Unit 5: Factoring Polynomials

Important Topics and *Vocabulary*

- Finding the *prime factorization* of *composite numbers*
- Finding the *greatest common factor* and the *least common multiple* of integers and monomials
- Simplifying the quotients of monomials
- Dividing polynomials by monomials
- Finding monomial factors of polynomials
- Finding the product of two binomials mentally
- Simplifying *conjugates* ie. products of the form (a+b)(a-b)
- Factoring the *difference of two squares*
- Finding the *squares of binomials*
- Recognizing and factoring *perfect square trinomials*
- Factoring *quadratic trinomials* of various forms, including *integral coefficients*
- Factoring polynomials by *grouping*
- Factoring polynomials completely using a variety of methods and steps
- Solving polynomial equations by factoring and using the *zero-product property*
- Solving word problems represented by quadratic equations
- Solving *quadratic type equations*
- Quadratics and Quadratic types with variable exponents (Advanced Only)

Sample Problems with Answers

1. Find the prime factorization of 1089 Answer: $3^2 \cdot 11^2$

2. Find the greatest common factor of 56, 98, 126 Answer: 14

3. Find the greatest common factor of $25p^2q^3,\ 15p^2q^2,\ 35pq^4$ Answer: $5pq^2$

4. Simplify $\dfrac{\left(3t^2\right)^3}{\left(3t^3\right)^2}$ Answer: 3

5. Simplify $\dfrac{x^{2n+1}y^{n+1}}{\left(xy^2\right)^n}$ Answer: $\dfrac{x^{n+1}}{y^{n-1}}$

6. Simplify $\dfrac{28r^3s^2+42r^2s^3-56r^3s^3}{-7r^2s^2}$ Answer: $-4r-6s+8rs$

7. Simplify $\dfrac{x^2y-3x^2y^2}{xy}+\dfrac{6xy+9xy^2}{3y}$ Answer: 3x

8. Factor $7y^3-21y^2-14y$ Answer: $7y\left(y^2-3y-2\right)$

9. Factor $96wx^3y^2z^2-144w^3xy^2z^2$ Answer: $48wxy^2z^2\left(2x^2-3w^2\right)$

10. Factor $49a^2-9b^2$ Answer: $(7a+3b)(7a-3b)$

11. Factor x^8-y^8 Answer: $\left(x^4+y^4\right)\left(x^2+y^2\right)(x+y)(x-y)$

12. Factor $2a^5-162a$ Answer: $2a\left(a^2+9\right)(a+3)(a-3)$

13. Factor $(s+2)^2-(s-2)^2$ Answer: 8s

14. Factor $rt^{4n}-16r$ Answer: $r\left(t^{2n}+4\right)\left(t^n+2\right)\left(t^n-2\right)$ (Advanced Only)

15. Factor $4s^2-36st+81t^2$ Answer: $(2s-9t)^2$

16. Factor $36p^4-48p^3+16p^2$ Answer: $4p^2(3p-2)^2$

17. Factor a^2-b^2+6b-9 Answer: $(a+b-3)(a-b+3)$

18. Factor $(x+1)^2-2(x+1)+1$ Answer: x^2

19. Factor $x^2-16xy+45y^2$ Answer: prime

20. Factor $124-35y+y^2$ Answer: $(4-y)(31-y)$

21. Factor $(y+3)^2+6(y+3)+9$ Answer: $(y+6)^2$

22. Factor x^4-3x^2-4 Answer: $(x+2)(x-2)\left(x^2+1\right)$

23. Factor $4x^4-17x^2+4$ Answer: $(2x+1)(2x-1)(x+2)(x-2)$

24. Factor $\left(y^2+3y-1\right)^2-9$ Answer: $(y+2)(y+1)(y+4)(y-1)$

25. Factor $p^2 - 2pq + pr - 2qr$ Answer: $(p+r)(p-2q)$

26. Factor $p^2 + q^2 - r^2 - 2pq + 2r - 1$ Answer: $(p-q+r-1)(p-q-r+1)$

27. Factor $x^2(x+2) - x(x+2) - 12(x+2)$ Answer: $(x+2)(x+3)(x-4)$

28. Write as a trinomial $(x^n - y^n)(2x^n + 3y^n)$ Answer: $2x^{2n} + x^n y^n - 3y^{2n}$ (Advanced only)

29. Find the values of *p*, *q*, and *r*, given $(px+q)(2x+5) = 6x^2 + 11x + r$

Answer: $p = 3$, $q = -2$, $r = -10$ (Advanced only)

30. Solve $(2x-3)(x+3) = (x-3)^2 + (x+3)^2$ Answer: 9

31. Solve $7x^2 = 18x - 11$ Answer: $\left\{\frac{11}{7}, 1\right\}$

32. Solve $y^4 - 10y^2 + 9 = 0$ Answer: $\{3, -3, 1, -1\}$

33. Solve $(z+1)(z-5) = 16$ Answer: $\{7, -3\}$

34. Solve $(x+2)^2 - (x-2)^2 = (x-1)^2 - (x-3)^2$ Answer: –2

35. The sum of two numbers is 25 and the sum of their squares is 313. Find the numbers.
Answer: 12 and 13

36. A 50 m by 120 m park consists of a rectangular lawn surrounded by a path of uniform width. Find the dimensions of the lawn if its area is the same as the area of the path.

Answer: 30 m by 100 m

37. The perimeter of a square garden is 12 m greater than the perimeter of a smaller square garden. The area of the larger garden is 105 m^2 greater than that of the smaller garden. Find the dimensions of the larger garden.

Answer: 19 m by 19 m

38. Write an expression in factored form for the area of the shaded region:

Answer: $2r^2(2\pi - 1)$