Mathematics 5 SN

CONICS

568536 - Mathematics

Question Booklet

1 A circle with centre O(2, 3) passes through point A(5, 6). Diameter AB is obtained by extending radius OA.

What is the equation of the tangent to the circle that passes through point B?

Show all your work.

Show all your work.

Answer: The equation of the tangent to the circle passing through point B is

____.

2 The line whose equation is $4x + 3y - 43 = 0$ is tangent to a circle with centre C(3, 2).

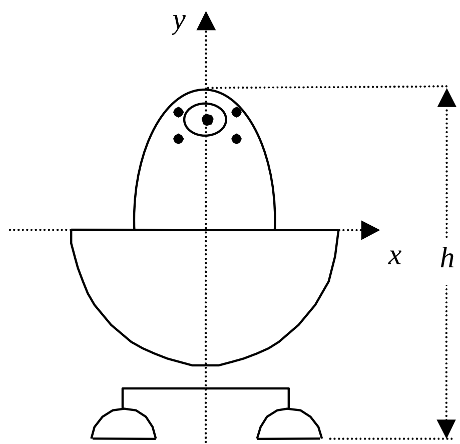
What is the equation of this circle?

Show all your work.

Show all your work.

Answer: The equation of the circle is ______________________________________.

3 Some Mechanical Engineering students decide to build a miniature robot as a class project. Here is their design :



- The middle of the single eye is situated at the focus of the semi-ellipse, centre (0, 0). This focus is located 4 units from the minor axis of the semi-ellipse.

- The minor axis of the semi-elliptical head has a length of 4 units.

- The semi-circular main body of the robot is defined by the equation : $x^2 + y^2 - 25 = 0$

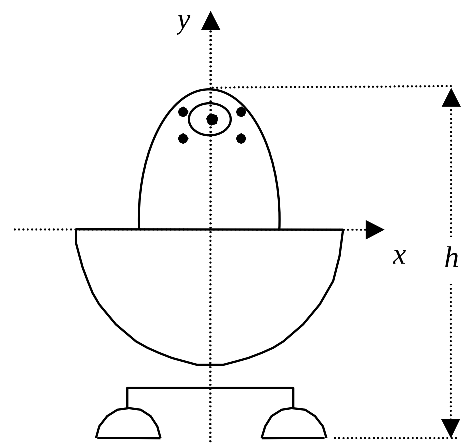
- The robot's semi-circular feet are defined by the equations

  $x^2 + y^2 + 6x + 16y + 72 = 0$ and $x^2 + y^2 - 6x + 16y + 72 = 0$

What is the overall height h of the robot?

Show your work.

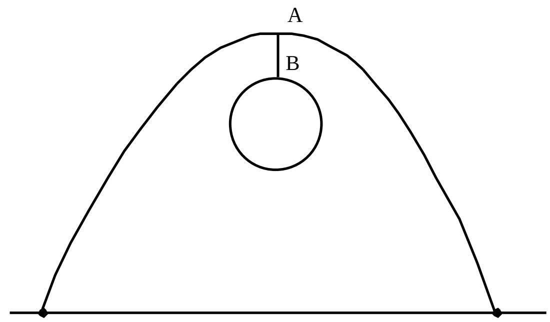
Work



Result : The height h of the robot is ________________.

4 In a circus act, a lion has to leap through a circular ring. A chain fastens the ring to a metal support that is parabolic in shape. The ring and the support are drawn in the Cartesian plane in the following manner :

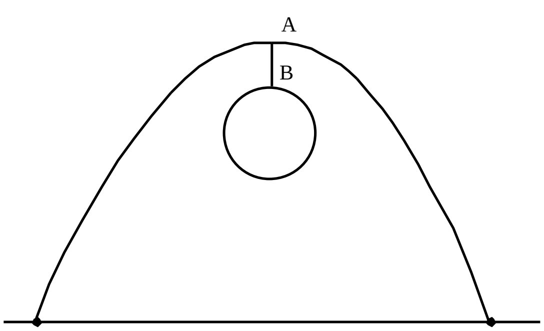
- point A is located at the vertex of the parabola and point B coincides with its focus;
- chain AB has the same measure as the radius of the ring;
- the equation of the circle representing the ring is $x^2 + (y - 2)^2 = 0.25$.

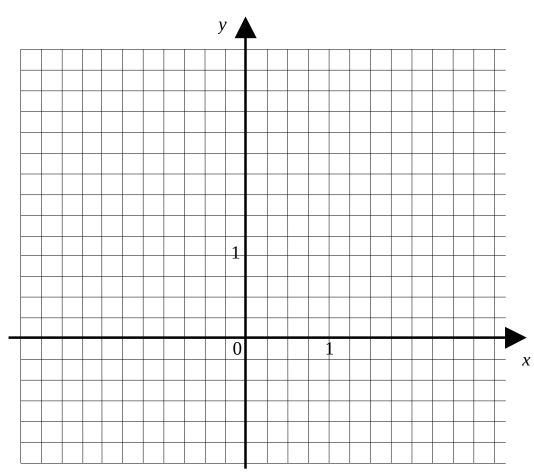


What is the equation of the parabola in this situation?

Show your work.

Work





Result : The equation of the parabola representing the metal support is ________________.

6

Match each definition on the left with the correct term on the right.

DEFINITION		**TERM**
1.	The set of all points in a plane for which the sum of the distances to two fixed points is constant.	(A) Circle
2.	The set of all points in a plane for which the difference of their distances to two fixed points is constant.	(B) Ellipse
3.	The set of all points in a plane that are equidistant from one fixed point.	(C) Hyperbola
		(D) Parabola

1. ________________ 2. ________________ 3. ________________

7

Given the hyperbola with the equation $9x^2 - 4y^2 - 36 = 0$. What is the equation of the circle centered at the origin which passes through the foci of this hyperbola?

A) $x^2 + y^2 = 5$

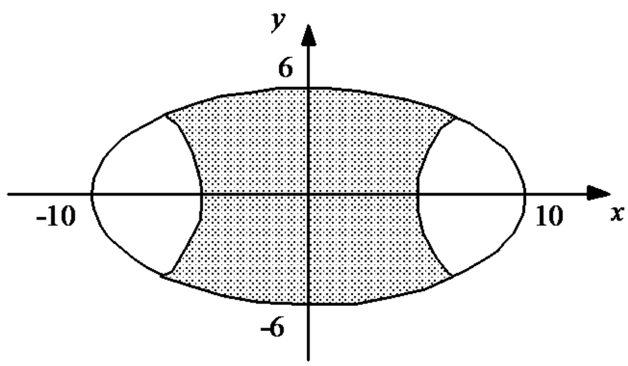
B) $x^2 + y^2 = 13$

C) $x^2 + y^2 = 20$

D) $x^2 + y^2 = 52$

8

The logo used by a football team is represented in the Cartesian plane. It comprises an ellipse and two parts of a hyperbola. The shaded region is defined by two relations.



The vertices of the hyperbola coincide with the foci of the ellipse.

The foci of the hyperbola coincide with the endpoints of the major axis of the ellipse.

What relation defines the shaded region of the hyperbola?

A) $\frac{x^2}{64} - \frac{y^2}{36} \leq 1$

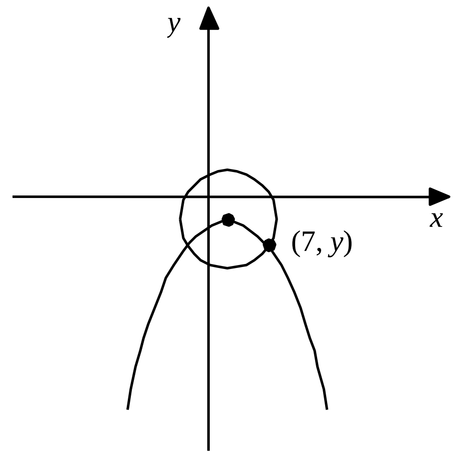
C) $\frac{x^2}{100} - \frac{y^2}{36} \leq 1$

B) $\frac{-x^2}{64} + \frac{y^2}{36} \leq 1$

D) $\frac{x^2}{100} + \frac{y^2}{36} \leq 1$

9

As illustrated in the adjacent diagram, a circle whose centre coincides with the vertex of a parabola, also cuts it at a point with coordinates (7, y). The equation of the parabola is :



$$x^2 - 6x + 4y + 17 = 0.$$

What is the equation of the circle?

A) $(x - 3)^2 + (y + 2)^2 = 32$

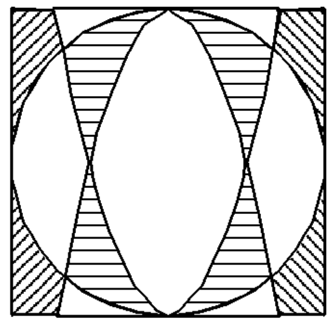
B) $(x - 3)^2 + (y + 2)^2 = 49$

C) $(x - 3)^2 + (y - 2)^2 = 32$

D) $(x - 3)^2 + (y - 2)^2 = 49$

10 A packaging company decorates its square boxes according to the model shown below.

The equation of the circle in the model has the form :



$$(x-h)^2 + (y-k)^2 = r^2$$

The equations of the parabolas are :

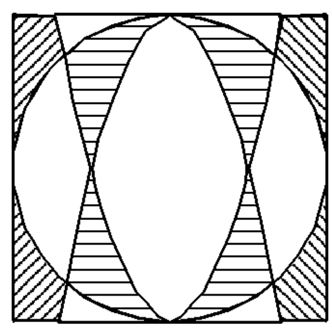
$$x^2 - 8x - y + 16 = 0 \text{ and}$$

$$x^2 - 8x + y + 8 = 0$$

If the cartesian unit of measure is 1 cm, calculate the length of the diagonal of this box.

Show your work.

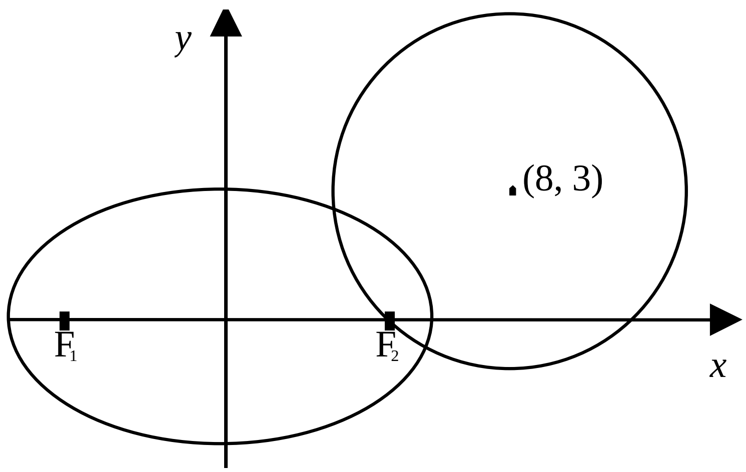
Work



Result : The length of the diagonal is _______________.

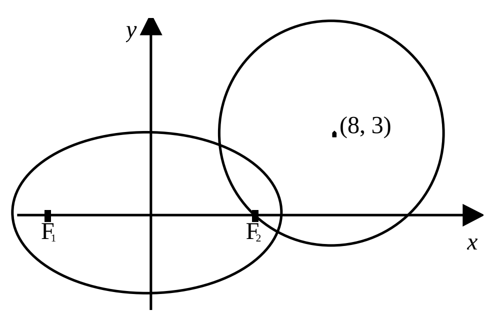
11

An ellipse whose equation is $\frac{x^2}{25}+\frac{y^2}{9}=1$ overlaps a circle, centre (8, 3). The circle passes through the focus F_2 of the ellipse, as shown below.



What is the equation of the circle?

Work



Answer : The equation of the circle is ____.

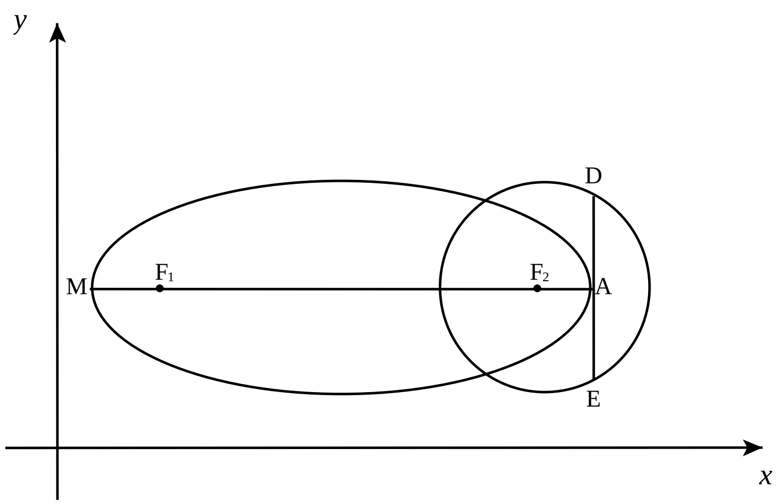
12 In the figure below, whose measures are in centimetres:

- the equation of the ellipse is $\frac{(x-12)^2}{100}+\frac{(y-8)^2}{36}=1$

- the centre of the circle is focus F_2 of the ellipse

- the radius of the circle measures 4 cm

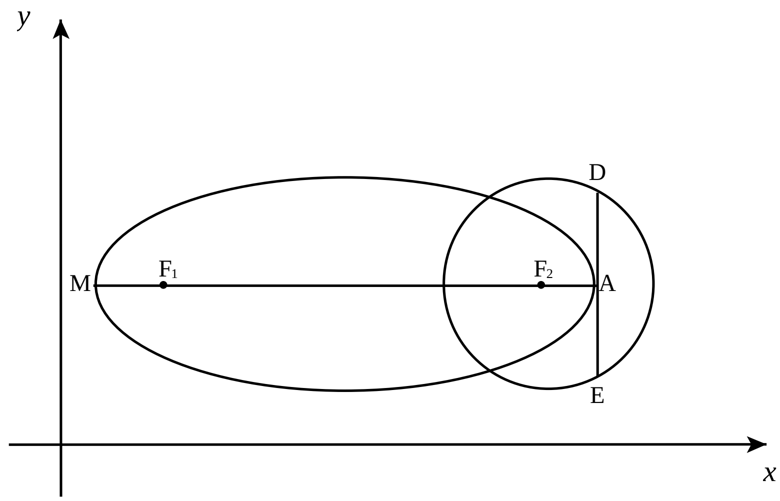
- chord DE is tangent to vertex A of the ellipse and $\overline{DE} \perp \overline{MA}$



To the nearest hundredth of a centimetre, what is the length of chord DE?

Show all your work.

Show all your work.



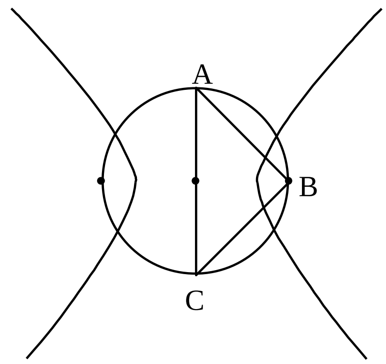
Answer: The length of chord DE is ___________ cm.

13

A figure is formed by two conic sections whose measures are in centimetres.

The equation of the first conic is $\dfrac{(x-3)^2}{64} - \dfrac{(y-4)^2}{36} = 1$.

The second conic is a circle that passes through the foci of the first.

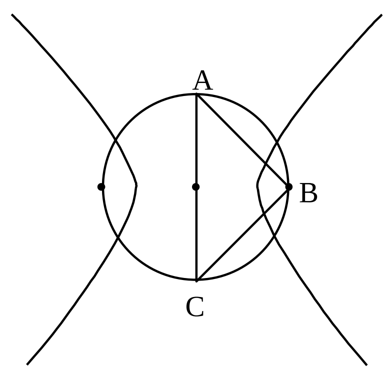


A right isosceles triangle ABC is inscribed in the semi-circle, as illustrated in the diagram on the right.

What is the perimeter of triangle ABC?

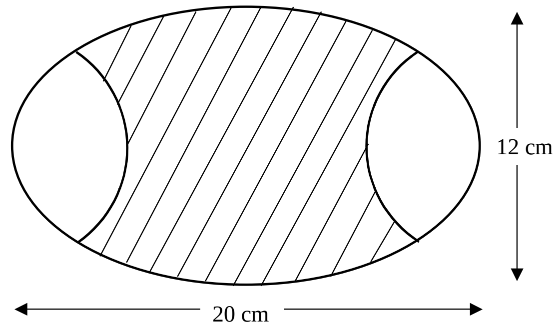
Show all your work.

Show all your work.



Answer: The perimeter of right isosceles triangle ABC is __________ cm.

14 John asked his mother to embroider a rugby ball on his sweater. He drew the following sketch:



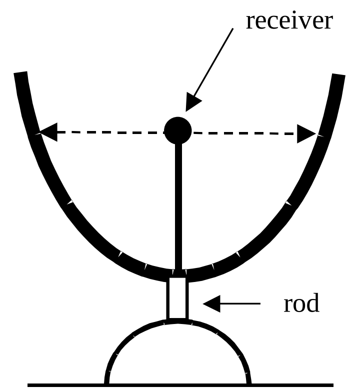
The ball is formed by an ellipse and a hyperbola. John wants the striped area to be embroidered in yellow. The vertices of the hyperbola coincide with the foci of the ellipse. The foci of the hyperbola coincide with the vertices of the ellipse.

What inequality corresponds to the striped area in relation to the hyperbola?

The inequality of the hyperbola is ____________________________.

15

An antenna in the shape of a parabola is attached to a rod whose length is 1 m. This rod is attached to a base in the shape of a hemisphere. For the best reception, a receiver must be placed at the same level as the focus of the parabola. The figure on the right represents a cross-section of the antenna.

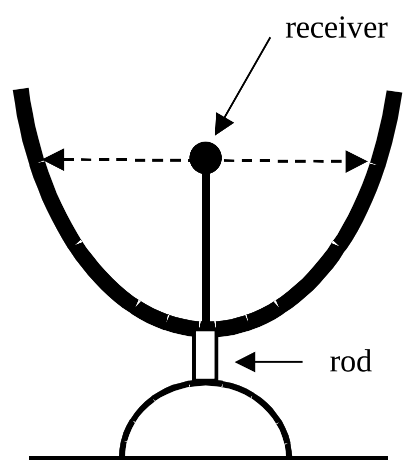


The equation of the circle representing the hemisphere is $(x-4)^2 + y^2 = 100$. The parabola, which represents the antenna, passes through point (20, 15).

What is the measure of the width of the antenna opening at the level of the receiver?

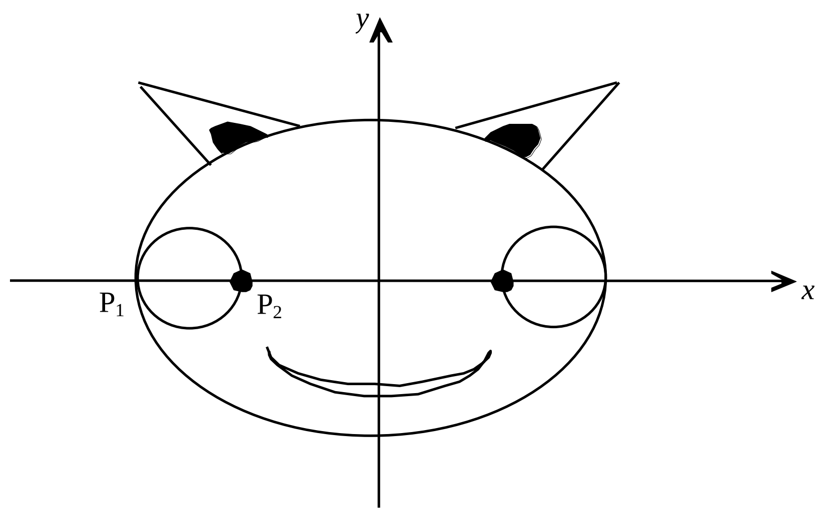
Show all your work.

Show all your work.



Answer: The measure of the width of the antenna is _____________________.

16 A graphic artist is using a computer to create a cartoon figure. The figure is shown in the Cartesian plane below.



The two eyes of the figure are circles and the equation of the circle with diameter $\overline{P_1P_2}$ is

$$x^2 + 36x + y^2 + 320 = 0$$

The face of the figure is an ellipse. The point P_1 corresponds to a vertex of the ellipse and the point P_2 corresponds to a focus.

Which of the following is the equation of the ellipse?

A) $\dfrac{x^2}{576} + \dfrac{y^2}{772} = 1$

C) $\dfrac{x^2}{400} + \dfrac{y^2}{656} = 1$

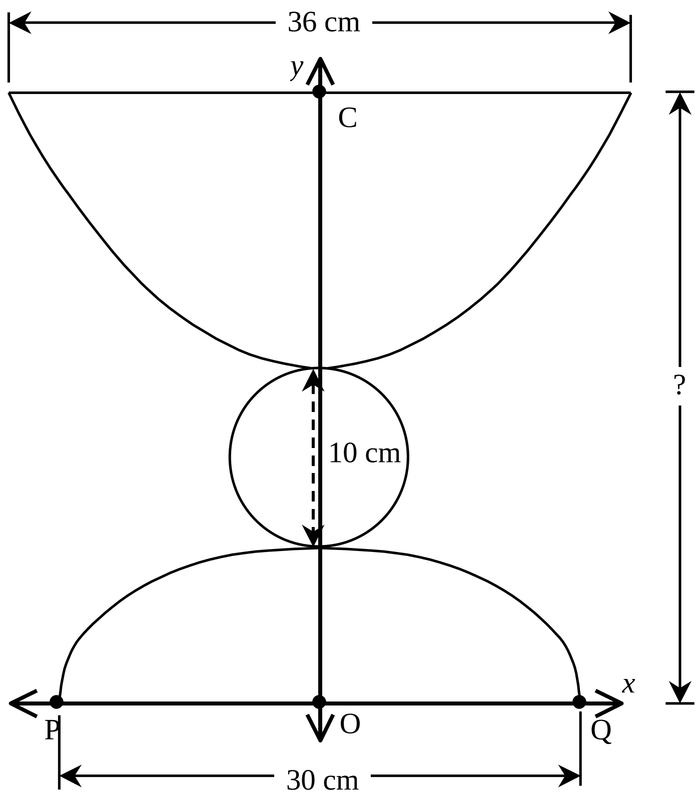
B) $\dfrac{x^2}{576} + \dfrac{y^2}{380} = 1$

D) $\dfrac{x^2}{400} + \dfrac{y^2}{144} = 1$

17

The cross-sectional view of a punchbowl is shown below. The view of the punchbowl's base is in the shape of a semi-ellipse, whose major axis, PQ, measures 30 cm. The foci of this semi-ellipse are 12 cm from its centre.

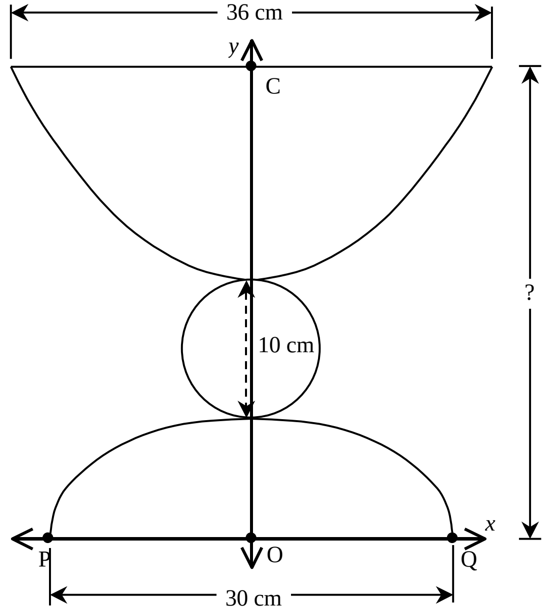
Directly above the semi-ellipse lies a circle whose diameter is 10 cm. On top of the circle lies a parabola whose vertex touches the circle and whose directrix passes through the centre of the circle. The width of the punchbowl at the top is 36 cm.



What is the height, CO, of the punchbowl?

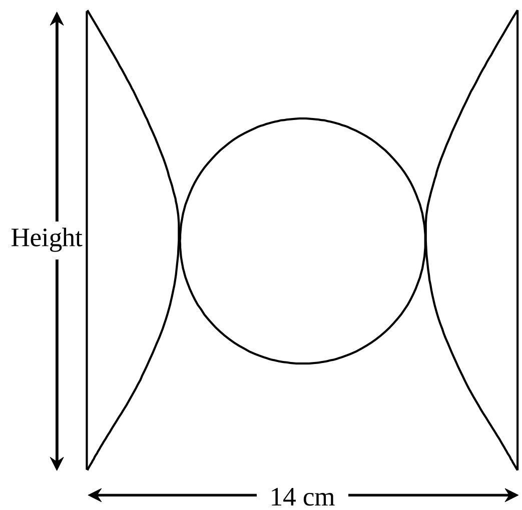
Show all your work.

Show all your work.



Answer The height, CO, of the punchbowl is ________ cm.

18 A modern picture frame is in the shape of a circle between the two branches of a hyperbola, as shown in the diagram below.

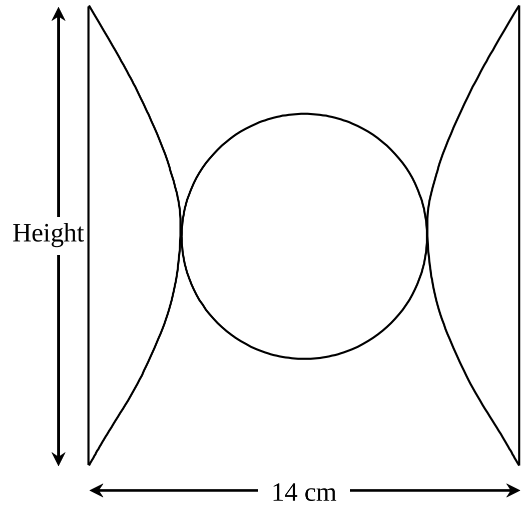


The equation of the circle is $(x - 13)^2 + (y - 10)^2 = 16$. The centre of the circle and the centre of the hyperbola coincide. The vertices of the hyperbola are the endpoints of the horizontal diameter of the circle and the vertical edges of the picture frame pass through the foci of the hyperbola. The total length of the frame is 14 cm.

What is the height of the frame?

Show all your work.

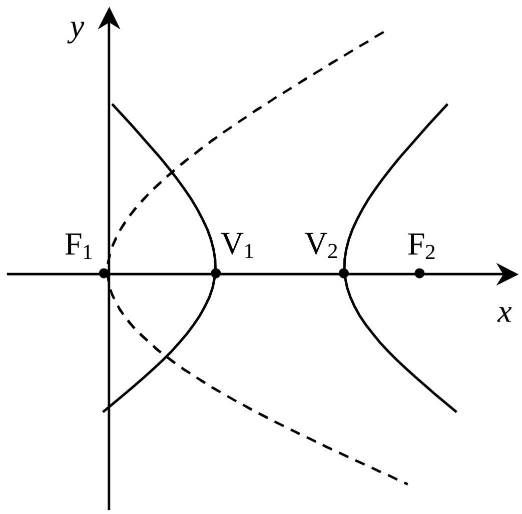
Show all your work.



Answer: The height of the frame is __________ cm.

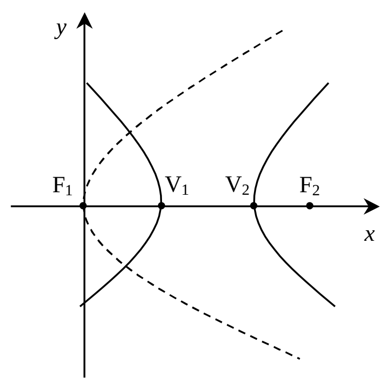
19

In a dual lens system one focus of the hyperbola is at the origin. The other focus coincides with the focus of the parabola. One of the vertices of the hyperbola is at (7, 0). The equation of the parabola is $y^2 = 32x$.



What is the equation of the hyperbola?

Show all your work.



Answer: The equation of the hyperbola is

____.

20 Courtney has been hired to paint lines on a field. The lines consist of a large circle C1, whose centre coincides with the centre of the rectangular field, two small (congruent) circles, and two (congruent) parabolas. The rectangular field, along with the lines Courtney must paint, are shown on the Cartesian plane below, which is scaled in metres.

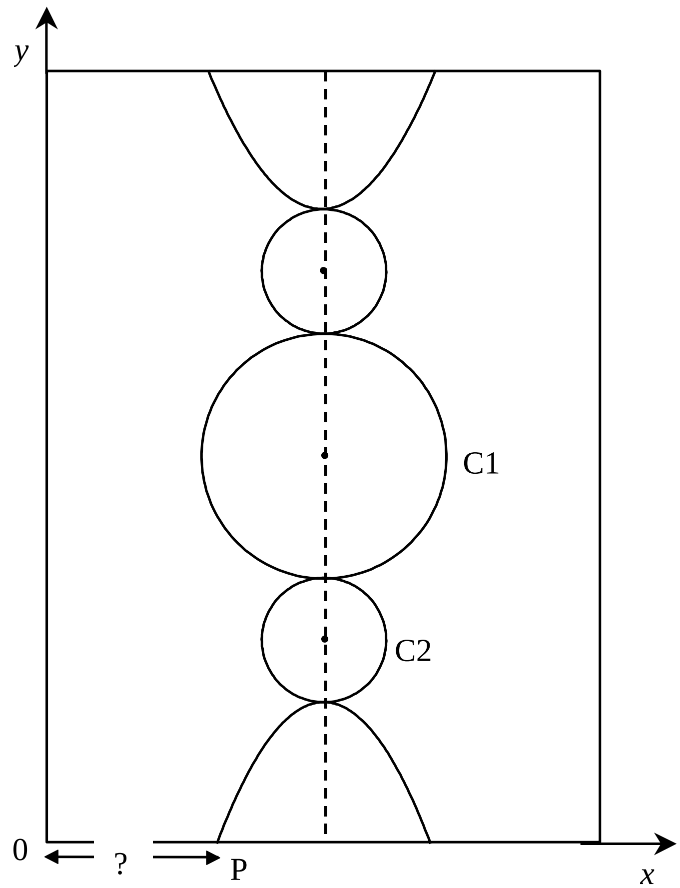
The equation of the large circle (C1) drawn on the field is:

$$x^2 + y^2 - 30x - 50y + 825 = 0$$

In addition,

- Circles C1 and C2 are tangent to one another. Their centres are vertically aligned 9 metres apart.
- The lower parabola is tangent to C2 at its vertex, which is directly below the center of C2.
- The center of C2 is a point on the directrix of the lower parabola.

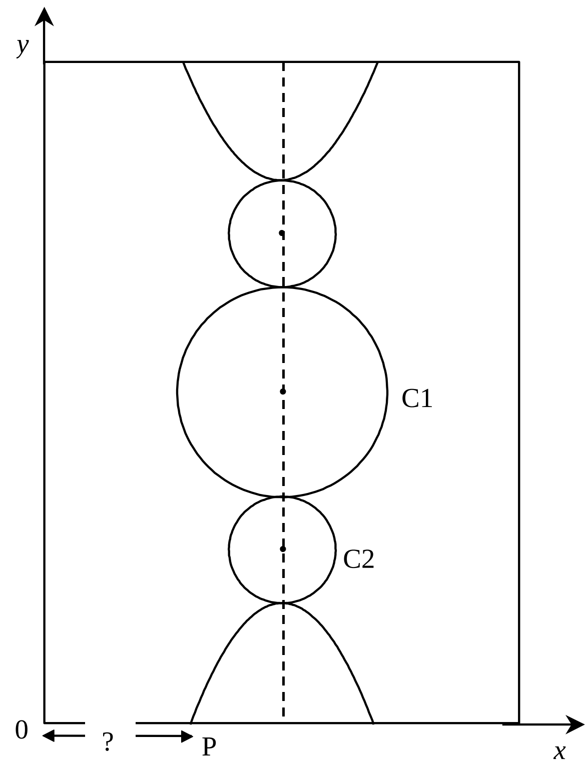
Courtney must begin the paint job at point P and she needs to know how far away from point 0 she should start.



What is the distance from point 0 to point P?

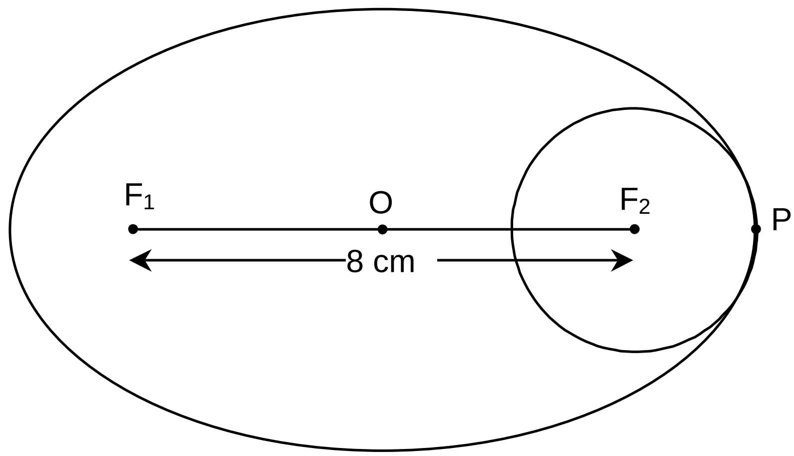
(Round your answer to the nearest hundredth metre.)

Show all your work.



Answer: The distance from point 0 to point P is ___________ m.

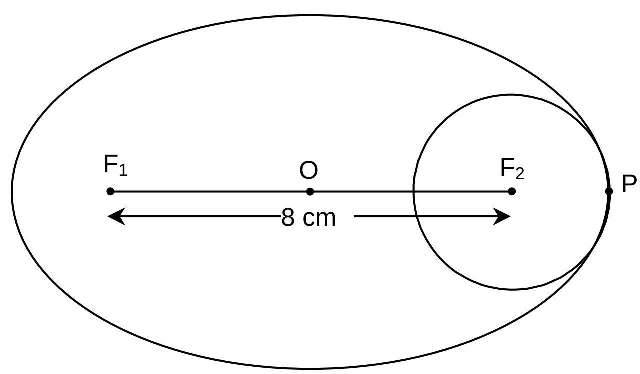
21 In the diagram below, the equation of the circle is $(x - 5)^2 + (y - 6)^2 = 4$.



The centre of the circle is the focus, F_2, of the ellipse. The distance between the foci is 8 cm. Point P is common to both the circle and the ellipse and is a vertex of the ellipse.

What is the equation of the ellipse?

Show all your work.

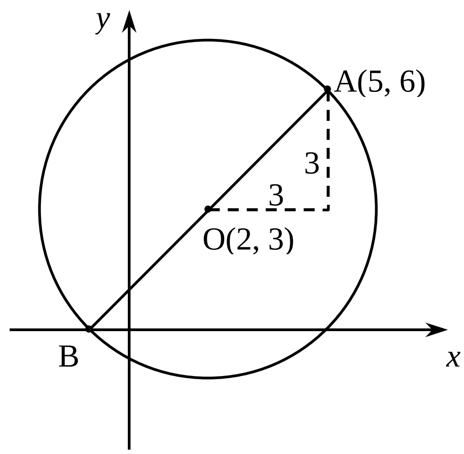


Answer: The equation of the ellipse is ______________________________.

2- Correction key

1 Example of an appropriate method

Slope of radii OA and OB



$$m = \frac{6-3}{5-2} = 1$$

Radius of the circle

$$r = \sqrt{3^2 + 3^2} = \sqrt{18}$$

Equation of the circle

$(x-2)^2 + (y-3)^2 = 18$

Equation of radius OB

$$\frac{y-3}{x-2} = 1$$

$y = x + 1$

Solving the system of equations:

$(x-2)^2 + (y-3)^2 = 18$ and $y = x + 1$

By substituting y with $x + 1$ in the equation of the circle, we find $x = 5$ or $x = -1$

If $x = -1$, the value of y is 0.

The coordinates of B are (-1, 0).

The equation of the tangent to the circle:

The slope of the tangent is -1 since it is perpendicular to the radius OB whose slope is 1.

$$\frac{y - 0}{x + 1} = -1$$

$y = -x - 1$

Answer: The equation of the tangent to the circle passing through point B is $x + y + 1 = 0$.

Note Accept all equations representing the same tangent.

2 Example of an appropriate method

Equation of the tangent line

$4x + 3y - 43 = 0$ or $y =$

$\frac{-4}{3}x + \frac{43}{3}$

Equation of the perpendicular line passing through C(3, 2)

$y = \frac{3}{4}x - \frac{1}{4}$

Intersection of the two lines

$\frac{-4}{3}x + \frac{43}{3} = \frac{3}{4}x - \frac{1}{4}$

$\frac{25}{12}x = \frac{175}{12}$

$x = 7$

If $x = 7$, $y = \frac{3}{4}(7) - \frac{1}{4} = 5$

The coordinate at the point of intersection is P(7, 5).

Measure of the radius of the circle

$$d(C, P) = \sqrt{(7-3)^2 + (5-2)^2} = \sqrt{25} = 5$$

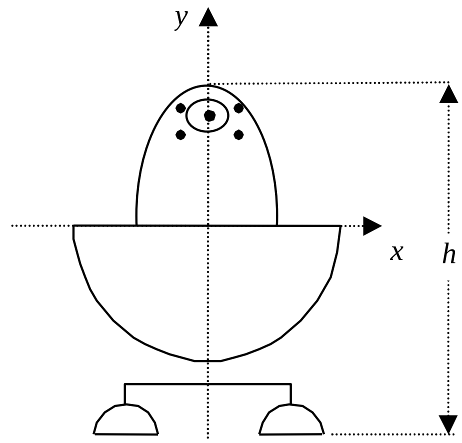
Equation of the circle $(x-3)^2 + (y-2)^2 = 25$

Answer The equation of the circle is $(x-3)^2 + (y-2)^2 = 25$.

The general form of the equation, $x^2 + y^2 - 6x - 4y - 12 = 0$, should also be accepted.

3

Work : (example)



Distance of eye from body :

4 units

Distance from vertex to body :

value b of the ellipse

Value a of the ellipse :

$4 \div 2 = 2$ units

And $a^2 + c^2 = b^2$

$2^2 + 4^2 = b^2$

$4 + 16 = 20 = b^2 \Rightarrow b = 2\sqrt{5} \approx 4.5$

Translation of semi-circular feet

$x^2 + 6x + y^2 + 16y + 72 = 0$

$x^2 + 6x + 9 + y^2 + 16y + 64 = -72 + 9 + 64$

$(x + 3)^2 + (y + 8)^2 = 1$

Translation (-3, -8) and radius = 1

or

(3, -8) for the right foot

There are 8 units from foot to neck

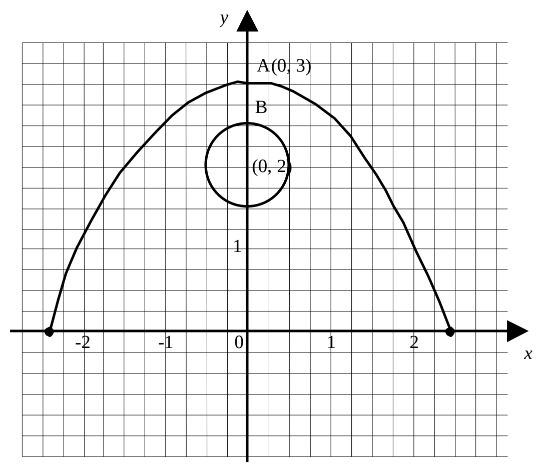
Total height : 8 + 4.5 = 12.5

Result : The height h of the robot is 12.5 units.

4 Work : (example)

To find the center of circle (h, k)

and radius :



$(x - h)^2 + (y - k)^2 = r^2$:

$(x - 0)^2 + (y - 2)^2 = 0.25$

<u>C(0, 2) and $r = 0.5$</u>

Coordinates of B

<u>the focus of the parabola : (0, 2.5)</u>

because B is on the circle.

<u>Coordinates of vertex A : (0, 3)</u>

because m $\overline{\text{AB}}$ = 0.5

The equation of the parabola S, with vertex (h, k), opening downward is :

$(x - h)^2 = 4c(y - k)$ where $(h, k) = (0, 3)$; coordinates of A and

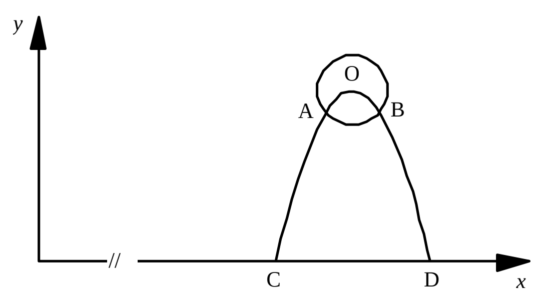
c = -0.5 because the parabola opens downward :

$(x - 0)^2 = 4(-0.5)\ (y - 3)$

The equation is $x^2 = -2(y - 3)$.

Result : The equation of the parabola representing the metal support is $x^2 = -2(y - 3)$.

5 A sculpture in the garden of a contemporary art museum consists of a circle and a parabola, as shown below.



The vertex of the parabola coincides with the centre O of the circle which has the equation

$$x^2 + y^2 - 32x - 10y + 279 = 0$$

where the unit of measure is the metre.

The parabola, whose axis of symmetry is vertical, is constructed such that angle AOB is a right angle.

Rounded to the nearest hundredth, what distance is there between bases C and D of the sculpture?

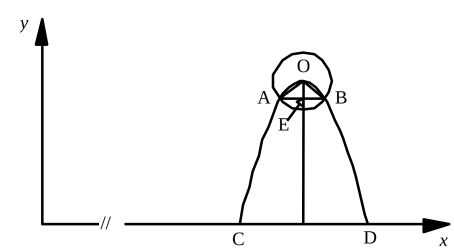
Show your work.

5 Work : (example)

Co-ordinates of O and measure of r

$x^2 + y^2 - 32x - 10y + 279 = 0$

in canonic form



$(x - 16)^2 + (y - 5)^2 = 2$

thus O(16, 5) and $r = \sqrt{2}$

Measure of segment AB

$\text{m}\,\overline{\text{AB}} = \sqrt{(\text{m}\,\overline{\text{AO}})^2 + (\text{m}\,\overline{\text{OB}})^2}$

$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$= \sqrt{2 + 2} = \sqrt{4} = 2$ Pythagorean theorem

Co-ordinates of point A

In isosceles triangle AOB, which is right-angled, the axis of symmetry of the parabola

$\text{m}\,\overline{\text{OE}} = \text{m}\,\overline{\text{AE}} = 1$

divides segment AB, at the point of intersection E, into two equal parts.

x-co-ordinate of A = 16 – 1 = 15

y-co-ordinate of A = 4

Point A belongs to the circle with centre O whose equation is known.

Equation of the parabola

$y = a(x - h)^2 + k$

$4 = a(15 - 16)^2 + 5$

The vertex of the parabola is O(16, 5).

$a = -1$

$y = -(x - 16)^2 + 5$

The co-ordinates of point A are used to calculate the value of a.

x-co-ordinate of points C and D

$0 = -(x - 16)^2 + 5$

$x^2 - 32x + 251 = 0$

$x_1 = 16 - \sqrt{5}$ and $x_2 = 16 + \sqrt{5}$

Measure of segment CD

$$\text{m}\,\overline{\text{CD}} = \left(16 + \sqrt{5}\right) - \left(16 - \sqrt{5}\right) = 2\sqrt{5} \approx 4.47$$

Result : Rounded to the nearest hundredth, the distance between C and D is 4.47 m.

Note.- Also accept 4.48 m.

6 1. (B) 2. (C) 3. (A)

7 B

8 A

9 A

10 Work : (example)

Translating the two parabolas

$x^2 - 8x - y + 16 = 0$

$x^2 - 8x + \underline{16} = y - 16 + \underline{16}$

$(x - 4)^2 = y$

$t(4, 0)$

$x^2 - 8x + y + 8 = 0$

$x^2 - 8x + \underline{16} = -y - 8 + \underline{16}$

$(x - 4)^2 = -y + 8$

$(x - 4)^2 = -(y - 8)$

$t(4, 8)$

The origin of the Cartesian plane is located at the lower left corner of the drawing.

The centre of the circle is at (4, 4) and the diameter of the circle measures 8 units which corresponds to the sides of the box.

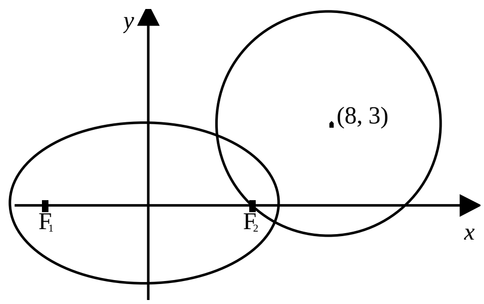
The diagonal of the box will measure

$$\mathrm{d} = \sqrt{8^2 + 8^2}$$

$$\mathrm{d} = \sqrt{128} \Rightarrow \mathrm{d} \approx 11.31$$

Result : The length of the diagonal is 11.31 cm.

11 Example of an appropriate solution



Coordinates of focus F_2

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$a = 5$ $b = 3$ $c = ?$

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c = 4$

therefore F_2 (4, 0)

Radius of circle

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(8 - 4)^2 + (3 - 0)^2}$$

$$r = \sqrt{16 + 9}$$

$r = 5$

Equation of circle

$(x - 8)^2 + (y - 3)^2 = 25$

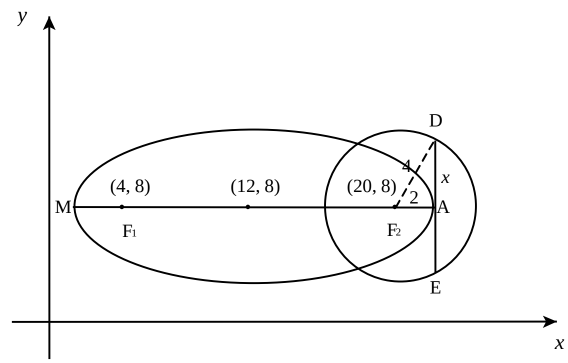
or $x^2 - 16x + y^2 - 6y + 48 = 0$

Answer : The equation of the circle is $(x-8)^2 + (y-3)^2 = 25$ or $x^2 - 16x + y^2 - 6y + 48 = 0$

12 Example of an appropriate solution

Centre C of the ellipse

(12, 8)



Co-ordinates of Focus F_2

$c^2 = a^2 - b^2$

$c^2 = 10^2 - 6^2$

$c^2 = 64$

$c = 8$

The co-ordinates of F_2 are (20, 8).

Measure of chord DE

$$(m\ \overline{DA})^2 = (m\ \overline{FD})^2 - (m\ \overline{FA})^2$$

$$(m\ \overline{DA})^2 = 4^2 - 2^2$$

$$(m\ \overline{DA})^2 = 12$$

$$m\ \overline{DA} = \sqrt{12}$$

$$m\ \overline{DE} = 2(m\ \overline{DA})$$

$$m\ \overline{DE} = 2\sqrt{12}$$

$$m\ \overline{DE} \approx 6.93$$

Answer: The length of chord DE is 6.93 cm.

13 Example of an appropriate solution

The equation $\frac{(x-3)^2}{64} - \frac{(y-4)^2}{36} = 1$ is that of a hyperbola with centre (3, 4).

Co-ordinates of the foci of the hyperbola

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 6^2$

$c = 10$

The co-ordinates of the foci are (13, 4) and (-7, 4).

Distance between the two foci

$d = |13 - (-7)|$

$d = 20$

Diameter of the circle

$D = 20$

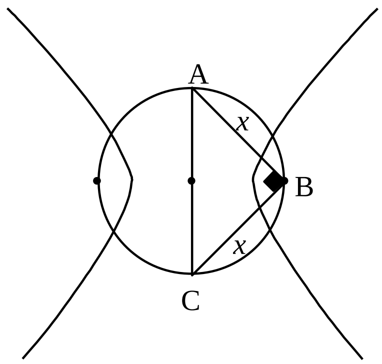
Measure of one of the sides adjacent to the right angle of triangle ABC

$2x^2 = 20^2$

$2x^2 = 400$

$x^2 = 200$

$x \approx 14.14$



Perimeter of the triangle

$P \approx 14.14 + 14.14 + 20$

$P \approx 48.28$

Answer: The perimeter of the right isosceles triangle ABC is 48.28 cm.

14 The inequality of the hyperbola is $\dfrac{x^2}{64} - \dfrac{y^2}{36} < 1$ or $\dfrac{x^2}{64} - \dfrac{y^2}{36} \leq 1$.

Accept any other equivalent inequality.

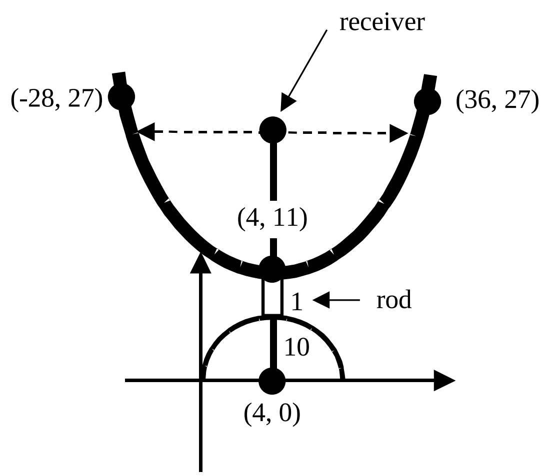
15 Example of an appropriate solution

Center of the circle: (4, 0)

Radius of the circle: $\sqrt{100} = 10$

Vertex of the parabola: (4, 11)

Equation of the parabola: $(x-h)^2 = 4c(y-k)$



$$(x-4)^2 = 4c(y-11)$$

Distance from the vertex to the focus of the parabola

Replacing point (20,15) into the equation

$$(20-4)^2 = 4c(15-1)$$
$$16^2 = 16c$$
$$16 = c \text{ (Distance from the vertex to the focus.)}$$

Equation of the parabola

$$(x-4)^2 = (16 \times 4)(y-11)$$
$$(x-4)^2 = 64(y-11)$$

Height of receiver

$$h = 10 + 1 + 16 = 27$$

Value of *x* when *h* = 27

$$\begin{aligned}(x-4)^2 &= 64(27-11)\\ (x-4)^2 &= 64(16)\\ \sqrt{(x-4)^2} &= \sqrt{64(16)}\\ x-4 &= \pm 32\\ x_1 = 36 \quad & x_2 = -28\end{aligned}$$

Measure of the width of the parabola

$$\begin{aligned}d &= 36-(-28)\\ &= 64\end{aligned}$$

Answer: The measure of the width of the antenna is **64** m.

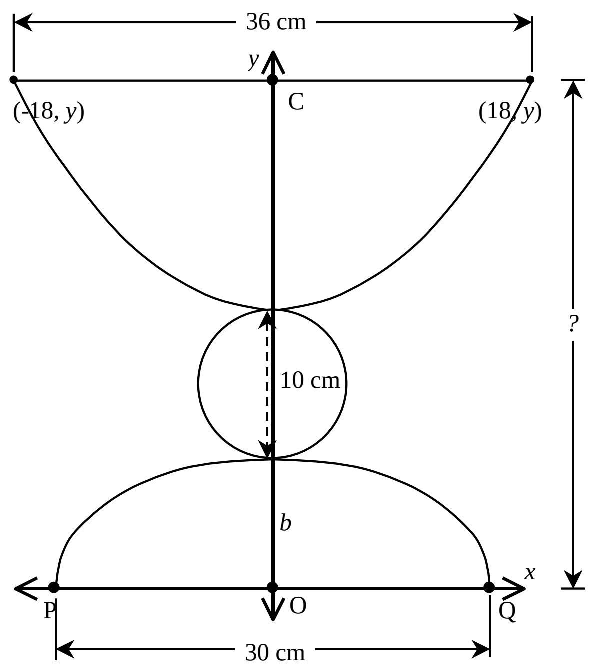
16

D

17 Example of an acceptable solution

Calculate the length of the Semi-Minor Axis (b)

c = 12 (Given)



Length of Major Axis = 2 a

$$\begin{aligned} 2a &= 30 \\ a &= 15 \end{aligned}$$

$$\begin{aligned} a^2 &= b^2 + c^2 \\ b &= \sqrt{a^2 - c^2} \\ b &= \sqrt{(15)^2 - (12)^2} \\ b &= \sqrt{225 - 144} \\ b &= \sqrt{81} \\ b &= 9 \end{aligned}$$

Find the equation of the parabola

Equation of Parabola: $(x - h)^2 = 4c(y - k)$ with Vertex V(h, k)

Note: Do not penalize students for having determined the rule in the form $y = a(x - h)^2$.

$h = 0$; $k = b + 10$

$k = (9) + 10$

$k = 19$

⇨ Equation of Parabola

$$\begin{aligned} (x - 0)^2 &= 4c(y - 19) \\ x^2 &= 4c(y - 19) \end{aligned}$$

With point O as the origin, the coordinates of the centre of the circle are (0, 14). Since the vertex of the parabola is V(0, 19), the vertical distance between these two points must be the value of c.

Hence $c = 19 - 14$

$c = 5$

Therefore the equation of the parabola is

$x^2 = 4(5)(y - 19)$

$x^2 = 20(y - 19)$

Find the height of the punchbowl

Substitute the point P(18, y) in the equation of the parabola

$$(18)^2 = 20(y - 19)$$
$$324 = 20(y - 19)$$
$$\frac{324}{20} + 19 = y$$
$$y = 35.2 \text{ cm}$$

Answer: The height of the punchbowl is **35.2** cm.

18 Example of an appropriate solution

Circle

Centre: (13, 10) radius: 4 cm

End points of the diameter

(9, 10) and (17, 10)

Hyperbola

The vertices are (9, 10) and (17, 10) and therefore $a = 4$.

Half of the total length is 7 cm. So the foci are (6, 10) and (20, 10) and therefore $c = 7$.

Equation of hyperbola

$$c^2 = a^2 + b^2$$
$$7^2 = 4^2 + b^2$$
$$33 = b^2$$

$$\therefore \quad \frac{(x-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$

To find the height let $x = 6$ and find the y coordinate

$$\frac{(6-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$

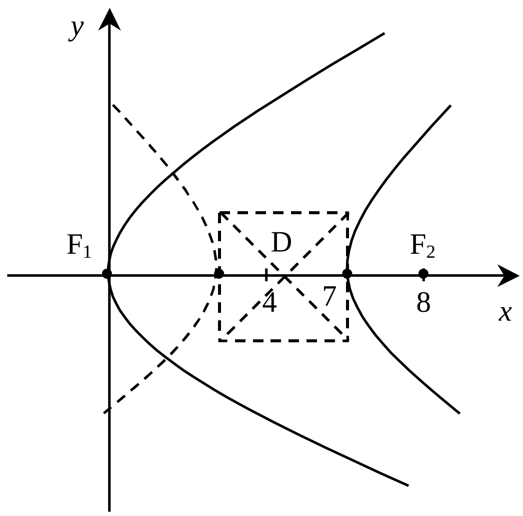
$$\begin{aligned} 33(-7)^2 - 16(y-10)^2 &= 16(33) \\ 1617 - 528 &= 16(y-10)^2 \\ 1089 &= 16(y-10)^2 \\ 68.0625 &= (y-10)^2 \\ \pm 8.25 &= (y-10) \end{aligned}$$

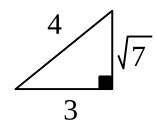
So $\begin{aligned} y &= 10 + 8.25 \\ &= 18.25 \end{aligned}$ and $\begin{aligned} y &= 10 - 8.25 \\ &= 1.75 \end{aligned}$

Answer: The height of the frame is **16.5** cm.

Note: Students who use an appropriate method in order to determine a correct equation of the hyperbola have shown they have a partial understanding of the problem.

19 Example of an appropriate solution





$y^2 = 32x$

$4c = 32$

$c = 8$

Coordinates of F_2 are (8, 0).

Coordinates of centre D of the hyperbola are (4, 0)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$(h,\ k) = (4,0)$$

$$\begin{aligned} a &= 7 - 4 \\ &= 3 \end{aligned}$$

$$\begin{aligned} c &= 8 - 4 \\ &= 4 \end{aligned}$$

$$\begin{aligned} b^2 &= 4^2 - 3^2 \\ b^2 &= 16 - 9 \\ b^2 &= 7 \\ b &= \sqrt{7} \end{aligned}$$

$$\frac{(x-4)^2}{9} - \frac{y^2}{7} = 1$$

Answer: The equation of the hyperbola is $\mathbf{\frac{(x-4)^2}{9} - \frac{y^2}{7} = 1}$.

Note: Students who use an appropriate method in order to determine **the coordinates (4, 0) of centre D of the hyperbola** have shown they have a partial understanding of the problem.

20 Example of an appropriate solution

To convert from general form to standard form:

$$x^2 + y^2 - 30x - 50y + 825 = 0$$
$$x^2 - 30x + y^2 - 50y = -825$$
$$(x-15)^2 - 225 + (y-25)^2 - 625 = -825$$
$$(x-15)^2 + (y-25)^2 = 25$$

- Formula for large circle C1: $(x - 15)^2 + (y - 25)^2 = 25$.

  Therefore the center is at (15, 25) and its radius is 5 m.

- Vertical distance from center of C1 to center of C2 is 25 m – 9 m = 16 m.

  Therefore the coordinates for the center of C2 are (15, 16) and its radius is 4 m.

- Vertex for the lower parabola is center of C2 – Radius of C2.

  Therefore coordinates for the vertex (h, k) of the lower parabola are **(15, 12)**

- Since center of C2 is also point on the directrix for lower parabola, then the value of parameter "c" ~~value~~ is vertical distance from vertex of lower parabola to center of C2.

  Therefore **"c" = -4**

Substituting in $(x-h)^2 = 4c(y-k)$ we get:

$$(x-15)^2 = -16(y-12)$$

- The y-coordinate of P is 0; the x-coordinate must be found.

  Therefore, substitute y = 0 in the equation above:

$$\begin{aligned}
(x-15)^2 &= -16\,(0-12) \\
(x-15)^2 &= 192 \\
x-15 &= \pm\sqrt{192} \\
x &= 15 \pm\sqrt{192} \\
x &\cong 1.14 \text{ or } x \cong 28.86
\end{aligned}$$

Answer: The distance from point 0 to point P is **1.14** m.

Note: Students who use an appropriate method to determine the vertex of the parabola, (15, 12) have shown they have a partial understanding of the problem.

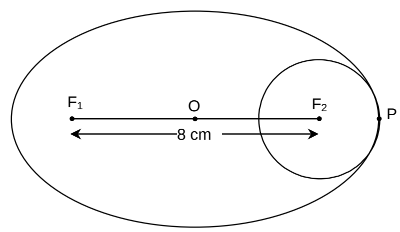
Do not penalize students who did not round or rounded incompletely.

21 Example of an appropriate solution

In the circle whose equation is $(x - 5)^2 + (y - 6)^2 = 4$,

the coordinates of F_2, the centre of the circle, are (5, 6) and the radius is 2.

The equation of the ellipse is in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



Where:

(h,k) is centre O

$a = \text{m}\,\overline{\text{OP}}$

$b = \sqrt{a^2 - c^2}$ where

$c = \text{m}\,\overline{\text{OF}_2}$

To find centre O, the values of parameters h and k, consider the coordinates of point F_2 (5,6) and the distance between the foci (8 cm).

Since $\text{m}\,\overline{\text{F}_1\text{F}_2}$ = 8 cm, $\text{m}\,\overline{\text{OF}_2}$ = 4 cm

The coordinates of point O are

(5 – 4, 6) = (1, 6)

Therefore parameter h = 1 and parameter k = 6

To find the $m\,\overline{OP}$, the value of parameter *a*, consider the radius of the circle (2 cm) and the coordinates of points O (1,6) and P.

The coordinates of point P are

(5 + 2, 6) = (7, 6)

$m\,\overline{OP}$ = d[(1,6),(7,6)]

$m\,\overline{OP}$ = 6 cm

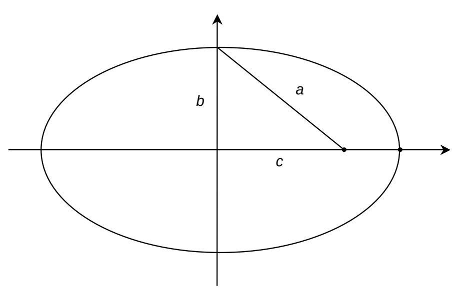
Therefore, the value of parameter *a* is 6 and *a*^{2} is 36.

To find the value of parameter *b*, consider the value of parameter *c* (4) and the relation $b = \sqrt{a^2 - c^2}$

$\boldsymbol{b = \sqrt{a^2 - c^2}}$

$\boldsymbol{b^2 = a^2 - c^2}$

$\boldsymbol{b^2 = 6^2 - 4^2}$



$b^2 = 36 - 16$
$b^2 = 20$
$b = \sqrt{20}$

Therefore, the value of parameter *b* is $\sqrt{20}$ and *b*^{2} is 20.

Substitute the values of the parameters into the equation:

$$\frac{(x-1)^2}{36}+\frac{(y-6)^2}{20}=1$$

Answer: The equation of the ellipse is $\boldsymbol{\frac{(x-1)^2}{36}+\frac{(y-6)^2}{20}=1}$.

Note: Students who used an appropriate method to determine the coordinates of the centre of the ellipse (1, 6) have shown they have a partial understanding of the problem.