MCR3U

Exponential Practice Test (3)

+ Logs

QUESTION ONE

Simplify these expressions. Show your working and leave you answers with positive, rational powers.

a) $\dfrac{3x^{-3}y^{2}}{15x^{2}y^{-1}}$

b) $\dfrac{(-8pq^{3}r^{2})(-6p^{2}r^{4})}{12p^{3}q^{2}r}$

c) $\left(\dfrac{x^{-3}y^{2}}{z^{-1}}\right)^{2}\left(\dfrac{z^{-3}}{x^{-4}y}\right)^{-2}$

d) $\left(\dfrac{12a^{2}b^{-3}}{9a^{-4}b^{1}}\right)^{-1}$

e) $m^{-1}\cdot m^{\frac{1}{6}}\cdot\sqrt[3]{m^{4}}$

f) $(16x)^{\frac{3}{4}}(81x^{8})^{-\frac{1}{4}}$

QUESTION TWO

Simplify these expressions, express the answer in radical form.

a) $x^{\frac{3}{2}}$

b) $\dfrac{x^{\frac{7}{2}}\cdot x^{\frac{3}{4}}}{x^{3}}$

QUESTION THREE

Express as a single, positive exponent, then evaluate and express answer in rational form.

a) $\dfrac{10^{-2}}{5^{-3}}$

b) $(\sqrt[3]{8})^{-2}$

c) $64^{\frac{2}{3}}\cdot 64^{-\frac{1}{2}}$

d) $81^{\frac{3}{4}}-16^{\frac{3}{2}}$

QUESTION FOUR

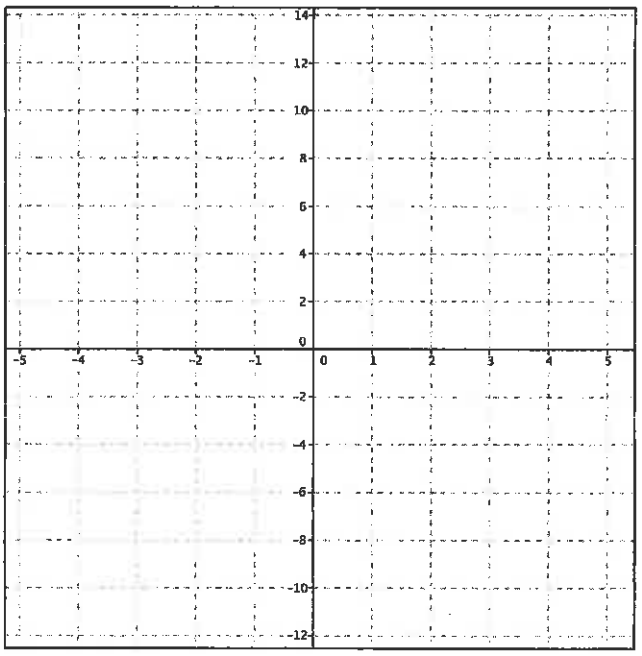
Graph the following exponential function.

Remember to label your graphs.

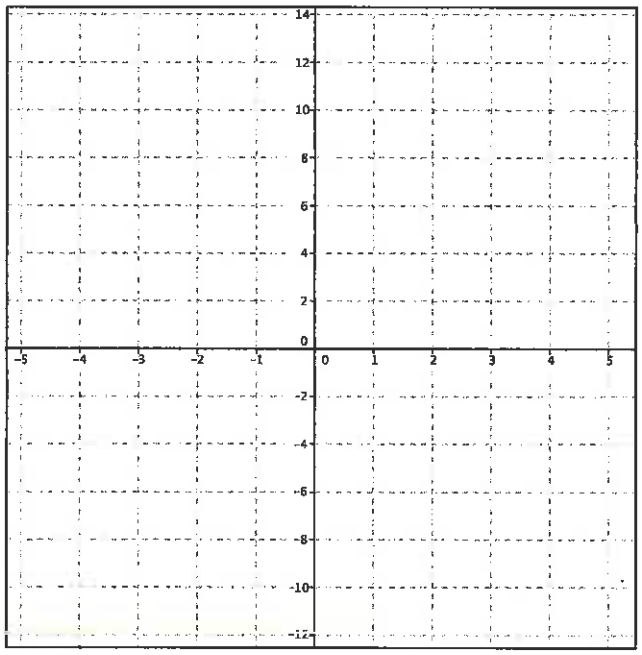
For each graph:

a) Graph the parent function, $f(x)$
b) State the transformations
c) Graph $g(x)$

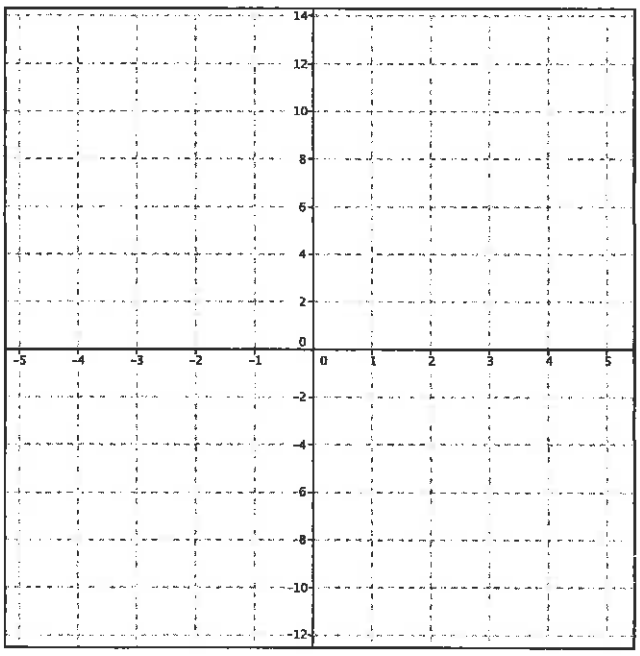
i) $g(x) = -(2)^{\frac{x}{2}}$



ii) $g(x) = 2\left(\frac{1}{3}\right)^{x} - 10$



iii) $g(x) = 3\left(\frac{1}{2}\right)^{x-4}$



QUESTION FIVE

a) Describe the following features of the graph.

x-intercept: _____

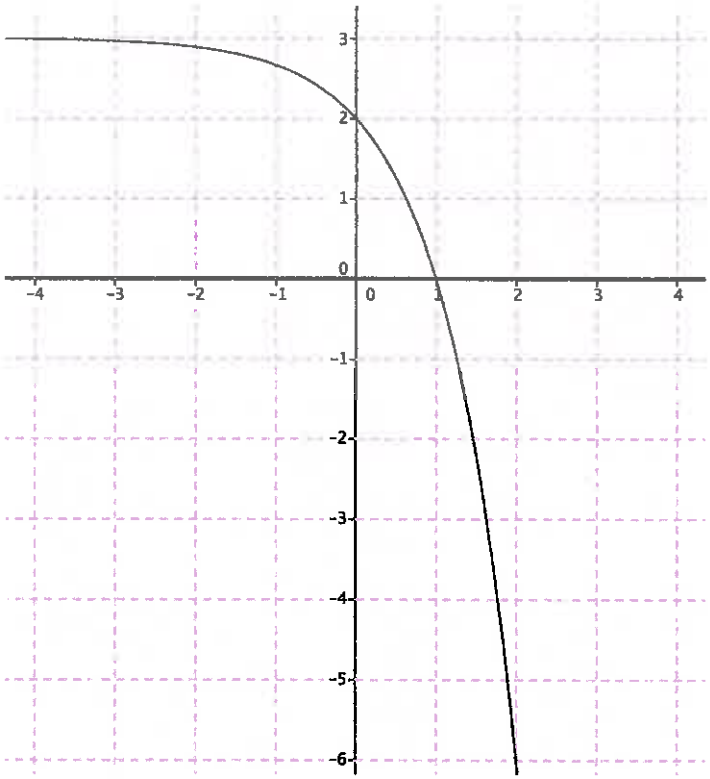
y-intercept: _____

Asymptote: _____

Parent function: _____

Domain: _____

Range: _____



b) State the transformations

c) Determine the equation of the following graph

QUESTION SIX

The temperature of hot soup decreases exponentially while it sits in the fridge. Approximately 6% of its heat is lost every 5 minutes.

a) If the initial temperature of the soup is 80°C, write an exponential function to model to describe the soup's cooling.

b) What is the temperate of the soup after 1 hour? Round to two decimal places.

c) How many minutes it take the temperature to drop to 10 °C? Round to the nearest minute.

QUESTION SEVEN

We know that bacteria grow at an exponential rate. A sample of chicken is left out at room temperature. After 5 hours, they find 3500 E.coli bacterium growing on the chicken. You can assume that bacteria double every 30 minutes.

a) Determine the initial number of E.coli in the sample. Round to the nearest whole number.

b) An agent is then added to the chicken which disrupts the reproduction process of the E.coli bacteria. After another 2 hours, the number of bacteria is measured to be 5000. Assuming they still divide every 30 minutes, what is the new rate? Ensure you consider what the initial number of bacteria should be.

QUESTION EIGHT

Show how the model $f(x) = 3(2)^{2x+6}$ can be rewritten as another equation in the form $f(x) = a(b)^x$ by simplifying the equation using powers laws.

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Exponential Practice Test

+
Logs

QUESTION ONE

Simplify these expressions. Show your working and leave you answers with positive, rational powers.

a) $\dfrac{3x^{-3}y^2}{15x^2y^{-1}}$

$\dfrac{1x^{-5}y^3}{5} = \dfrac{y^3}{5x^5}$

b) $\dfrac{(-8pq^3r^2)(-6p^2r^4)}{12p^3q^2r}$

$\dfrac{48p^3q^3r^6}{12p^3q^2r}$

$4qr^5$

c) $\left(\dfrac{x^{-3}y^2}{z^{-1}}\right)^2\left(\dfrac{z^{-3}}{x^{-4}y}\right)^{-2}$

$\dfrac{x^{-6}y^4}{z^{-2}} \cdot \dfrac{z^6}{x^8y^{-2}}$

$x^{-14}y^6z^8$

$\dfrac{y^6z^8}{x^{14}}$

d) $\left(\dfrac{12a^2b^{-3}}{9a^{-4}b^1}\right)^{-1}$

$\left(\dfrac{4a^6b^{-4}}{3}\right)^{-1}$

$\left(\dfrac{4}{3}\right)^{-1}a^{-6}b^4$

$\dfrac{3b^4}{4a^6}$

e) $m^{-1}\cdot m^{\frac{1}{6}}\cdot\sqrt[3]{m^4}$

$m^{-1}\cdot m^{\frac{1}{6}}\, m^{\frac{4}{3}}$

$m^{-\frac{6}{6}}\cdot m^{\frac{1}{6}}\cdot m^{\frac{8}{6}}$

$m^{\frac{3}{6}}$

$m^{\frac{1}{2}}$

or $\sqrt{m}$

f) $(16x)^{\frac{3}{4}}(81x^8)^{-\frac{1}{4}}$

$\dfrac{(16x)^{3/4}}{(81x^8)^{\frac{1}{4}}}$

$= \dfrac{(16^{\frac{1}{4}})^3\cdot x^{3/4}}{(81)^{\frac{1}{4}}\cdot(x^8)^{\frac{1}{4}}}$

$= \dfrac{8x^{3/4}}{3x^2} = \dfrac{8x^{-5/4}}{3} = \dfrac{8}{3x^{5/4}}$

QUESTION TWO

Simplify these expressions, express the answer in radical form.

a) $x^{\frac{3}{2}}$

$\sqrt{x^3}$

b) $\dfrac{x^{\frac{7}{2}}\cdot x^{\frac{3}{4}}}{x^3} = \dfrac{x^{14/4}\cdot x^{3/4}}{x^{12/4}} = \dfrac{x^{17/4}}{x^{12/4}}$

$= x^{5/4} = \sqrt[4]{x^5}$

QUESTION THREE

Express as a single, positive exponent, then evaluate and express answer in rational form.

a) $\dfrac{10^{-2}}{5^{-3}}$

$\dfrac{5^3}{10^2} = \dfrac{5^3}{5^2\cdot 2^2} = \dfrac{5}{2^2}$

$= \dfrac{5}{4}$

$10^2 = (5\times 2)^2$
$= 5^2\cdot 2^2$

b) $(\sqrt[3]{8})^{-2}$

$\dfrac{1}{(8^{\frac{1}{3}})^2} = \dfrac{1}{8^{2/3}}$

$= \dfrac{1}{\sqrt[3]{8^2}}$

$= \dfrac{1}{4}$

c) $64^{\frac{2}{3}}\cdot 64^{-\frac{1}{2}}$

$64^{4/6}\cdot 64^{-3/6}$

$64^{\frac{1}{6}}$

$= \sqrt[6]{64} = 2$

d) $81^{\frac{3}{4}} - 16^{\frac{3}{2}}$

$((81)^{\frac{1}{4}})^3 - (16^{\frac{1}{2}})^3$

$27 - 64$

$= -37$

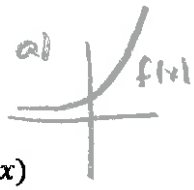
QUESTION FOUR

Graph the following exponential function.
Remember to label your graphs.
For each graph:

a) Graph the parent function, $f(x)$
b) State the transformations
c) Graph $g(x)$

i) $g(x) = -(2)^{\frac{x}{2}}$

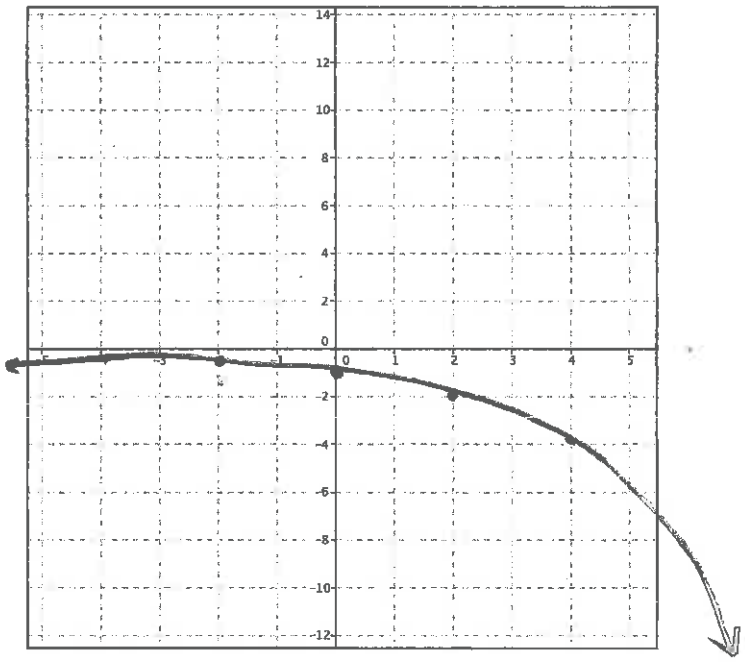
a)



x	-2	0	2	4	6
g(x)	$-\frac{1}{2}$	-1	-2	-4	-8

b) reflected across x axis
$b = \frac{1}{2}$

c)

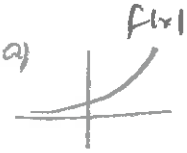


ii) $g(x) = 2\left(\frac{1}{3}\right)^{x} - 10$

decay

HA: $g(x) = -10$

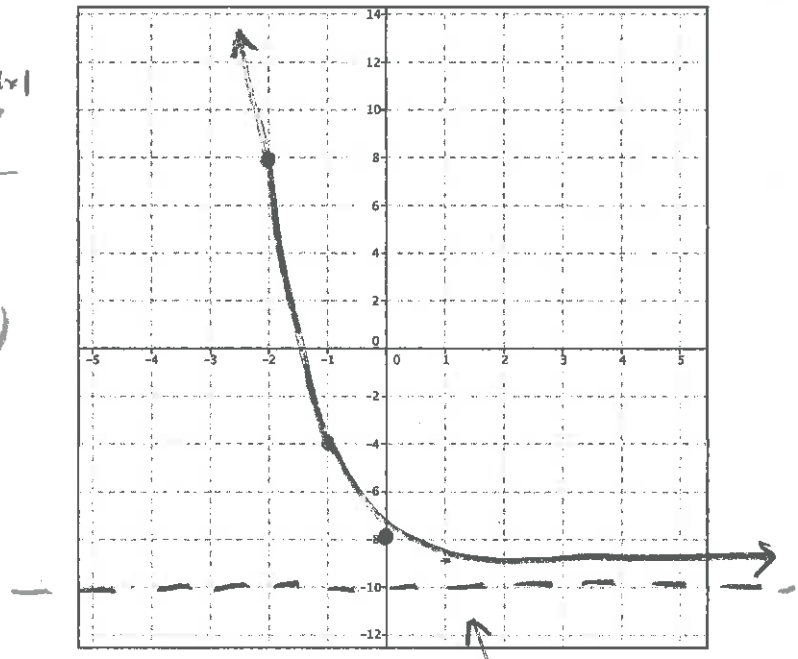
a)



x	0	1	-1	-2
g(x)	-8	-9.33	-4	8

b) decay → reflected across y axis
translate 10 units down

c)



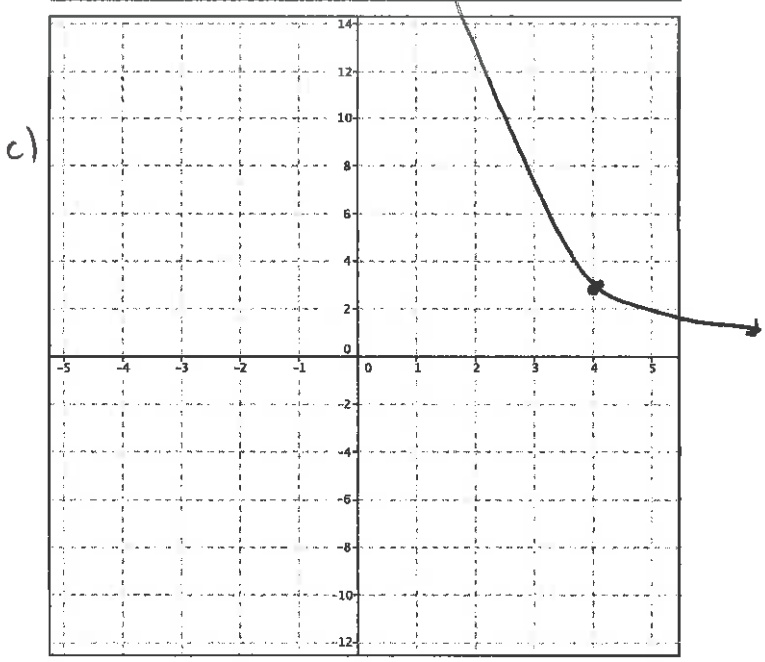
iii) $g(x) = 3\left(\frac{1}{2}\right)^{x-4}$

a)

b) decay → reflected across y axis
translate 4 units to right

x	4	0
y	3	48

c)



QUESTION FIVE

a) Describe the following features of the graph.

x-intercept: 1

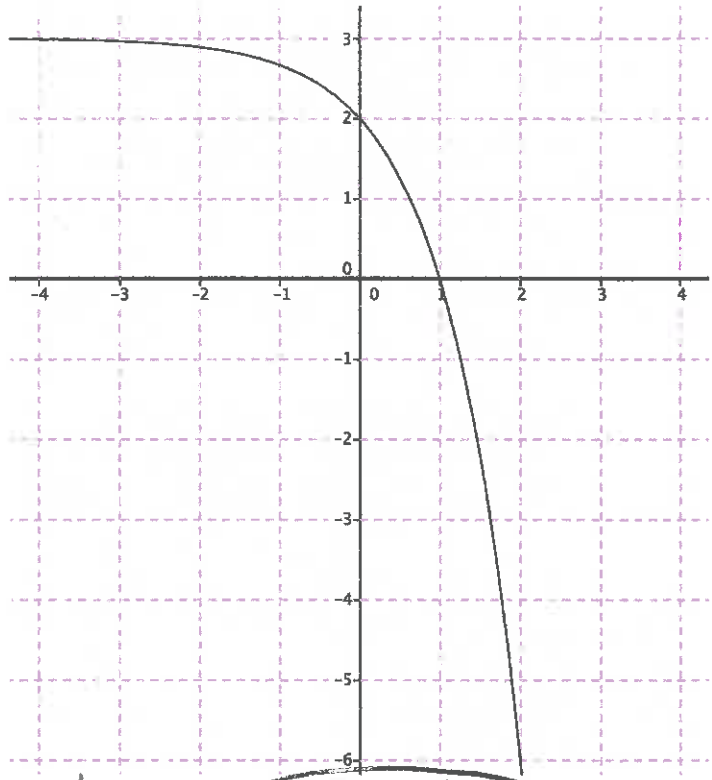
y-intercept: 2

Asymptote: $y = 3$

Parent function: $y = 3^x$

Domain: $\mathbb{R}$

Range: $]-\infty, 3[$



b) State the transformations

- reflected across the x axis
- translated 3 units up

c) Determine the equation of the following graph

$y = ac^x + k$ $(0,2)$ $(1,0)$ $2 = ac^0 + 3$

$-1 = ac^0$

$\boxed{a = -1}$

$0 = -1(c)^1 + 3$

$\boxed{c = 3}$

$y = -1(3)^x + 3$

QUESTION SIX

The temperature of hot soup decreases exponentially while it sits in the fridge. Approximately 6% of its heat is lost every 5 minutes.

a) If the initial temperature of the soup is 80°C, write an exponential function to model to describe the soup's cooling.

$x = t$ (min)

$y =$ temp (°C)

$y = 80(0.94)^{x/5}$

b) What is the temperate of the soup after 1 hour? Round to two decimal places.

let $x = 60$ min

$\therefore y = 80(0.94)^{60/5} = \boxed{38.07\,°C}$

c) How many minutes it take the temperature to drop to 10 °C? Round to the nearest minute.

$\frac{10}{80} = \frac{80(0.94)^{x/5}}{80}$

$0.125 = 0.94^{x/5}$

$\log_{0.94} 0.125 = \frac{x}{5}$

$\frac{5 \log 0.125}{\log 0.94}$

168 minutes

QUESTION SEVEN

We know that bacteria grow at an exponential rate. A sample of chicken is left out at room temperature. After 5 hours, they find 3500 E.coli bacterium growing on the chicken. You can assume that bacteria double every 30 minutes.

a) Determine the initial number of E.coli in the sample. Round to the nearest whole number.

$x = t$ (hr)
$y =$ bacteria

$y = a(2)^{2x}$

$3500 = a(2)^{(2)(5)}$

$3500 = a \cdot 2^{10}$

$a = 3.41$

Round down $\rightarrow$ 3

Rule
$y = 3(2)^{2x}$

b) An agent is then added to the chicken which disrupts the reproduction process of the E.coli bacteria. After another 2 hours, the number of bacteria is measured to be 5000. Assuming they still divide every 30 minutes, what is the new rate? Ensure you consider what the initial number of bacteria should be.

$\frac{5000}{3500} = 3500(c)^4$

$1.4286 = c^4$

$1.09 = c$

$c = 1.09$ $\therefore$ growth rate = 9%

QUESTION EIGHT

Show how the model $f(x) = 3(2)^{2x+6}$ can be rewritten as another equation in the form $f(x) = a(b)^x$ by simplifying the equation using powers laws.

$3(2)^{2x+6}$

$= 3 \cdot 2^{2x} \cdot 2^6$

$= 3 \cdot (2^2)^x \cdot 64$

$= 3 \cdot 4^x \cdot 64$

$f(x) = 192(4)^x$