**58** Functions $f$ and $g$ are defined by $f(x) = (0.5)^x$ et $g(x) = -3(0.5)^{x+2}$.

***a)*** State the rule for the following functions:

1) $f + g$ 2) $f - g$ 3) $f \cdot g$ 4) $\frac{f}{g}$

***b)*** Which of these functions are exponential?

**59** **PLANET EARTH: NO VACANCY**

The table below shows the human population on earth in different years. It is apparent that the human population is increasing at an alarming rate, so much so that scientists are now looking for ways to deal with the problems of overpopulation. Maybe one day in the not-so-distant future, humans will begin inhabiting other planets.

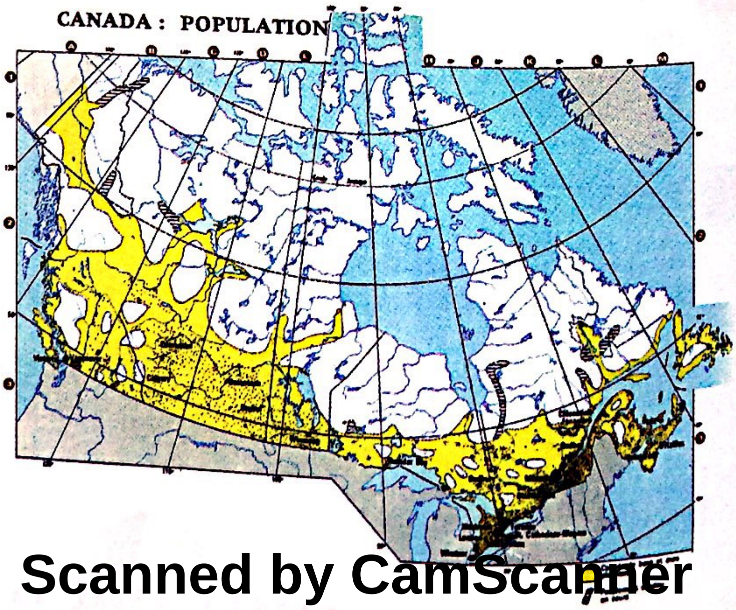
| Year | Population (billions) |
|---|---|
| 1959 | 3 |
| 1974 | 4 |
| 1986 | 5 |
| 1997 | 6 |

***a)*** The rule $p(x) = 10^{(0.007953x - 15.1)}$ estimates the earth's population, in billions, on the first day of January in a given year. According to this empirical model, what will the earth's population be in 2010?

***b)*** According to some scientists, the earth can support 27 billion people. In what year will the earth's population reach this critical point?

**60** **POPULATIONS COMPARED**

On January 1, 1994, Canada had a population of 29 094 400 people, increasing by 1.1% each year. At the same time, Québec had a population of 7 258 400, increasing by 0.6% each year. On January 1, 2010, assuming this trend persists, what fraction of the Canadian population will the Québec population represent?



## COMPOUND INTEREST

When a principal $P$ is invested with compound interest, the interest is added to the principal after a specified time period. At the end of each period, when the interest is added, we say that the interest has been "capitalized" (turned into capital). Below is a formula that calculates the final amount $A$ obtained when a principal $P$ is invested for $t$ years at an annual interest rate of $r$. The interest is compounded $n$ times a year.

$$A = C\left(1 + \frac{r}{n}\right)^{nt}$$

One dollar is invested at an annual rate of 100% for one year. The final amount $A$ of this investment is calculated when interest is compounded over different periods of time.

a) Complete the table.

b) What happens as the number of capitalization periods increases?

c) When you continually increase the number of capitalizations, what value does $A$ approach?

Value $A$ is calculated from a principal $P$ invested at an annual interest rate $r$ compounded continuously for $t$ years according to the formula

$$A = Ce^{rt}$$

| Capitalization frequency | $n$ | $A = C\left(1 + \frac{r}{n}\right)^{nt}$ | $A$ (\$) after 1 year |
|---|---|---|---|
| Yearly | 1 | $A = 1(1 + 1)^1$ | 2 |
| Semi-annually | 2 | $A = 1\left(1 + \frac{1}{2}\right)^2$ | 2.25 |
| Quarterly | 4 | $A = 1\left(1 + \frac{1}{4}\right)^4$ | 2.44141 |
| Monthly | 12 | $A = 1\left(1 + \frac{1}{12}\right)^{12}$ | 2.61304 |
| Weekly | 52 | $A = 1\left(1 + \frac{1}{52}\right)^{52}$ | 2.69260 |
| Daily | 365 | | |
| Hourly | | | |
| Every second | | | |

d) A sum of \$1000 is invested for 5 years at an interest rate of 8%. At the end of this period, what would the difference in returns be if the interest was compounded continuously as opposed to monthly?

e) A financial institution guarantees an interest rate of 8.5% for a 10-year period. How much capital must be invested to yield a return of \$15 000 at the end of this period:

1) If the interest is compounded every three months?

2) If the interest is compounded continuously?