

Lesson 9-2: Graphs of Polynomial Functions

Once again, here we are; back to graphing. And, once again I'll tell you, it isn't as bad as it seems. ☺ And, once again I'll tell you that graphing is very important because being able to visualize your data is extremely helpful. Seeing where things are increasing or decreasing, where the zero values are, etc. is ***much*** better than staring at a huge list of numbers.


The basic characteristics of polynomial graphs

All polynomial graphs have the following four characteristics:

1. They are ***continuous***. In other words, they have no gaps or breaks.
2. They are ***smooth***. There are no sharp corners or elbows.
3. A function of degree n has ***at most*** $n - 1$ turns.
4. Sign of leading coefficient tells if it rises or falls on the left and the right:

The basic shape of polynomial graphs

The basic shape of a polynomial graph is determined by the polynomial's degree:

- Even degree: basically U shaped...a parabola shape based on $f(x) = x^2$
- Odd degree: basically a twist  shape...based on $f(x) = x^3$

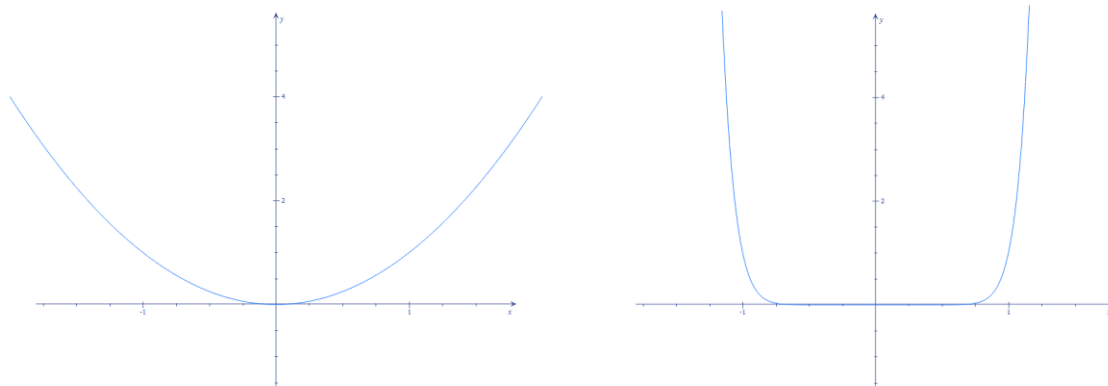
So all of the following functions are basically U shaped:

$$f(x) = x^2 \quad f(x) = x^4 \quad f(x) = x^6 \quad f(x) = x^8 \quad f(x) = x^{10}$$

...and all the following functions are basically twist shaped:

$$f(x) = x^3 \quad f(x) = x^5 \quad f(x) = x^7 \quad f(x) = x^9 \quad f(x) = x^{11}$$

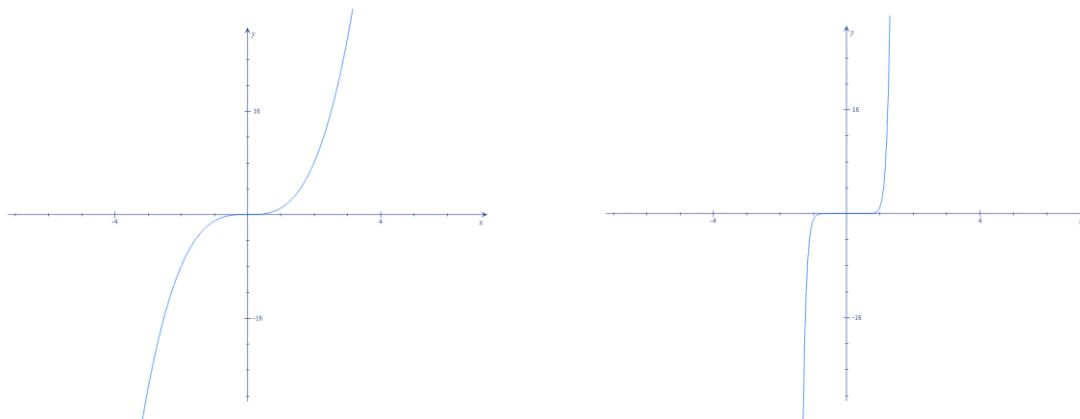
The higher the degree, the flatter the graph gets around the vertex. Below on the left are the graphs of $f(x) = x^2$ and $g(x) = x^{12}$ for comparison:



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You can see how the graph of $g(x) = x^{12}$ is much flatter, almost squared off around the vertex $(0, 0)$.

Below are the graphs of $f(x) = x^3$ and $g(x) = x^{13}$ for comparison:



Once again, you can see how the graph of the higher degree polynomial is much flatter, and squared off around the vertex.

How to tell the number of turns

The maximum number of turns in the graph of a polynomial is one less than the degree of the polynomial:




- At most 2 turns: $f(x) = 2x^3 - x^2 + 5$ because it is degree 3
- At most 3 turns: $f(x) = -3x^4 - x^3 + 9$ because it is degree 4
- At most 4 turns: $f(x) = 8x^5 - x$ because it is degree 5

Basic graph transformations

The basic transformations you know and love <cough> work for the graphs of polynomials.

1. Up / down shifts: determined by the constant coefficient.
2. Left / right shifts: determined by adding/subtracting a constant to the leading term **inside** parentheses with the variable.
3. Up-side-down: with polynomials we typically talk about the rise/fall behavior of the left and right side. This differs with an odd degree polynomial verses an even degree polynomial:

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- Even degree (U shaped):
 - Positive leading coefficient \rightarrow U \rightarrow rises on left and right.
 - Negative leading coefficient \rightarrow \cap \rightarrow falls on left and right.
- Odd degree ( shaped):
 - Positive leading coefficient \rightarrow  \rightarrow falls on left, rises right.
 - Negative leading coefficient \rightarrow  \rightarrow rises on left, falls right.

Consider the following polynomial functions; describe the transformations and left/right behavior:

$$f(x) = (x - 2)^4$$

$$g(x) = (x + 3)^7$$

$$h(x) = -x^4 + 2$$

$$j(x) = -(x - 5)^7 - 3$$

...

$$f(x) = (x - 2)^4$$

right 2

rise left

rise right

$$g(x) = (x + 3)^7$$

left 3

fall left

rise right

$$h(x) = -x^4 + 2$$

up 2

fall left

fall right

$$j(x) = -(x - 5)^7 - 3$$

right 5, down 3


rise left

fall right

Sketching graphs of simple polynomials

First, please note I said **simple polynomials**. A simple polynomial is one in which the variable (usually x) appears once. In other words, there is only one thing being raised to a power. In the prior paragraph about transformations, all the example functions are simple polynomials.

To sketch the graph of a simple polynomial:

1. Use the transformations to plot the vertex.
2. Determine basic shape (odd degree , even degree U).
3. Determine left and right behavior (rise, fall).

A note on calculators

Many times you will be asked to sketch the graph of a polynomial. You can not use your graphing calculator to sketch **unless** you are told you can. You can use it to double check your sketch however.