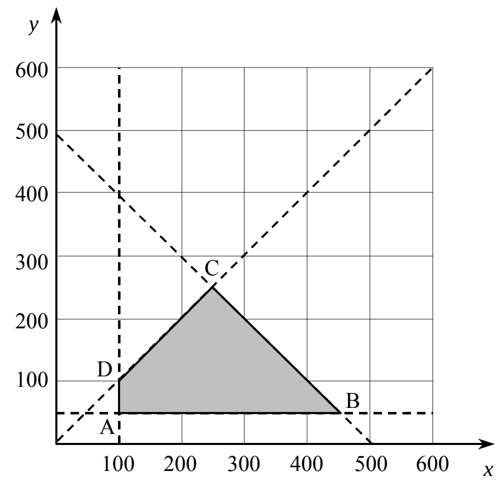
Let x: number of hours per month at first job
y: number of hours per month at second job

Because of a shortage of employees, Murray was later advised that he could increase the number of hours he worked at the second job.

By how much did Murray's maximum possible salary increase because of the employee shortage?

33 To raise money, the Graduation Committee decides to sell cases of fruit. The following polygon represents the constraints that must be respected.

If x represents the number of cases of oranges for sale and y, the number of cases of grapefruit for sale, the constraints are:



① $x \leq y$
② $x + y \leq 500$
③ $x \geq 100$
④ $y \geq 50$

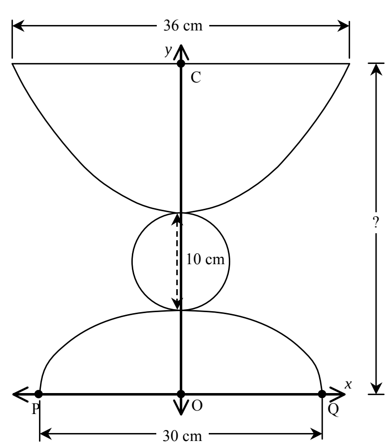
For each case of oranges and grapefruit sold, the Graduation Committee makes a profit of $1.00 and $1.50, respectively.

Yesterday, the head of the committee received a call from the supplier. Because of a recent flood, the supplier can deliver a maximum of 400 cases of fruit.

By how much will the maximum possible revenue decrease because of the flood?

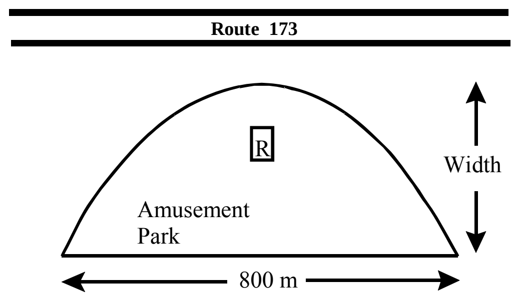
34 The cross-sectional view of a punchbowl is shown below. The view of the punchbowl's base is in the shape of a semi-ellipse, whose major axis, PQ, measures 30 cm. The foci of this semi-ellipse are 12 cm from its centre.

Directly above the semi-ellipse lies a circle whose diameter is 10 cm. On top of the circle lies a parabola whose vertex touches the circle and whose directrix passes through the centre of the circle. The width of the punchbowl at the top is 36 cm.



What is the height, CO, of the punchbowl?

35 The fence surrounding an amusement park is in the shape of a parabola. Each point that lies on this fence is the same distance from Route 173 as it is from restaurant **R**, located inside the park. The distance between Route 173 and restaurant **R** is 200 metres. The length of the amusement park is 800 metres.



What is the width of the amusement park?

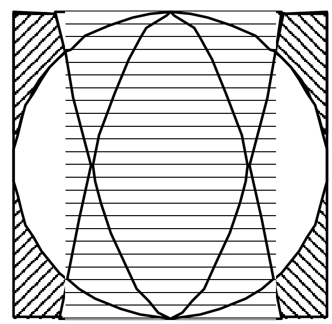
36 A packaging company decorates its square boxes according to the model shown below.

The equation of the circle in the model has the form :

$$(x-h)^2 + (y-k)^2 = r^2$$

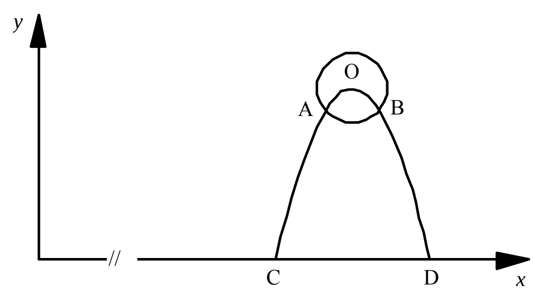
The equations of the parabolas are :

$x^2 - 8x - y + 16 = 0$ and
$x^2 - 8x + y + 8 = 0$



If the cartesian unit of measure is 1 cm, calculate the length of the diagonal of this box.

37 A sculpture in the garden of a contemporary art museum consists of a circle and a parabola, as shown below.



The vertex of the parabola coincides with the centre O of the circle which has the equation

$$x^2 + y^2 - 32x - 10y + 279 = 0$$

where the unit of measure is the metre.

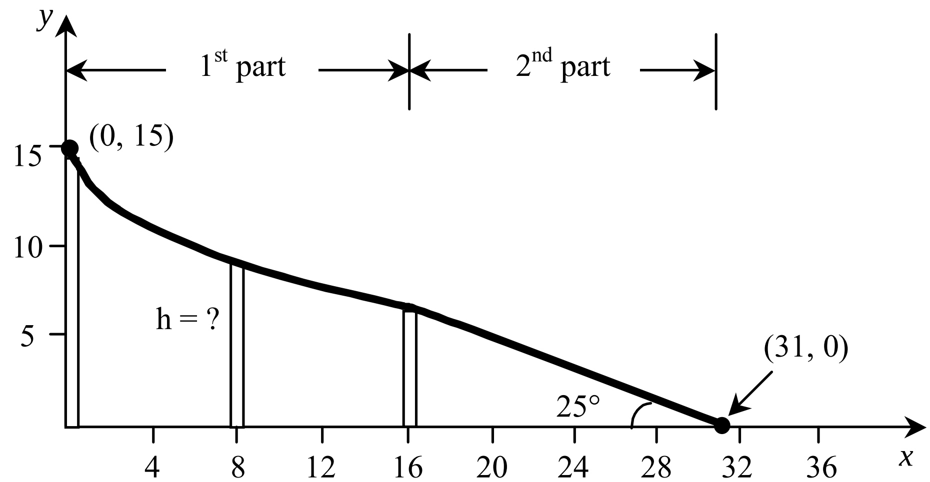
The parabola, whose axis of symmetry is vertical, is constructed such that angle AOB is a right angle.

Rounded to the nearest hundredth, what distance is there between bases C and D of the sculpture?

38 The graph shown below, with the axes graduated in meters, represents a slide that spans 31 metres in length.

Three posts placed 8 meters apart support the first part of the slide. This part of the slide is represented by a square root function with vertex (0, 15).

The second part of the slide is linear and forms an angle of 25° with the horizontal.



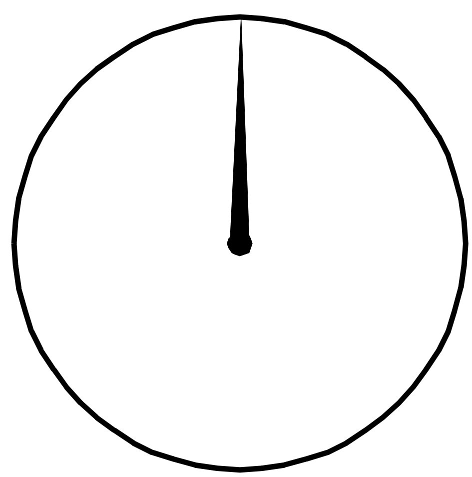
As shown in the graph above, the second post is 8 meters away from the first, which is located on the *y*-axis.

What is the height of the second post?
Round your answer to the nearest hundredth of a metre.

39 The given clock only has a second hand.

At noon, Maria sees the tip of the second hand at the top of the clock and notes the height of the tip of second hand in relation to the bottom of the clock. The second hand is 10 cm long.

What is the function rule describing the height (h) in cm of the tip of the second hand compared to the time (t) elapsed in seconds since noon?



40 A fountain in a shopping centre has a single jet of water. The height of the jet of water varies according to a sinusoidal function. Joel notes that, in exactly one minute, the jet goes from a minimum height of 1 m to a maximum height of 5 m and back to 1 m.

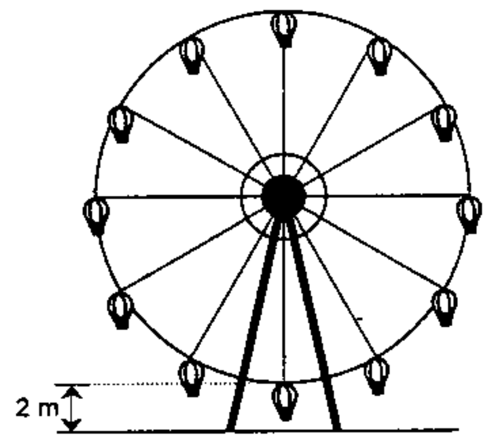
At 13:00, the jet of water is at a height of 1 m.

What will be the height of the jet of water, to the nearest tenth of a metre, when the clock reads 13:12:40? (13 hours, 12 minutes, 40 seconds)

41 The gondolas on a Ferris wheel are secured at a point 10 m from its centre.

Mark entered a gondola when it was resting at the bottom of the Ferris wheel. At that time, the point at which the gondola is secured to the wheel was 2 m off the ground.

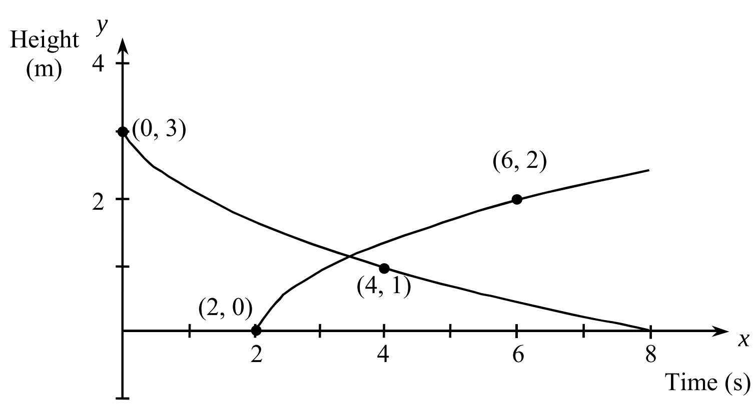
The Ferris wheel then began turning at a constant speed. It takes this wheel 3 minutes to make a complete revolution.



The distance between the ground and the point at which Mark's gondola is secured to the wheel in relation to the time elapsed from the moment he got on the ride is represented by a sinusoidal function.

Seven minutes after Mark got on the ride, what was the distance between the ground and the point at which his gondola is secured to the wheel?

42 Two missiles are launched 2 seconds apart. The paths they follow over a span of 8 seconds can be represented by two different square root functions, as illustrated below:

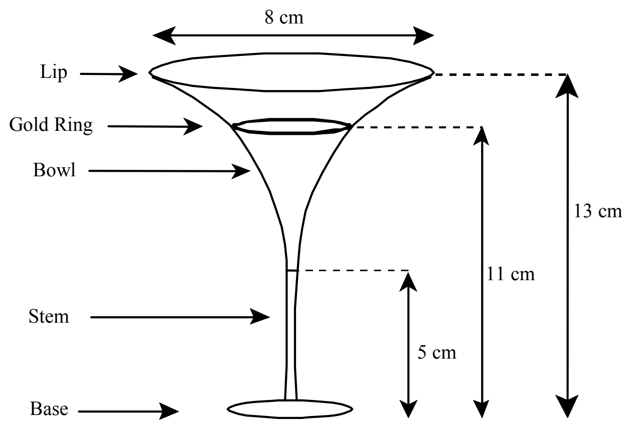


How many seconds after the 2nd projectile has been launched, will it be higher than the 1st projectile?

43 A new glass has been designed by rotating part of a graph of a square root function about the axis containing the stem of the glass. (Assume the width of the stem to be zero.)

As illustrated in the diagram, the diameter of the lip of the bowl is 8 centimetres. The glass stands 13 centimetres in height and the top of the stem of the glass is 5 centimetres high.

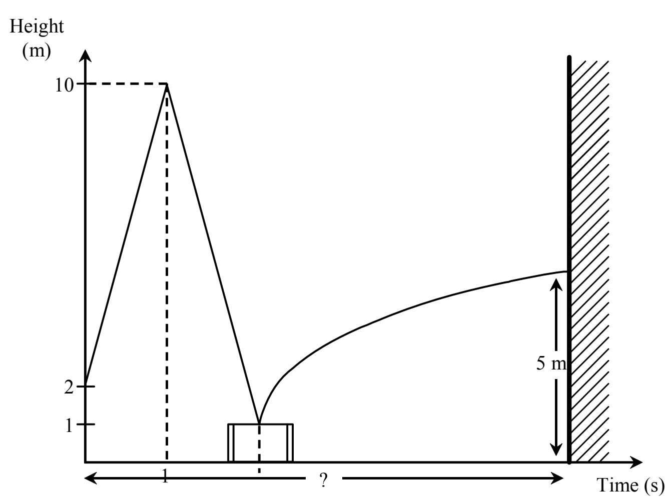
A decorative gold ring is to be painted around the bowl 11 centimetres from the bottom of the glass, at a cost of 2 cents per centimetre.



How much will it cost to paint the gold ring around the bowl?

Round your answer to the nearest cent.

44 A tennis ball is hit by a racket from a height of 2 metres and follows the path of an absolute value function. One second later the ball hits the ceiling, which has a height of 10 metres. On its way down, the ball bounces off a table that is 1 metre high. After the bounce, its path is a semi parabola. One second after the ball hits the table, it reaches a height of 3 metres before hitting a wall at a height of 5 metres.



How many seconds after the ball was hit by the racket did it hit the wall?

45 Cindy is in charge of buying food for the school cafeteria. She wants to buy two types of bread: raisin bread and olive bread. Cindy must respect the following constraints:

- she can store at most 1000 loaves of bread;
- she must buy at least 200 loaves of raisin bread;
- she must buy at least 100 loaves of olive breads but no more than 350;
- she must buy at least as many loaves of raisin bread as olive bread.

The cafeteria makes a \$0.10 profit on each loaf of raisin bread and a \$0.20 profit on each loaf of olive bread.

How many loaves of bread of each type must Cindy buy to maximize the cafeteria's profits?

46 Prove the following trigonometric identity :

$$\text{cosec A}\,(\text{cosec A} + \cot \text{A}) = \frac{1}{1 - \cos \text{A}}$$

47 Prove the following identity :

$$\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} = 2\sec\theta\,.$$

48 Prove the following identity :

$$\frac{\sec\theta}{1 - \cos\theta} - \frac{\sec\theta}{1 + \cos\theta} = 2\,\text{cosec}^2\theta\,.$$

49 Prove the following identity :

$$\frac{2}{\text{cosec a}} + 2\sin a\,\tan^2 a = \frac{2\tan a}{\cos a}$$

50 Suzanne is a sea captain. Her knowledge of trigonometry helps her calculate distances at sea. She must simplify a trigonometric expression to verify that her calculations are correct.

Show that Suzanne's calculations are correct by proving the following identity :

$$\frac{2 - 2\sin A \cos A \times \tan A}{2} = \cos^2 A$$

51 Prove the given trigonometric identity.

$$\frac{\operatorname{cosec} A - \sin A}{\cos A} = \cot A$$

52 A plane goes from city A to city B. In a Cartesian plane, city A is at the origin and city B has coordinates (100, 150). If there is no wind, the flight lasts one hour.

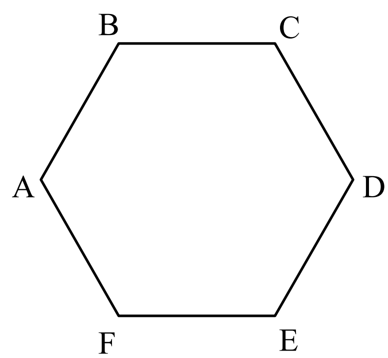
Unfortunately, there is a wind. If the pilot does not adjust his flight path, he will be at point (120, 160) after an hour.

What is the speed of the wind?

53 Given the regular hexagon on the right

where $\overrightarrow{AB} = \vec{a}$

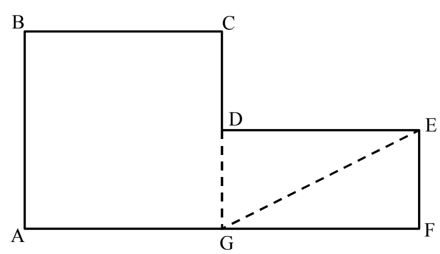
and $\overrightarrow{BC} = \vec{b}$.



Prove the following identity: $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{DE} + \overrightarrow{EF} = \vec{a}$.

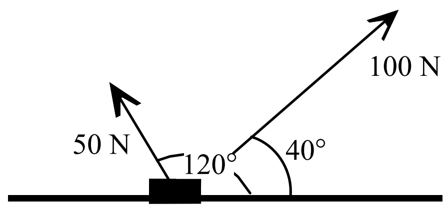
54 In the polygon below, ABCG is a square.

D and G are the midpoints of sides CG and AF, respectively. Side AB is parallel to side EF.



Using the properties of vectors, show that $\overrightarrow{CB} + \overrightarrow{AC} - \overrightarrow{FE} + \overrightarrow{GF} = \overrightarrow{GE}$.

55 Peter and Marie are pulling on an object. The forces they applied are 100 N and 50 N respectively but in different directions: 40° and 120°. The situation is represented below.



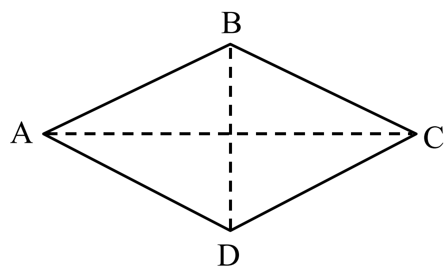
Tim is going to replace them.

What force must Tim apply to produce the same effect on the object (strength and direction)?

56 Given the adjacent rhombus ABCD.

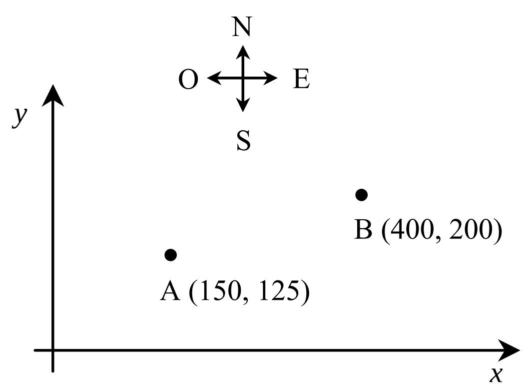
Use vectors to prove the following statement:

« The diagonals of the rhombus are perpendiculars. »



57 An airplane leaves airport A and must fly to airport B. In the Cartesian plane on the right, these airports are represented by points A and B respectively. The scale of the graph is in kilometres.

During the flight, the plane encounters a steady wind. This wind is represented by the vector $\vec{v}$ = (20, -15).



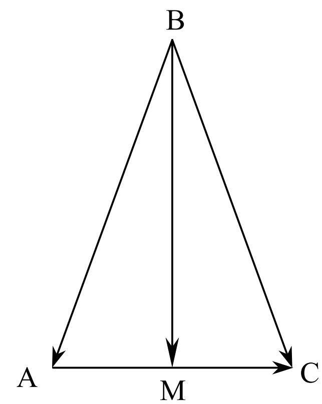
The pilot steers the plane so as to negate the effect of the wind.

To the nearest degree, at what angle relative to the east should the pilot point the plane in order to reach airport B?

58 Triangle ABC, shown on the right, is isosceles.

Using vectors, show that median $\overrightarrow{BM}$ is equal to

$$\overrightarrow{BM} = \frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{BC}\right).$$



59 If $x \in [0, 2\pi[$, find the solution set of the following equations :

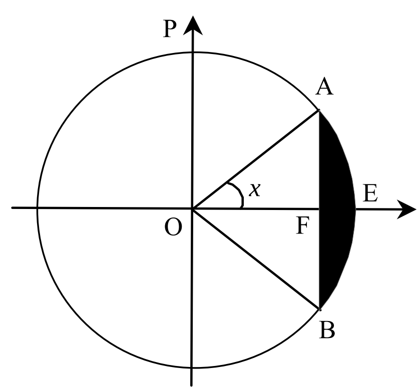
a) $\sin^2 x + \cos x = 1$

b) $2\sin^2 x - \cos x - 2 = 0$

60 $\text{Sin } x = \frac{\sqrt{3}}{2}$ in the circle with centre O, show on the right.

Furthermore, $\overline{AB} \,//\, \overline{OP}$ and m $\overline{OA}$ = 1 unit.

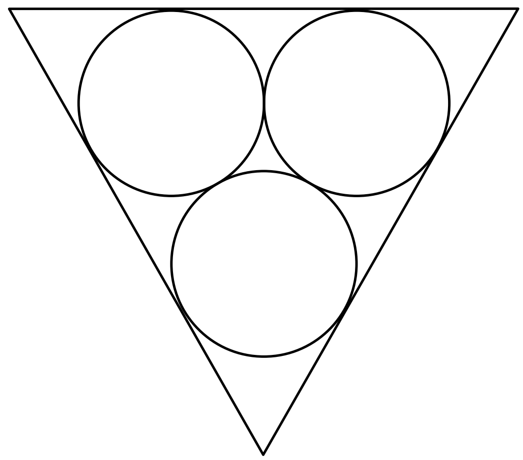
What is the area of the shaded region?



61 Find all the solutions of the equation $2\cos^2 x - 3\sin x - 3 = 0$ such that

$x \in \left[\frac{3\pi}{2}, 2\pi\right]$. Give your solution in exact radian measure.

62 Strange geometric formations, known as crop circles, have appeared in fields around the world. The creators of the crop circle shown below would like to surround their design with a border, in the shape of an equilateral triangle.



The circles that were used to make the design are congruent to the circle whose equation is:

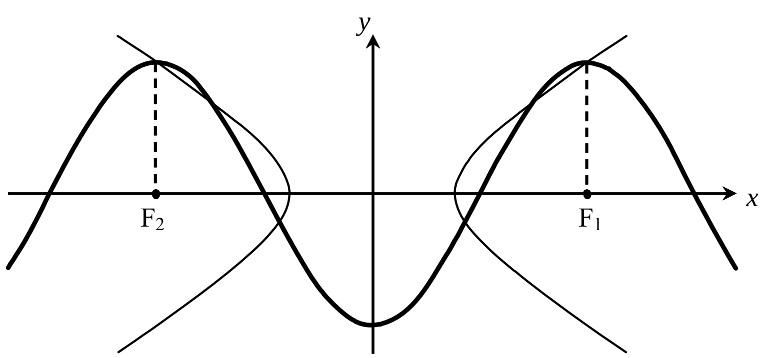
$$x^2 + y^2 - 6x - 2y - 26 = 0$$

The circles are externally tangent to one another and tangent to the border.

What is the length of the border?
Round your answer to the nearest hundredth of a unit.

63 What is the solution set of the equation $2\sin^2 x - 4\cos^2 x + 1 = 0$ where $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$?

64 A hyperbola and a trigonometric function are drawn on the same Cartesian plane. The equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{144} = 1$.



The foci of the hyperbola are directly below two of the maxima of the trigonometric function.

Give an equation of the trigonometric function.

Math 5 SN June Review Answer Key

1 General equation of the absolute value function

$t(x) = a|x - h| + k$

Determining the value of ***a*** knowing one point and the minimum

$-1 = a|0 - 8| + (-5)$ thus $a = 0.5$

Equation of function t

$t(x) = 0.5|x - 8| - 5$

Calculating the length of time

$0.5|x - 8| - 5 \leq -3$

thus $x \geq 4$ and $x \leq 12$

therefore the length of time is $12 - 4 = 8$

Result : The length of time the temperature was less than or equal to -3 °C was 8 hours.

2 The situation can be represented by an absolute value function of the form $f(x) = a|x - h| + k$

where x represents the number of months elapsed and $f(x)$ represents the value of the share, in $.

Share's initial value: (0, 25)
Share's value May 31st: (5, 10) = (h, k)

Value of parameter a, using ordered pair (0, 25)

$f(x) = a|x - 5| + 10$

$25 = a|0 - 5| + 10$

$a = 3$

Rule of the function

$f(x) = 3|x - 5| + 10$

Value of f(x) on December 31st

$f(x) = a|12 - 5| + 10$

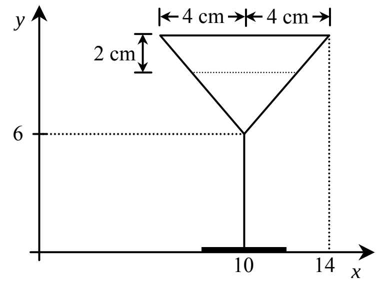
$f(x) = 31$

Answer On December 31st 1999, the share's value was $31.

3 Height of the cocktail glass

If $x = 10 + 4 = 14$, then $f(14) = \frac{5}{4}|14 - 10| + 6 = 11$.

The height of the cocktail glass is 11 cm.



Point on the glass where the gold leaf strip is applied

$11 - 2 = 9$ cm

Diameter of the glass at the point where the gold leaf strip is applied

$f(x) = 9$ therefore $9 = \frac{5}{4}|x - 10| + 6$

$3 = \frac{5}{4}|x - 10|$

$2.4 = |x - 10|$

$x - 10 = 2.4$ and $|x - 10| = -2.4$

$x = 12.4$ $x = 7.6$

The diameter of the glass at the point where the gold leaf strip is applied:

$12.4 - 7.6 = 4.8$

Answer: The diameter of the glass at the point where the gold leaf strip will be applied is **4.8** cm.

4 Let x: time measured in weeks

$f(x)$: percentage of popular vote

Coordinates of points P(0, 28) S(10, 43)

Find the equation

$f(x) = a|x - 10| + 43$ domain $f = [0, 26]$

Substitute P(0, 28) into the equation

$28 = a|0 - 10| + 43$

$-15 = a \bullet 10$

$-1.5 = a$

$f(x) = -1.5|x - 10| + 43$

Solve the equation

$f(x) = 25$

$-1.5|x - 10| + 43 = 25$

$-1.5|x - 10| = -18$

$|x - 10| = 12$

$|x - 10| = 12$

$x - 10 = 12$ or $-x + 10 = 12$

$x = 22$ $-x = 2$

$x = -2$ before the poll

Answer: Party "A" has one quarter of the popular vote at **22** weeks.

5 $\sin A \cot A + 2\cos^2 A = 1$

$$\sin A \frac{\cos A}{\sin A} + 2\cos^2 A = 1$$
$$\cos A + 2\cos^2 A = 1$$
$$2\cos^2 A + \cos A - 1 = 0$$
$$(2\cos A - 1)(\cos A + 1) = 0$$
$$\cos A = \frac{1}{2} \quad \text{or} \quad \cos A = -1$$

$A = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ or $A = \pi$

Answer $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

6 $V(t) = -25|t - 8| + 65$
$V(t) \geq 35$

$$-25|t - 8| + 65 \geq 35$$
$$-25|t - 8| \geq 35 - 65$$
$$|t - 8| \leq \frac{-30}{-25}$$
$$|t - 8| \leq 1.2$$

If $t - 8 \geq 0$, then $|t - 8| = t - 8$;
$$t - 8 \leq 1.2$$
$$t \leq 9.2$$

if $t - 8 < 0$, then $|t - 8| = -(t - 8)$
$$-(t - 8) \leq 1.2$$
$$-t + 8 \leq 1.2$$
$$-t \leq -6.8$$
$$t \geq 6.8$$

$$9.2 - 6.8 = 2.4$$

Answer The police officer should be on duty for 2.4 hours.
(Accept 2 hours and 24 minutes)

7

Slope of left ray

$$\frac{7-4}{0-(-2)} = \frac{3}{2}$$

$\therefore$ equation of left ray

$$y = \frac{3}{2}x + 7$$

Slope of right ray

$$\frac{-3}{2}$$

Equation of right ray

$$y = \frac{-3}{2}x + b$$

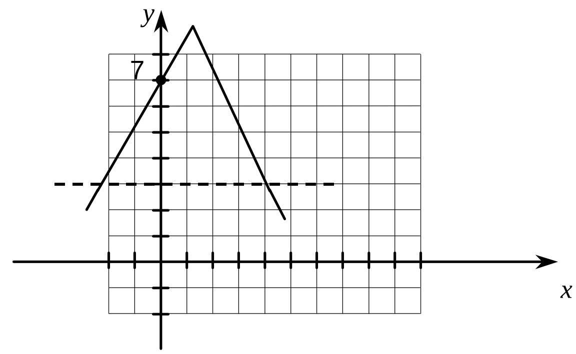
$$5 = \frac{-3}{2}(4) + b$$

$$5 = \frac{-12}{2} + b$$

$$5 = -6 + b$$

$$11 = b$$

$$y = \frac{-3}{2}x + 11$$



Vertex of absolute value function is the intersection of both rays

$$\frac{3}{2}x + 7 = \frac{-3}{2}x + 11 \qquad y = \frac{3}{2}\left(\frac{4}{3}\right) + 7$$

$$\frac{6}{2}x = 4 \qquad y = \frac{12}{6} + 7$$

$$x = \frac{4}{3} \qquad y = 9$$

Equation of absolute value function

$$y = a|x - h| + k$$

$$y = \frac{-3}{2}\left|x - \frac{4}{3}\right| + 9$$

The vertex of the birdhouse is $\left(\frac{4}{3}, 9\right)$, which yields the altitude of the triangle to be $9 - 3 = 6$ units.

The base is the intersection of the absolute value function and the constant function

$$\frac{-3}{2}\left|x-\frac{4}{3}\right|+9=3$$

$$\frac{-3}{2}\left|x-\frac{4}{3}\right|=-6$$

$$\left|x-\frac{4}{3}\right|=4$$

x

$\therefore \quad x-\frac{4}{3}=4 \quad$ or $\quad x-\frac{4}{3}=-4$

$x=\frac{16}{3} \qquad\qquad x=\frac{-8}{3}$

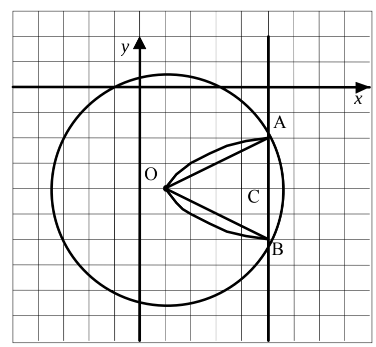
and base has length $\frac{16}{3} - \frac{-8}{3} = 8$ units

$\therefore$ Area of front face $= \frac{1}{2}(6)(8)$

$= 24u^2$

Answer: The area of the shaded triangular section of the birdhouse is $\mathbf{24u^2}$.

8



Co-ordinates of the vertex of the parabola

Transform the equation $y^2 - x + 8y + 17 = 0$ to the form

$(y-k)^2 = 4c(x-h)$, which gives $(y+4)^2 = 4(\frac{1}{4})(x-1)$.

The co-ordinates of the vertex are (1, -4).

The points of intersection of the parabola and the line

The equation of the line is $x = 5$. Replace x by 5 in the equation $(y+4)^2 = x - 1$, which gives $(y+4)^2 = 5 - 1$

$(y+4)^2 = 4$

$y_1 = -6$ and $y_2 = -2$

The points of intersection are (5, -6) and (5, -2).

Measure of arc AB

(Arc AB has the same measure as central angle AOB.)

Draw $\overline{OC} \perp \overline{AB}$.

$\tan \angle AOC = \frac{m\,\overline{AC}}{m\,\overline{OC}} = \frac{2}{4}$

$m\angle AOC \approx 26.565°$

$m\angle AOB \approx 53.13°$

$m\,\overset{\frown}{AB} \approx 53.13°$

Result : The measure of the arc is approximately 53.13°.

9 Equation of the large circle in standard form

$$x^2 + y^2 - 12x - 16y = 44$$
$$(x^2 - 12x + 36) + (y^2 - 16y + 64) = 44 + 36 + 64$$
$$(x - 6)^2 + (y - 8)^2 = 144$$
$$(x - 6)^2 + (y - 8)^2 = 12^2$$

The large circle is centered at point (6, 8) and its radius is 12 cm.

Radius of the small circle

$12 - 8 = 4$ cm

Centre of the small circle

Two concentric circles have the same centre : (6, 8)

Equation of the small circle

$$(x - 6)^2 + (y - 8)^2 = 4^2$$

or

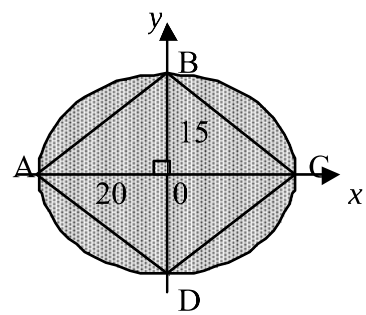
$$(x - 6)^2 + (y - 8)^2 = 16$$

or

$$x^2 + y^2 - 12x - 16y + 84 = 0$$

Result : The equation of the small circle is $(x - 6)^2 + (y - 8)^2 = 4^2$.

10 Using the equation of the ellipse $\dfrac{x^2}{400} + \dfrac{y^2}{225} = 1$



$a = \sqrt{400} = 20$
$b = \sqrt{225} = 15$

Since the axes of the ellipse intersect at right angles, triangle AOB is right-angled at O.

Axis AC bisects angle A. By trigonometric proportions :

$$\tan \frac{A}{2} = \frac{15}{20}$$
$$\frac{m\angle A}{2} \approx 36.9°$$
$$m\angle A = 73.8°$$

To the nearest degree, m ∠A is 74°.

Consecutive angles in a quadrilateral are supplementary.

Therefore ∠B = 180° – m ∠A
= 180° – 74°
= 106°

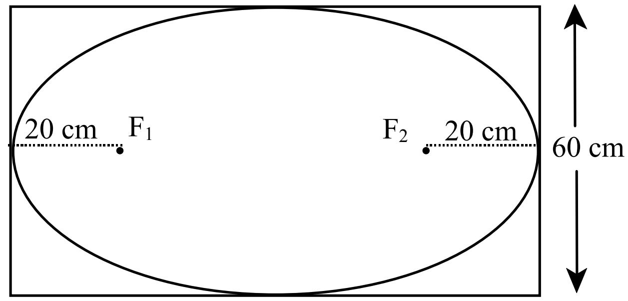
Opposite angles of a rhombus are congruent.

m ∠A = m ∠C = 74°
m ∠B = m ∠D = 106°

Result : Angles A and C measure 74°.
Angles B and D measure 106°.

11 Since width of rectangle is 60 cm, minor axis measures 60 cm. So $b = 30$ cm.

If c represents distance between centre and focal point, then $c + 20$ equals the length of the semi-major axis, a.



Solving for c:

$$a^2 = b^2 + c^2$$
$$(c + 20)^2 = 30^2 + c^2$$
$$c^2 + 40c + 400 = 900 + c^2$$
$$40c = 500$$
$$c = 12.5$$

Therefore length of plywood is $2(c + 20) = 2(12.5 + 20) = 65$

Area of plywood: $65 \times 60 = 3900$

Answer Rounded to the nearest cm^2, the area of the plywood is 3900 cm^2.

12 $2000 \times 2^t = 2\,048\,000 \times 0.5^t$

$$2^t = \frac{2\,048\,000}{2000} \times 0.5^t$$
$$2^t = 1024 \times 0.5^t$$
$$t \log 2 = t \log 0.5 + \log 1024$$
$$t \log 2 = -t \log 2 + \log 1024$$
$$2t \log 2 = \log 1024$$
$$2t = \frac{\log 1024}{\log 2} = 10$$
$$2t = 10$$
$$t = 5$$

Result : The two populations will be equal in number after 5 days.

13

Mathematizing the situation

$$f(x) = 200 \times 2^{2x}$$

Calculate the number of years

$$18\,500 = 200 \times 2^{2x}$$

$$\frac{18\,500}{200} = 2^{2x}$$

$$\frac{185}{2} = 2^{2x}$$

$$92.5 = 2^{2x}$$

$$\log 92.5 = 2x \log 2$$

$$\frac{\log 92.5}{\log 2} = 2x$$

$$6.5314 \approx 2x$$

$$3.2657 \approx x$$

Answer : This species of birds will no longer be threatened with extinction in about 3.3 years.
Accept an answer in [3, 4[.

14

1. $1400 = 1000 \times \left(1 + \frac{t}{2}\right)^{3 \times 2}$
2. $1.4 = \left(1 + \frac{t}{2}\right)^{3 \times 2}$
3. $\log 1.4 = 6 \log \left(1 + \frac{t}{2}\right)$
4. $\frac{0.146128}{6} \approx \log\left(1 + \frac{t}{2}\right)$
5. $0.0243 \approx \log \left(1 + \frac{t}{2}\right)$
6. $10^{0.0243} \approx 1 + \frac{t}{2}$
7. $1.0576 \approx 1 + \frac{t}{2}$
8. $1.0576 \approx \frac{t}{2}$
9. $0.1152 \approx t$

Answer : The annual rate of interest is 11.52 %.
Accept all answer in [11.51 ; 11.54].

15 Given : Q(*x*) is the quantity of dry ice left and *x* is the number of hours that have elapsed since the beginning of the trip.

$$Q(x) = 25 \times 0.91^{\frac{x}{12}}$$

Calculate the number of hours that elapsed:

$$10 = 25 \times 0.91^{\frac{x}{12}}$$

$$\frac{10}{25} = \frac{25 \times 0.91^{\frac{x}{12}}}{25}$$

$$0.4 = 0.91^{\frac{x}{12}}$$

$$\frac{x}{12} = \log_{0.91} 0.4$$

$$\frac{x}{12} = \frac{\log 0.4}{\log 0.91}$$

$$x = 116.58796$$

Result : The duration of the trip was 116.6 hours.

16

$$\text{P(t)} = (70\ 000)2^{\frac{t}{50}}$$

$$\text{P}_i = 70\ 000$$

$$\text{P}_f = 100\ 000$$

$$100\ 000 = (70\ 000)2^{\frac{t}{50}}$$

$$2^{\frac{t}{50}} = \frac{100\ 000}{70\ 000} = \frac{10}{7}$$

$$\frac{t}{50}\log 2 = \log\left(\frac{10}{7}\right)$$

$$t = 50\left(\log\left(\frac{10}{7}\right) \div \log 2\right) = 25.7286$$

$$t \approx 25.7 \text{ years}$$

$$1990 + 25.7 = 2015.7$$

Result : The number of deer will be 100 000 in the year 2015.

17 Value of the car after 4 years

$$28\ 000 \times 0.85^4 = \$14\ 616.18$$

Mathematize the situation for $t > 4$

$$V(t) = 14\ 616.18 \times 0.9^t$$

where $V(t)$ is the value of the car after t years.

Value of t for $V(t) = 6632.13$

$$6632.13 = 14\ 616.18 \times 0.9^t$$
$$0.453752 \approx 0.9^t$$
$$\log 0.453752 \approx t \log 0.9$$
$$t \approx \frac{\log 0.453\ 752}{\log 0.9}$$
$$t \approx 7.5$$

Number of years elapsed

$$4 + 7.5 = 11.5 \text{ years}$$

Result : A \$28 000 car will be worth \$6 632.13 after 11.5 years.

18 Let t: time after 1995 (years)

$V(t)$: value of the car (\$)

$V(t) = 17\ 500(r)^t$ where r is the rate at which the value declines and

$$10\ 000 = 17\ 500(r)^3$$
$$0.5714285 = (r)^3$$
$$r \approx \sqrt[3]{0.571\,428}$$
$$r \approx 0.83$$

When $V(t) = 5000$

$$5000 \approx 17\ 500(0.83)^t$$
$$\frac{5000}{17\,500} \approx (0.83)^t$$
$$t \approx \frac{\ln\left(\frac{5000}{17\,500}\right)}{\ln(0.83)}$$
$$t \approx 6.72$$

Answer The value of the car falls below \$5000 when it is 6.72 years ≈ 6 years 9 months.

19 Let t be the number of years after 1990

$$5.5(1 + 0.019)^t = 9$$
$$(1 + 0.019)^t = 1.\overline{63}$$
$$t \log (1.019) = \log 1.\overline{63}$$
$$t = \frac{\log 1.\overline{63}}{\log 1.019} \approx 26.165...$$

Answer: The population will reach 9 billion in the year **2016**.

20 Let t: number of days

$f(t)$: amount of the compound remaining (g)

$$f(t) = 150c^t$$
$$123 = 150c^{10}$$
$$0.82 = c^{10}$$
$$c = \sqrt[10]{0.82}$$
$$c \approx 0.98$$

Time for 75 g to remain:

$$f(t) = 150(0.98)^t$$
$$75 = 150(0.98)^t$$
$$0.5 = (0.98)^t$$
$$t = \log_{0.98}(0.5)$$
$$t = \frac{\log(0.5)}{\log(0.98)}$$
$$t \approx 34.3$$

Answer: To the nearest day, half of the compound will remain after **34** days.

21

$$f(x) = \mathrm{M}\left(\frac{4}{9}\right)^{\frac{t}{2}}$$
$$f(x) = 60 \times \left(\frac{4}{9}\right)^{\frac{12}{2}}$$
$$f(x) = 60 \times 0.0077$$
$$f(x) = 0.46$$

Result : 0.46 grams

22 Form of the equation of the hyperbola

$$-\frac{(x-\mathrm{h})^2}{\mathrm{a}^2}+\frac{(y-\mathrm{k})^2}{\mathrm{b}^2}=1$$

Parameters h and k

The coordinates of A are A(0, 0).
The coordinates of C are C(12, 0), because the circle is 12 units in diameter.
The coordinates of point B are B(6, 0), because it is the midpoint of $\overline{\mathrm{AC}}$.
Since point B is the centre of the hyperbola, h = 6 and k = 0.

Parameter a and b

$2\mathrm{b} = \mathrm{m}\ \overline{\mathrm{ED}} = 12$ therefore, $\mathrm{b} = 6$

The distance 2c between the foci is 20 units; therefore, c = 10.

$$\mathrm{a}^2+\mathrm{b}^2=\mathrm{c}^2$$
$$\mathrm{a}^2+6^2=10^2$$
$$\mathrm{a}=8$$

Answer The equation of the hyperbola with vertices D and E is $-\frac{(x-6)^2}{64}+\frac{y^2}{36}=1$.

23 Equation of the tangent line

$4x+3y-43=0$ or $y=\frac{-4}{3}x+\frac{43}{3}$

Equation of the perpendicular line passing through C(3, 2)

$$y=\frac{3}{4}x-\frac{1}{4}$$

Intersection of the two lines

$$\frac{-4}{3}x+\frac{43}{3}=\frac{3}{4}x-\frac{1}{4}$$
$$\frac{25}{12}x=\frac{175}{12}$$
$$x=7$$

If $x = 7$, $y=\frac{3}{4}(7)-\frac{1}{4}=5$

The coordinate at the point of intersection is P(7, 5).

Measure of the radius of the circle

$$\mathrm{d(C, P)}=\sqrt{(7-3)^2+(5-2)^2}=\sqrt{25}=5$$

Equation of the circle $(x-3)^2+(y-2)^2=25$

Answer The equation of the circle is $(x-3)^2+(y-2)^2=25$.
The general form of the equation, $x^2+y^2-6x-4y-12=0$, should also be accepted.

24 The equation $\frac{(x-3)^2}{64} - \frac{(y-4)^2}{36} = 1$ is that of a hyperbola with centre (3, 4).

Co-ordinates of the foci of the hyperbola

$c = 10$

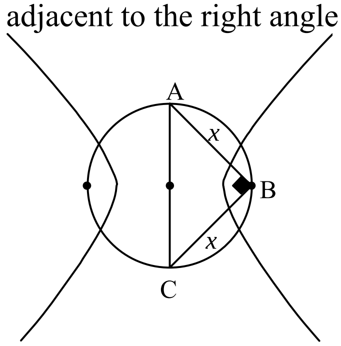
The co-ordinates of the foci are (13, 4) and (-7, 4).

Distance between the two foci $d = 20$

Diameter of the circle D = 20

Measure of one of the sides adjacent to the right angle of triangle ABC

$2x^2 = 20^2$
$2x^2 = 400$
$x^2 = 200$
$x \approx 14.14$



Perimeter of the triangle

P ≈ 14.14 + 14.14 + 20

Answer: The perimeter of the right isosceles triangle ABC is 48.28 cm.

25 Equation of the hyperbola

Value of a: 2a = (12 – 0)
a = 6

Value of b: if $x = 12$ then $y = \frac{4}{3}(12) - 8 = 8$
b = 8

$$\frac{(x-6)^2}{36} - \frac{y^2}{64} = 1$$

Distance between the centre of the hyperbola and the focus (c)

$a^2 + b^2 = c^2$
$6^2 + 8^2 = c^2$
$10 = c$

Coordinates of points P and Q x-coordinate: 6 + 10 = 16

y-coordinates: $\frac{(16-6)^2}{36} - \frac{y^2}{64} = 1$

$y = \frac{-32}{3}$ and $y = \frac{32}{3}$

$P\left(16, \frac{32}{3}\right)$ and $Q\left(16, \frac{-32}{3}\right)$

Distance between the statues

$$m\overline{PQ} = \frac{32}{3} - \left(\frac{-32}{3}\right) = 21.3333...$$

Answer: The distance between the two statues to the nearest metre is **21** m.

26

Circle
Centre: (13, 10)   radius: 4 cm

End points of the diameter
(9, 10) and (17, 10)

Hyperbola
The vertices are (9, 10) and (17, 10) and therefore $a = 4$.

Half of the total length is 7 cm. So the foci are (6, 10) and (20, 10) and therefore $c = 7$.

Equation of hyperbola

$$c^2 = a^2 + b^2$$
$$7^2 = 4^2 + b^2$$
$$33 = b^2$$

$$\therefore \quad \frac{(x-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$

To find the height let $x = 6$ and find the y coordinate

$$\frac{(6-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$
$$33(-7)^2 - 16(y-10)^2 = 16(33)$$
$$1617 - 528 = 16(y-10)^2$$
$$1089 = 16(y-10)^2$$
$$68.0625 = (y-10)^2$$
$$\pm 8.25 = (y-10)$$

So $y = 10 + 8.25 = 18.25$ and $y = 10 - 8.25 = 1.75$

Answer: The height of the frame is **16.5** cm.

27

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

$$\log_2 \frac{(x^2 + 5)}{5} = \log_2 6$$

$$\frac{x^2 + 5}{5} = 6$$

$$x^2 + 5 = 30$$

$$x^2 = 25$$

$$x = -5 \quad \text{ou} \quad x = 5$$

Result : $x = -5$ or $x = 5$

28 Gross profit of company A after the 12th month

$$a(t) = 1000 \log_4 12 = 1000 \frac{\log 12}{\log 4} \approx 1000 \times 1.792\ 481 \approx 1792.48$$

Gross profit of company B after the 12th month

$$b(t) = 1000 \log_5 12 = 1000 = \frac{\log 12}{\log 5} \approx 1000 \times 1.543\ 959 \approx 1543.96$$

Net profit of company A after the 12th month.

$1792.48 – $800.00 = $992.48

Net profit of company B after the 12th month.

$1543.96 – $500.00 = $1043.96

Result : Company B

29 $\log_5 (x - 1) + \log_5 (x + 3) - 1 = 0$

$\log_5 (x - 1)(x + 3) = 1$ Sum of logs = log of the product

$(x - 1)(x + 3) = 5^1$

$x^2 + 2x - 3 = 5$

$x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$x - 2 = 0$ or $x + 4 = 0$

$x = 2$ or -4

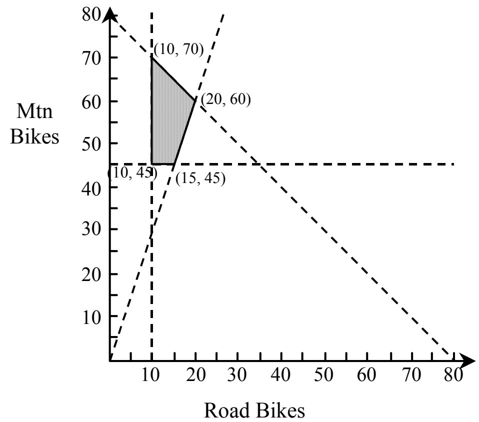
-4 is an extraneous root

Answer The solution of the equation is 2.

30

x = number of road bikes
y = number of mountain bikes

$x \geq 0$
$y \geq 0$
$x \geq 10$
$y \geq 45$
$x + y \leq 80$
$y \geq 3x$



Objective Function
Max. Profit = $250x + 175y$

Points (x, y)		Calculation	Profit
1.	(10, 45)	250(10) + 175(45)	$10 375
2.	(15, 45)	250(15) + 175(45)	$11 625
3.	(20, 60)	250(20) + 175(60)	$15 500
4.	(10, 70)	250(10) + 175(70)	$14 750

Answer The maximum weekly profit is $15 500.

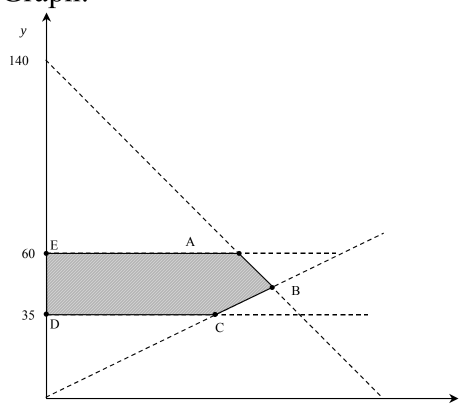
31

Let x: number of lobsters
y: number of crabs

$x \geq 0 \quad y \geq 0$
$y \geq 35$
$y \leq 60$
$x \leq 2y$
$x + y \leq 140$

Objective Function: R = $8.70x + 9.60y$

Graph:



Vertex	R = $8.70x + 9.60y$
A(80, 60)	1272 ← max
B($93.\bar{3}$, $46.\bar{6}$)	1259
C(70, 35)	945
D(0, 35)	336
E(0, 60)	576

Answer: The maximum revenue this fisherman can expect to make is **$1272**.

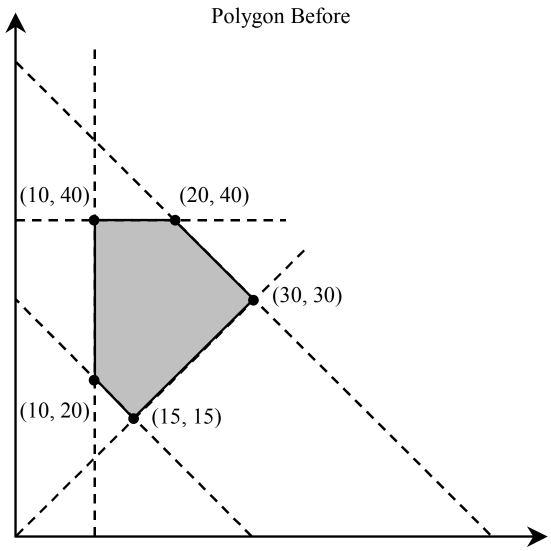
32 x: number of hours at first job per month
y: number of hours at second job per month

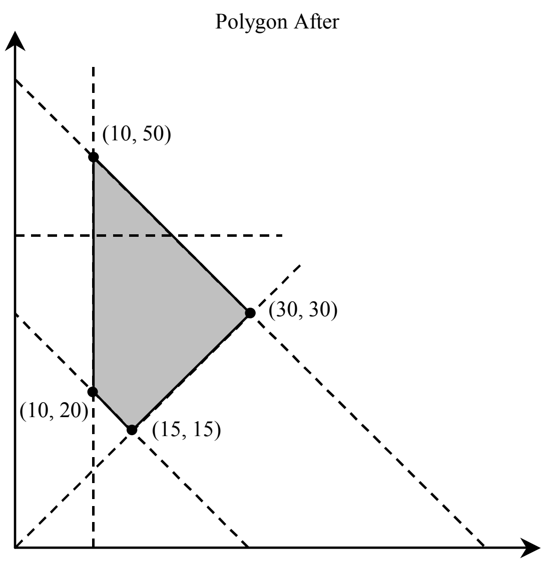
Constraints before

$x \geq 10$
$y \leq 40$
$y \geq 0$
$x + y \geq 30$
$x + y \leq 60$
$y \geq x$

Constraints after

$x \geq 10$
$y \geq 0$
$x + y \geq 30$
$x + y \leq 60$
$y \geq x$





Maximum Before

Vertices	S = 6.3x + 8y ($)
A(10, 40)	383
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
E(20, 40)	446

Maximum After

Vertices	S = 6.3x + 8y ($)
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
F(10, 50)	463

Difference in maximum salary

$463 – $446 = $17

Answer: Murray's maximum possible salary increased by $17

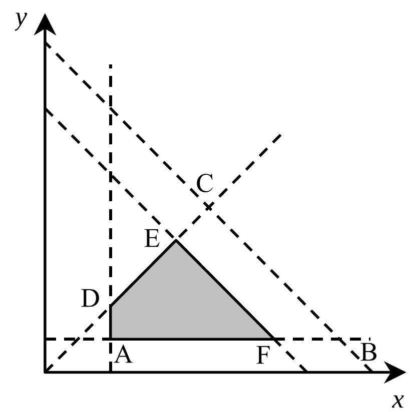
33 Calculation of revenue before the new constraint is considered

Vertices	$R(x, y) = 1.00x + 1.50y$
A(100, 50)	100 + 75 = $175
B(450, 50)	450 + 75 = $525
C(250, 250)	250 + 375 = $625
D(100, 100)	100 + 150 = $250

Maximum Revenue ⇒ $625 — 250 cases of oranges and 250 cases of grapefruit

Calculation of revenue after consideration of the constraint $x + y \leq 400$

Vertices	$R(x, y) = 1.00x + 1.50y$
A(100, 50)	$175
D(100, 100)	$250
E(200, 200)	$500
F(350, 50)	$425



Maximum Revenue ⇒ 500 — 200 cases of oranges, 200 cases of grapefruit

Decrease of revenue because of the flood

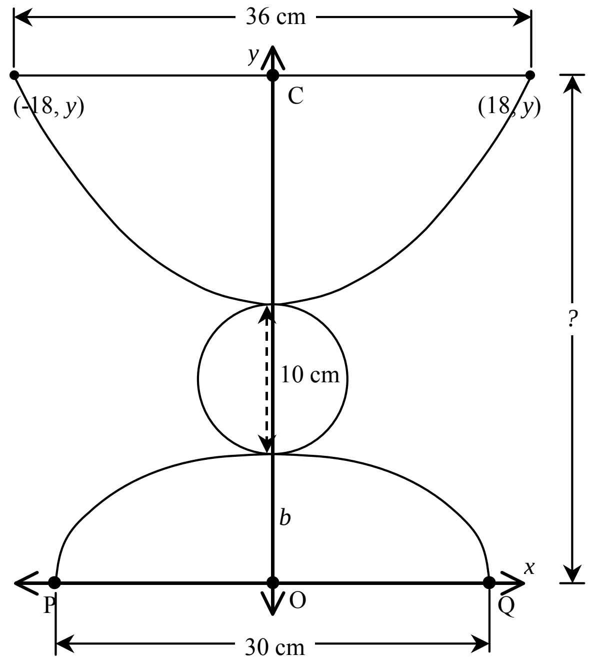
625 − 500 = 125

Answer: The decrease in revenue caused by the flood is $**125**.

34

Calculate the length of the Semi-Minor Axis (b)

$c = 12$ (Given)



Length of Major Axis = 2 a

$$2a = 30$$
$$a = 15$$

$$a^2 = b^2 + c^2$$
$$b = \sqrt{a^2 - c^2}$$
$$b = \sqrt{(15)^2 - (12)^2}$$
$$b = \sqrt{225 - 144}$$
$$b = \sqrt{81}$$
$$b = 9$$

Find the equation of the parabola

Equation of Parabola: $(x - h)^2 = 4c(y - k)$ with Vertex V(h, k)

Note: Do not penalize students for having determined the rule in the form $y = a(x - h)^2$.

$h = 0$; $k = b + 10$
$k = (9) + 10$
$k = 19$

⇨ Equation of Parabola $(x - 0)^2 = 4c(y - 19)$
$x^2 = 4c(y - 19)$

With point O as the origin, the coordinates of the centre of the circle are (0, 14). Since the vertex of the parabola is V(0, 19), the vertical distance between these two points must be the value of c.

Hence $c = 19 - 14$
$c = 5$

Therefore the equation of the parabola is
$x^2 = 4(5)(y - 19)$
$x^2 = 20(y - 19)$

Find the height of the punchbowl

Substitute the point P(18, y) in the equation of the parabola

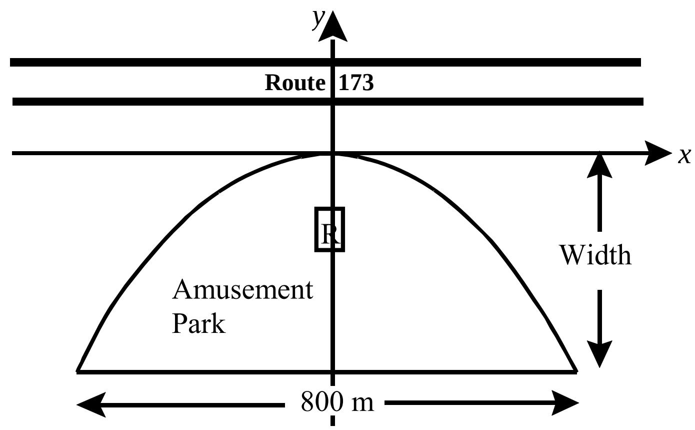
$$(18)^2 = 20(y - 19)$$
$$324 = 20(y - 19)$$
$$\frac{324}{20} + 19 = y$$
$$y = 35.2 \text{ cm}$$

Answer: The height of the punchbowl is **35.2** cm.

35 Equation of the parabola

$c = 100$ and $(h, k) = (0, 0)$

$x^2 = -4(100)y$



Value of y for $x = 400$

$x^2 = -4(100)y$

$400^2 = -400y$

$-400 = y$

Answer The width of the amusement park is 400 metres.

36 Translating the two parabolas

$x^2 - 8x - y + 16 = 0$
$x^2 - 8x + \underline{16} = y - 16 + \underline{16}$
$(x - 4)^2 = y$
$t(4, 0)$

$x^2 - 8x + y + 8 = 0$
$x^2 - 8x + \underline{16} = -y - 8 + \underline{16}$
$(x - 4)^2 = -y + 8$
$(x - 4)^2 = -(y - 8)$
$t(4, 8)$

The origin of the Cartesian plane is located at the lower left corner of the drawing.

The centre of the circle is at (4, 4) and the diameter of the circle measures 8 units which corresponds to the sides of the box.

The diagonal of the box will measure

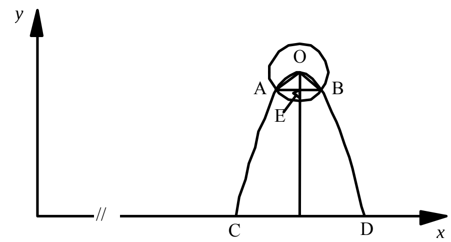
$d = \sqrt{8^2 + 8^2}$

$d = \sqrt{128} \Rightarrow d \approx 11.31$

Result : The length of the diagonal is 11.31 cm.

37

Co-ordinates of O and measure of r



$x^2 + y^2 - 32x - 10y + 279 = 0$

in canonic form

$(x - 16)^2 + (y - 5)^2 = 2$
thus O(16, 5) and $r = \sqrt{2}$

Measure of segment AB

$m\,\overline{AB} = \sqrt{(m\,\overline{AO})^2 + (m\,\overline{OB})^2}$

$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$= \sqrt{2+2} = \sqrt{4} = 2$

Pythagorean theorem

Co-ordinates of point A

$m\,\overline{OE} = m\,\overline{AE} = 1$

x-co-ordinate of A = 16 – 1 = 15

y-co-ordinate of A = 4

In isosceles triangle AOB, which is right-angled, the axis of symmetry of the parabola divides segment AB, at the point of intersection E, into two equal parts.

Point A belongs to the circle with centre O whose equation is known.

Equation of the parabola

$y = a(x - h)^2 + k$

$4 = a(15 - 16)^2 + 5$

The vertex of the parabola is O(16, 5).

$a = -1$

$y = -(x - 16)^2 + 5$

The co-ordinates of point A are used to calculate the value of a.

x-co-ordinate of points C and D

$0 = -(x - 16)^2 + 5$

$x^2 - 32x + 251 = 0$

$x_1 = 16 - \sqrt{5}$ and $x_2 = 16 + \sqrt{5}$

Measure of segment CD

$m\,\overline{CD} = \left(16 + \sqrt{5}\right) - \left(16 - \sqrt{5}\right) = 2\sqrt{5} \approx 4.47$

Result : Rounded to the nearest hundredth, the distance between C and D is 4.47 m.

38 Coordinates of point (16,?)

$y = 15 \tan 25° \approx 6.99$ or 7 m $\Rightarrow$ (16,7)

Equation of the square root function

$f(x) = a\sqrt{(x-h)} + k$ with b = 1

$f(x) = a\sqrt{(x-0)} + 15$ with b = 1

Value of a with the help of point (16,7)

$7 = a\sqrt{16} + 15$

$\Rightarrow a = -2$

$f(x) = -2\sqrt{x} + 15$

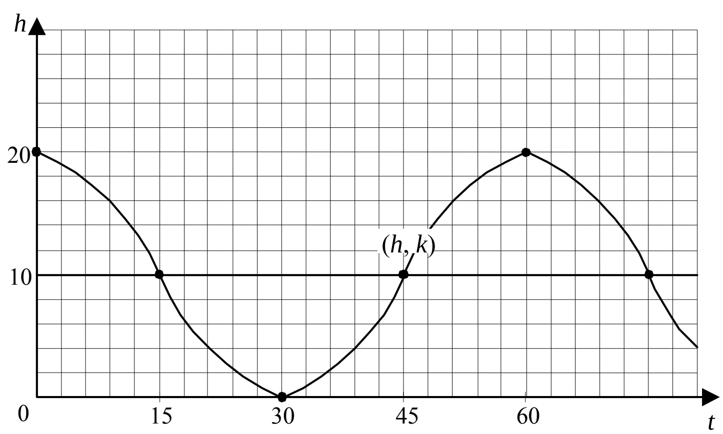
Height of the 2nd post where $x = 8$

$f(x) = -2\sqrt{8} + 15$

$f(x) \approx 9.34$

Answer: The height of the second post is 9.34 meters.

39 Graphic representation of the situation



The function rule is

$h(t) = \text{A} \sin \text{B}(t - h) + k$ or $h(t) = \text{A} \cos \text{B}(t - h) + k$

Value of B

The period is 60 seconds.

$\frac{2\pi}{\text{B}} = 60$ and $\text{B} = \frac{\pi}{30}$

Value of (h, k)

For a sine graph, there is a horizontal shift of 45 to right, $h = 45$

axis of symmetry is at 10 $k = 10$

Answer The rule describing the height (h) of the needle at time (t) elapsed since noon is:

$$h(t) = 10 \sin \frac{\pi}{30}(t - 45) + 10$$

or $$h(t) = 10 \cos \frac{\pi}{30}(t) + 10.$$

40 Let t: time, in seconds, that has past since 13:00

$f(t)$: height of the jet, in metres

The rule of correspondence

$$f(t) = a \sin b(t - h) + k$$

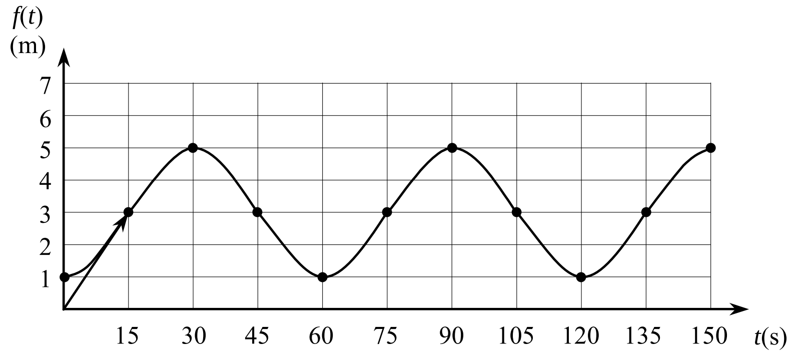
$$a = \frac{5 - 1}{2} = 2$$

$$p = \frac{2\pi}{|b|}$$

$$60 = \frac{2\pi}{|b|}$$

$$|b| = \frac{2\pi}{60}$$

$$|b| = \frac{\pi}{30}$$



$$f(x) = 2 \sin \frac{\pi}{30}(t - h + k)$$

Translation $(h, k) = (15, 3)$

$$f(t) = 2 \sin \frac{\pi}{30}(t - 15) + 3$$

Height at 13 h 12 min 40 s

Since the function has a period of one minute, the jet will be at the same height in 40 seconds.

$$f(40) = 2 \sin \frac{\pi}{30}(40 - 15) + 3$$

$$\approx 3.09$$

Answer: At 13 hours 12 minutes and 40 seconds, the water jet will be at a height of **4** m.

41

Rule of the function

x : time in minutes

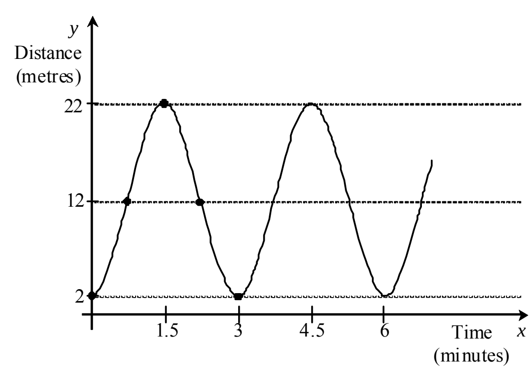
$f(x)$: distance in metres

$f(x) = \text{a} \sin \text{b}(x - \text{h}) + \text{k}$

$$|\text{a}| = \frac{22 - 2}{2} = 10$$

$f(x) = 10 \sin \text{b}\,(x - 0.75) + 12$

since the period is 3 minutes, $3 = \dfrac{2\pi}{|\text{b}|}$, $|\text{b}| = \dfrac{2\pi}{3}$.



$$f(x) = 10 \sin \frac{2\pi}{3}\ (x - 0.75) + 12$$

Distance when the Ferris wheel comes to a stop

$$f(7) = 10 \sin \frac{2\pi}{3}\ (7 - 0.75) + 12 = 17$$

Answer: Seven minutes after Mark got on the ride, the distance between the ground and the point at which his gondola is secured to the wheel was **17** metres.

42

Finding the rules of the form

$$y = a\sqrt{b(x-h)} + k$$

Function 1: $(h, k) = (0, 3)$
$(x, y) = (4, 1)$

Let b = 1

$$1 = a\sqrt{1(4-0)} + 3$$
$$a\sqrt{4} = -2$$
$$a = -1$$
$$y = -\sqrt{x} + 3$$

Function 2: $(h, k) = (2, 0)$
$(x, y) = (6, 2)$

Let b = 1

$$2 = a\sqrt{1(6-2)} + 0$$
$$a\sqrt{4} = 2$$
$$a = 1$$
$$y = \sqrt{x-2}$$

Finding the time at the intersection

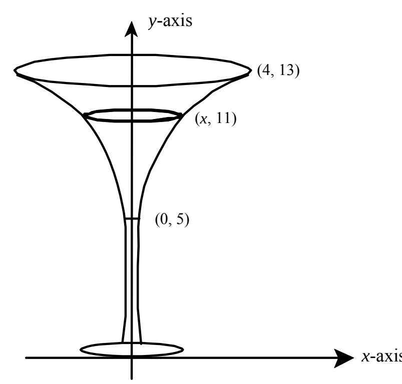
$$-\sqrt{x} + 3 = \sqrt{x-2}$$
$$\left(-\sqrt{x} + 3\right)^2 = \left(\sqrt{x-2}\right)^2$$
$$x - 6\sqrt{x} + 9 = x - 2$$
$$-6\sqrt{x} = -11$$
$$\sqrt{x} = \frac{11}{6}$$
$$x \approx 3.6$$

Time wanted

$$3.36 - 2 = 1.36$$

Answer: 1.36 seconds after it has been launched, the 2nd projectile will be higher than the 1st projectile.

43



The square root function must be in the form: $y = a\sqrt{x} + k$

Substituting (0, 5) we get: $5 = a\sqrt{0} + k$ So k = 5

Substituting (4, 13) we get: $13 = a\sqrt{4} + 5$ So a = 4

So the function is: $y = 4\sqrt{x} + 4$

At the ring, $y = 11$ $11 = 4\sqrt{x} + 5$ So $x = 2.25$

But 2.25 represents the radius of the gold ring in centimetres.

So the circumference is: $C = 2\pi\ (2.25) \approx 14.13$ cm

Therefore the gold ring will cost: $14.13 \times 2 = 28.26$ cents.

Answer Rounded to the nearest cent, the cost of the gold ring is 28 cents.

44

Find the equation of the absolute value function

Vertex (1, 10) point (0, 2)

$$y = a|x - 1| + 10$$
$$2 = a|-1| + 10$$
$$-8 = a$$
$$y = -8|x - 1| + 10$$

Find the x value when $y = 1$

$$y = -8|x - 1| + 10$$
$$1 = -8|x - 1| + 10$$
$$-9 = -8|x - 1|$$
$$\frac{9}{8} = |x - 1|$$

$\therefore \quad x - 1 = \frac{-9}{8} \quad$ or $\quad x - 1 = \frac{9}{8}$

$x = \frac{-1}{8} \qquad\qquad x = \frac{17}{8}$

$x = \frac{17}{8}$ or 2.125

Find the equation of the square root function

Starting point (2.125, 1) point (3.125, 3)

$$y = a\sqrt{x - 2.125} + 1$$
$$3 = a\sqrt{3.125 - 2.125} + 1$$
$$2 = a\sqrt{1}$$
$$2 = a$$

So $\quad y = 2\sqrt{x - 2.125} + 1$

Find the time when the height is 5 m

$$y = 2\sqrt{x - 2.125} + 1$$
$$5 = 2\sqrt{x - 2.125} + 1$$
$$4 = 2\sqrt{x - 2.125}$$
$$2 = \sqrt{x - 2.125}$$
$$4 = x - 2.125$$
$$6.125 = x$$

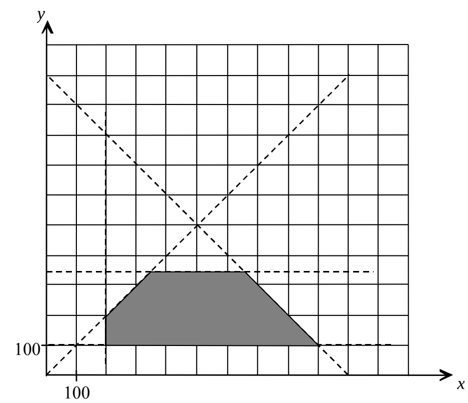
Answer: The ball hits the wall **6.125** seconds after it was hit by the racket.

45 Given x: the number of loaves of raisin bread
y: the number of loaves of olive bread

List of the constraints

$x \geq 0$	$x + y \leq 1000$	$y \geq 100$	$x \geq y$
$y \geq 0$	$x \geq 200$	$y \leq 350$	

Polygon of constraints



Function rule to be optimized: $Z = 0.10x + 0.20y$

Vertices that optimize the objective function

Vertices	Value of Z ($)
(200, 100)	40
(200, 200)	60
(350, 350)	105
(650, 350)	135
(900, 100)	110

Answer: Cindy must buy 650 loaves of raisin bread and 350 loaves of olive bread.

46

1. $\text{cosec A}\,(\text{cosec A} + \cot \text{A}) = \dfrac{1}{1 - \cos \text{A}}$

2. $\dfrac{1}{\sin \text{A}}\left(\dfrac{1}{\sin \text{A}} + \dfrac{\cos \text{A}}{\sin \text{A}}\right) = \dfrac{1}{1 - \cos \text{A}}$

3. $\dfrac{1}{\sin \text{A}}\left(\dfrac{1 + \cos \text{A}}{\sin \text{A}}\right) = \dfrac{1}{1 - \cos \text{A}}$

4. $\dfrac{1 + \cos \text{A}}{\sin^2 \text{A}} = \dfrac{1}{1 - \cos \text{A}}$

5. $\dfrac{1 + \cos \text{A}}{1 - \cos^2 \text{A}} = \dfrac{1}{1 - \cos \text{A}}$

6. $\dfrac{1 + \cos \text{A}}{(1 + \cos \text{A})\,(1 - \cos \text{A})} = \dfrac{1}{1 - \cos \text{A}}$

7. $\dfrac{1}{1 - \cos \text{A}} = \dfrac{1}{1 - \cos \text{A}}$

47

$$\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} = 2\sec\theta$$

$$\frac{(1 + \sin\theta)^2 + \cos^2\theta}{\cos\theta(1 + \sin\theta)} = 2\sec\theta$$

$$\frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1 + \sin\theta)} = 2\sec\theta$$

$$\frac{1 + 2\sin\theta + 1}{\cos\theta(1 + \sin\theta)} = 2\sec\theta \qquad (\text{because } \sin^2\theta + \cos^2\theta = 1)$$

$$\frac{2 + 2\sin\theta}{\cos\theta(1 + \sin\theta)} = 2\sec\theta$$

$$\frac{2(1 + \sin\theta)}{\cos\theta(1 + \sin\theta)} = 2\sec\theta$$

$$\frac{2}{\cos\theta} = 2\sec\theta \qquad \left(\text{because } \cos\theta = \frac{1}{\sec\theta}\right)$$

$$2\sec\theta = 2\sec\theta$$

48

$$\frac{\sec\theta}{1 - \cos\theta} - \frac{\sec\theta}{1 + \cos\theta} = 2\operatorname{cosec}^2\theta$$

$$\frac{\sec\theta(1 + \cos\theta) - \sec\theta(1 - \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)} = 2\operatorname{cosec}^2\theta$$

$$\frac{\sec\theta + \sec\theta\cos\theta - \sec\theta + \sec\theta\cos\theta}{(1 - \cos\theta)(1 + \cos\theta)} = 2\operatorname{cosec}^2\theta \qquad \text{as } \sec\theta\cos\theta = 1$$

$$\frac{2}{(1 - \cos\theta)(1 + \cos\theta)} = 2\operatorname{cosec}^2\theta$$

$$\frac{2}{1 - \cos^2\theta} = 2\operatorname{cosec}^2\theta$$

$$\frac{2}{\sin^2\theta} = 2\operatorname{cosec}^2\theta \qquad \text{as } \sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$2\operatorname{cosec}^2\theta = 2\operatorname{cosec}^2\theta$$

49

1. $$\frac{2}{\text{cosec } a} + 2\sin a \tan^2 a = \frac{2\tan a}{\cos a}$$

2. $$2\sin a + 2\sin a \tan^2 a = \frac{2\tan a}{\cos a}$$

3. $$2\sin a\,(1 + \tan^2 a) = \frac{2\tan a}{\cos a}$$

4. $$2\sin a \sec^2 a = \frac{2\tan a}{\cos a}$$

5. $$2\sin a \times \frac{1}{\cos^2 a} = \frac{2\tan a}{\cos a}$$

6. $$\frac{2\sin a}{\cos a} \times \frac{1}{\cos a} = \frac{2\tan a}{\cos a}$$

7. $$\frac{2\tan a}{\cos a} = \frac{2\tan a}{\cos a}$$

50

1. $$\frac{2 - 2\sin A \cos A \times \tan A}{2} = \cos^2 A$$

2. $$\frac{2 - 2\sin^2 A}{2} = \cos^2 A$$

3. $$\frac{2\left(1 - \sin^2 A\right)}{2} = \cos^2 A$$

4. $$\frac{2 \times \cos^2 A}{2} = \cos^2 A$$

5. $$\cos^2 A = \cos^2 A$$

51

$$\frac{\text{cosec } A - \sin A}{\cos A} = \frac{\frac{1}{\sin A} - \sin A}{\cos A}$$

$$= \frac{\frac{1 - \sin^2 A}{\sin A}}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\sin A \times \cos A}$$

$$= \frac{\cos^2 A}{\sin A \times \cos A}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

52

Let $\overrightarrow{(x, y)}$ be the wind vector

$$\overrightarrow{(100, 150)} + \overrightarrow{(x, y)} = \overrightarrow{(120, 160)}$$

$$\overrightarrow{(100 + x, 150 + y)} = \overrightarrow{(120, 160)}$$

$100 + x = 120$ and $x = 20$

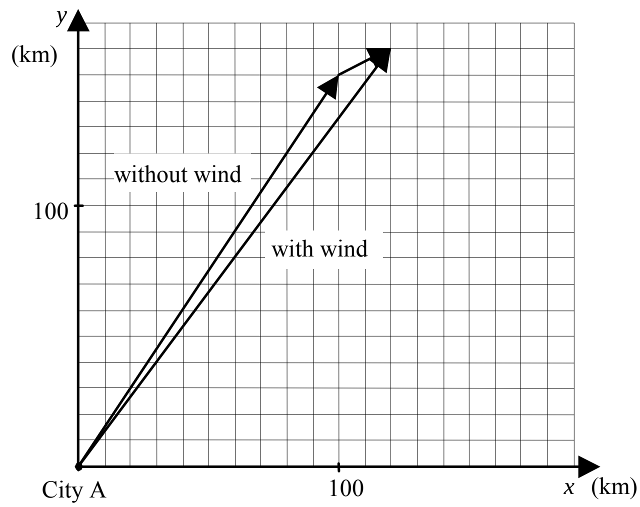
$150 + y = 160$ and $y = 10$

Therefore

$$\overrightarrow{(x, y)} = \overrightarrow{(20, 10)}$$

The speed of the wind:

$$\left\|\overrightarrow{(20, 10)}\right\| = \sqrt{20^2 + 10^2} \approx 22.36$$



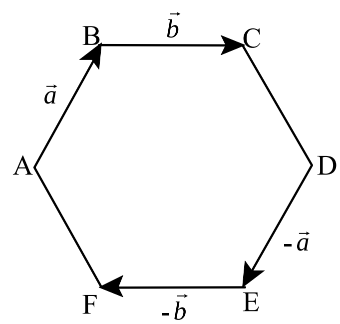
Answer The wind speed is approximately 22.36 km/h.

53 **Hypothesis:**

1. ABCDEF is a regular hexagon
2. $\overrightarrow{AB} = \vec{a}$

   $\overrightarrow{BC} = \vec{b}$

Conclusion : $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{DE} + \overrightarrow{EF} = \vec{a}$



Proof			Reasons		
1.	a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$	1.	a)	Chasles' relation
	b)	$\overrightarrow{AC} = \vec{a} + \vec{b}$		b)	By substitution
2.	a)	$\overrightarrow{DE} = -\overrightarrow{AB}$	2.	a)	$\overrightarrow{DE}$ and $\overrightarrow{AB}$ are non-collinear vectors by definition of a regular hexagon.
	b)	$\overrightarrow{DE} = -\vec{a}$		b)	By substitution
3.	a)	$\overrightarrow{EF} = -\overrightarrow{BC}$	3.	a)	$\overrightarrow{EF}$ and $\overrightarrow{BC}$ are non-collinear vectors by definition of a regular polygon.
	b)	$\overrightarrow{EF} = -\vec{b}$		b)	By substitution
4.		$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{DE} + \overrightarrow{EF} =$ $\vec{a} + \vec{a} + \vec{b} + -\vec{a} + -\vec{b} = \vec{a}$	4.		By vector addition

54 $\overrightarrow{CB} + \overrightarrow{AC} - \overrightarrow{FE} + \overrightarrow{GF} = \overrightarrow{GE}$ to be proved

1. $\overrightarrow{GA} + \overrightarrow{AC} - \overrightarrow{FE} + \overrightarrow{DE} = \overrightarrow{GE}$

   by substitution since $\overrightarrow{CB} = \overrightarrow{GA}$ and $\overrightarrow{GF} = \overrightarrow{DE}$

2. $\overrightarrow{GA} + \overrightarrow{AC} + \overrightarrow{EF} + \overrightarrow{DE} = \overrightarrow{GE}$

   because $\overrightarrow{EF}$ is the vector opposite to $\overrightarrow{FE}$

3. $\overrightarrow{GA} + \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{GE}$

   by substitution since $\overrightarrow{EF} = \overrightarrow{CD}$

4. $\overrightarrow{GC} + \overrightarrow{CE} = \overrightarrow{GE}$

   according to Chasles' Relation $\overrightarrow{GA} + \overrightarrow{AC} = \overrightarrow{GC}$ and $\overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{CE}$

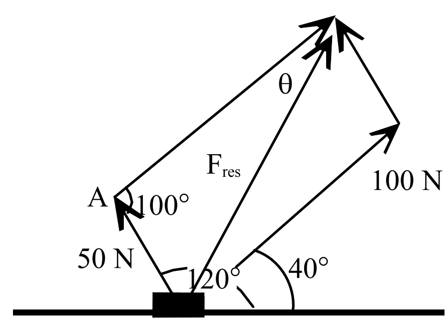
5. $\overrightarrow{GE} = \overrightarrow{GE}$

   according to Chasles' Relation $\overrightarrow{GC} + \overrightarrow{CE} = \overrightarrow{GE}$

55 Measure of angle A

m ∠A = 180° − 80° = 100°

since two consecutive angle in a parallelogram are supplementary.



Resultant force (strength)

$$\|\vec{F}_{res}\|^2 = 50^2 + 100^2 - 2(50)(100)\cos 100°$$

$$\|\vec{F}_{res}\| \approx 119.3\, N$$

Direction of resultant force

$$\frac{\sin 100°}{119.3} = \frac{\sin \theta}{50}$$

$$\sin \theta \approx 0.412\,74$$

$$\theta \approx 24.38°$$

The direction is 24.38° + 40°, so about 64.38°.

Answer Tim must apply a force of 119.3 N with a direction of 64.38°.

56 $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ Chasles Relation

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ Chasles Relation

Scalar product

$$\begin{aligned}
\overrightarrow{AC} \bullet \overrightarrow{BD} &= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) \bullet \left(\overrightarrow{BC} + \overrightarrow{CD}\right) \\
&= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) \bullet \left(\overrightarrow{BC} - \overrightarrow{AB}\right) \quad \text{as} \quad \overrightarrow{CD} = -\overrightarrow{AB} \quad \text{definition of a rhombus} \\
&= \overrightarrow{AB} \bullet \overrightarrow{BC} - \overrightarrow{AB}^2 + \overrightarrow{BC}^2 - \overrightarrow{BC} \bullet \overrightarrow{AB} \quad \text{by distributivity} \\
&= -\overrightarrow{AB}^2 + \overrightarrow{BC}^2 \\
&= -\left\|\overrightarrow{AB}\right\|^2 + \left\|\overrightarrow{BC}\right\|^2 \quad \text{definition of scalar product} \\
&= c^2 - c^2 \qquad c = \text{length of one side of the rhombus} \\
&= 0
\end{aligned}$$

Since $\overrightarrow{AC} \bullet \overrightarrow{BD} = 0$, $\overrightarrow{AC} \perp \overrightarrow{BD}$ Scalar product theorem

57 Components of vector AB

$\overrightarrow{AB} = (400 - 150, 200 - 125) = (250, 75)$

Components of the unknown vector

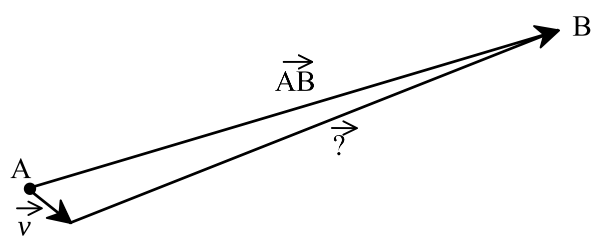
$\vec{v} + \vec{?} = \overrightarrow{AB}$

$\vec{?} = \overrightarrow{AB} - \vec{v}$

$\vec{?} = (250, 75) - (20, -15)$

$\vec{?} = (250 - 20, 75 + 15)$

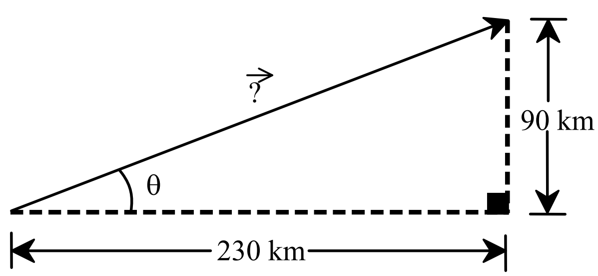
$\vec{?} = (230, 90)$



Direction of the unknown vector

$\tan\theta = \dfrac{90}{230}$

$\theta \approx 21.37°$



Answer: To the nearest degree, the pilot should point the plane at an angle of **21°** relative to the east in order to reach airport B.

58

1) According to Chasles' Relation

$$\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{AM}$$

2) According to Chasles' Relation

$$\overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM}$$

3) Hypothesis

$$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC} \qquad \text{(Isosceles Triangle)}$$

4) Similarly $\overrightarrow{CM} = \frac{-1}{2}\overrightarrow{AC}$

5) By adding (1) and (2), we get

$$2\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{AM} + \overrightarrow{CM}$$

6) Substituting in (3) and (4) , we get

$$2\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{BC} + \frac{1}{2}\overrightarrow{AC} + \frac{-1}{2}\overrightarrow{AC}$$

7) Simplifying (6), we get

$$2\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{BC}$$

8) Therefore,

$$\overrightarrow{BM} = \frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{BC}\right)$$

59

a) $\sin^2 x + \cos x = 1$

$$1 - \cos^2 x + \cos x = 1$$
$$\cos^2 x - \cos x = 0$$
$$\cos x (\cos x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \cos x = 1$$
$$x = \frac{\pi}{2} \quad \text{or} \quad \frac{3\pi}{2} \quad \text{or} \quad x = 0$$

Result : The solution set is $\left\{0, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$.

b) $2\sin^2 x - \cos x - 2 = 0$

$$2(1 - \cos^2 x) - \cos x - 2 = 0$$
$$2 - 2\cos^2 x - \cos x - 2 = 0$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x (2\cos x + 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \cos x = \frac{-1}{2}$$
$$x = \frac{\pi}{2} \quad \text{or} \quad \frac{3\pi}{2} \quad \text{or} \quad x = \frac{2\pi}{3} \quad \text{or} \quad \frac{4\pi}{3}$$

Result : The solution set is $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$

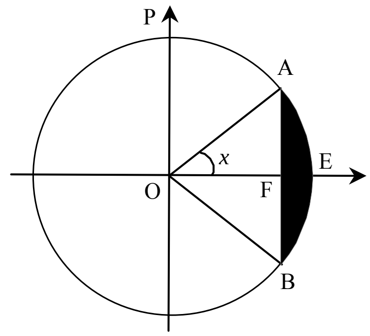
60

Measure of angle x

$$\sin x = \frac{\sqrt{3}}{2}$$

$$m\angle x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$m\angle x = 60° \quad \text{or} \quad m\angle x = \frac{\pi}{3} \text{ radians}$$



Base AB of triangle AOB

$$m\ \overline{AB} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

Height FO of triangle AOB

$$m\ \overline{FO} = \cos\ 60° = \frac{1}{2}$$

Area of triangle AOB

$$\text{Area} = \left(\sqrt{3} \times \frac{1}{2}\right) \div 2 = \frac{\sqrt{3}}{4} \approx 0.43 \text{ square units}$$

Area of sector AOB

$$\text{Area of sector AOB} = \pi(1)^2 \times \frac{120°}{360°} \approx 1.05 \text{ square units}$$

Area of the shaded region $\approx 1.05 - 0.43 = 0.62$ square units

Answer: The area of the shaded region is 0.62 square units.

61

$2\cos^2 x - 3\sin x - 3 = 0$

$2(1 - \sin^2 x) - 3\sin x - 3 = 0$

$2 - 2\sin^2 x - 3\sin x - 3 = 0$

$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0 \qquad \sin x + 1 = 0$

$2\sin x = -1 \qquad \sin x = -1$

$\sin x = \frac{-1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{3\pi}{2}$

Answer The values of x in the domain are $\frac{11\pi}{6}$ and $\frac{3\pi}{2}$.

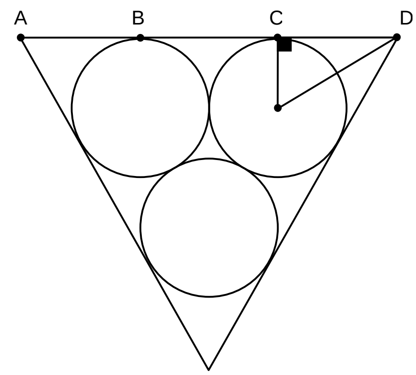
62 Equation of the circle in standard form

$$x^2 + 6x + y^2 - 2y = 26$$
$$(x-3)^2 + (y-1)^2 = 26 + 9 + 1$$
$$(x-3)^2 + (y-1)^2 = 36$$

Each radius measures 6 units

Since the three circles are congruent, the border forms an equilateral triangle.

According to the diagram on the right,



Note: Adjustments will have to be made for different labelling.

Δ COD is a right triangle, m $\overline{OC}$ = 6 units and m $\angle$ CDO = 30°

$$\tan 30° = \frac{m\ \overline{OC}}{m\ \overline{CD}} = \frac{6}{m\ \overline{CD}}$$
$$m\ \overline{CD} = 10.39$$
$$m\ \overline{AD} = m\ \overline{AB} + m\ \overline{BC} + m\ \overline{CD}$$
$$= 10.39 + 12 + 10.39$$
$$32.78$$
$$\text{Perimeter: } P = 3 \bullet 32.78$$
$$= 98.34$$

Answer: To the nearest hundredth of a unit, the border measures **98.34**.

63 $\left\{\frac{-\pi}{4}, \frac{\pi}{4}\right\}$

64 $y = -28.8 \cos\left(\frac{\pi}{13}x\right)$ or $y = 28.8 \cos\left(\frac{\pi}{13}(x+13)\right)$ or $y = 28.8 \sin\left(\frac{\pi}{13}(x-6.5)\right)$