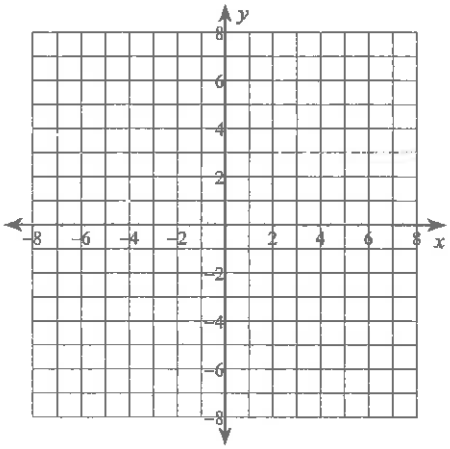
MATH 5 SN Name_____

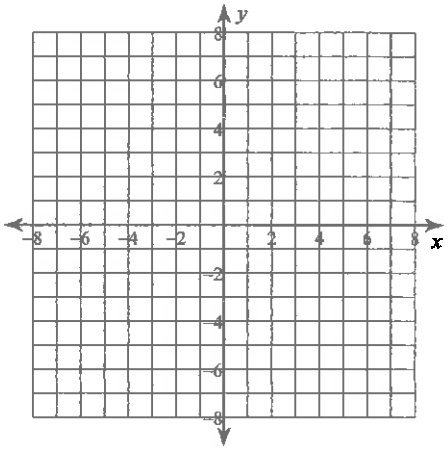
PRETEST - HYPERBOLAS and PARABOLAS

Identify the vertices, foci, and asymptotes of each. Then sketch the graph.

1) $\dfrac{(x-2)^2}{9} - (y+2)^2 = 1$

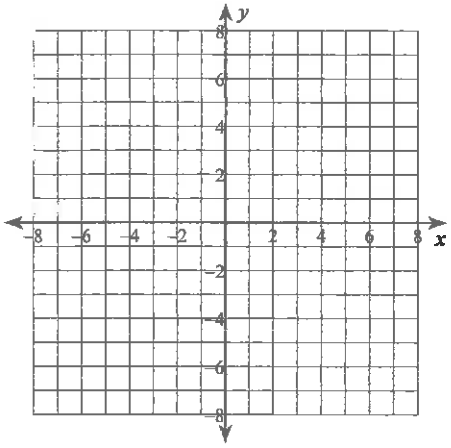


2) $-9x^2 + 25y^2 - 100y - 125 = 0$

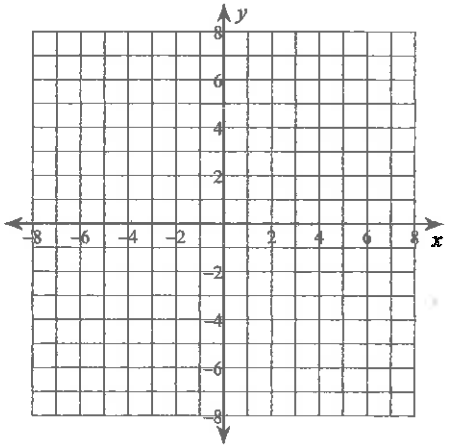


Identify the vertex and axis of symmetry of each. Then sketch the graph.

3) $-(y-4)=(x+3)^2$

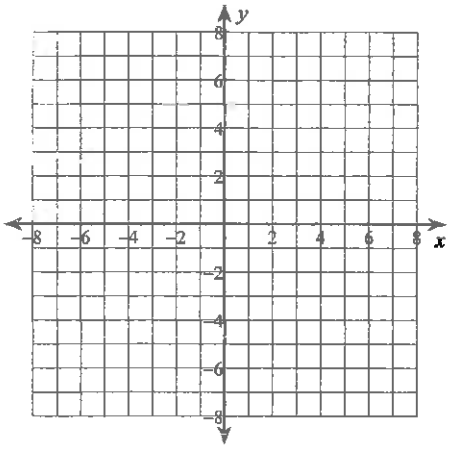


4) $2(y-3)=(x-4)^2$

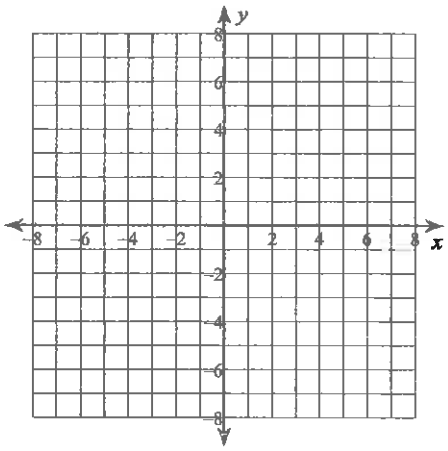


Identify the vertex, focus, axis of symmetry, and directrix of each. Then sketch the graph.

5) $7y^2 + x + 56y + 110 = 0$

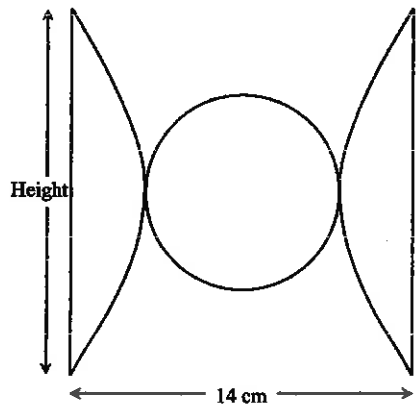


6) $-3y^2 + 4x + 6y + 9 = 0$



7

A modern picture frame is in the shape of a circle between the two branches of a hyperbola, as shown in the diagram below.



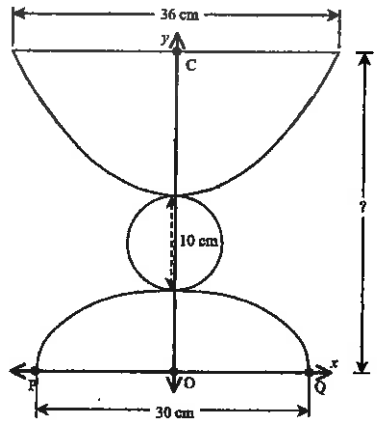
The equation of the circle is $(x - 13)^2 + (y - 10)^2 = 16$. The centre of the circle and the centre of the hyperbola coincide. The vertices of the hyperbola are the endpoints of the horizontal diameter of the circle and the vertical edges of the picture frame pass through the foci of the hyperbola. The total length of the frame is 14 cm.

What is the height of the frame?

8

The cross-sectional view of a punchbowl is shown below. The view of the punchbowl's base is in the shape of a semi-ellipse, whose major axis, PQ, measures 30 cm. The foci of this semi-ellipse are 12 cm from its centre.

Directly above the semi-ellipse lies a circle whose diameter is 10 cm. On top of the circle lies a parabola whose vertex touches the circle and whose directrix passes through the centre of the circle. The width of the punchbowl at the top is 36 cm.



What is the height, CO, of the punchbowl?

MATH 5 SN

Key

Name_____

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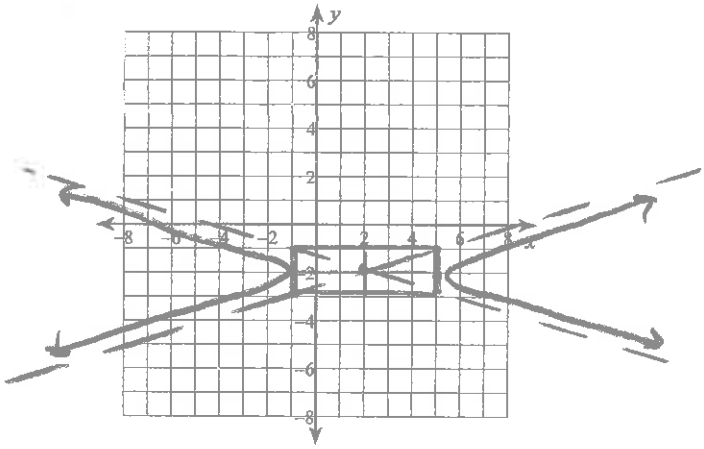
Identify the vertices, foci, and asymptotes of each. Then sketch the graph.

1) $\frac{(x-2)^2}{9} - (y+2)^2 = 1$



$(h,k) = (2,-2)$

$a = 3 \qquad b = 1$

$c^2 = a^2 + b^2$

$c^2 = 9 + 1 \qquad c = \sqrt{10}, or -\sqrt{10}$

Vertices $(5,-2)$ and $(-1,-2)$

foci $(2+\sqrt{10}, -2)$ and $(2-\sqrt{10}, -2)$

asymptotes $\frac{1}{3} = \frac{y+2}{x-2}$

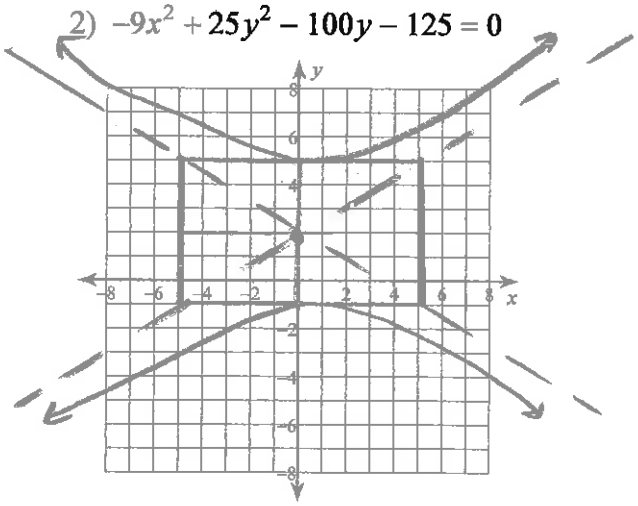
$\boxed{x-2 = 3(y+2)}$

$-\frac{1}{3} = \frac{y+2}{x-2}$

$\boxed{-(x-2) = 3(y+2)}$

point slope form: $\frac{b}{a} = \frac{y-k}{x-h}$

2) $-9x^2 + 25y^2 - 100y - 125 = 0$



$-9x^2 + 25y^2 - 100y \_\_\_\_ = 125$

$-9(x^2) + 25(y^2 - 4y + 4) = 125 + 100$

$\frac{-9x^2}{-225} + \frac{25(y-2)^2}{-225} = \frac{225}{-225}$

$\frac{x^2}{25} - \frac{(y-2)^2}{9} = -1$

$(h,k) = (0,2)$

$a = 5 \qquad b = 3$

$c^2 = a^2 + b^2 = 5^2 + 3^2$

$c = \sqrt{34}, or -\sqrt{34}$

Vertices $(0,5)$ and $(0,-1)$

foci $(0, 2+\sqrt{34})$ and $(0, 2-\sqrt{34})$

asymptotes $\frac{3}{5} = \frac{y-2}{x}$

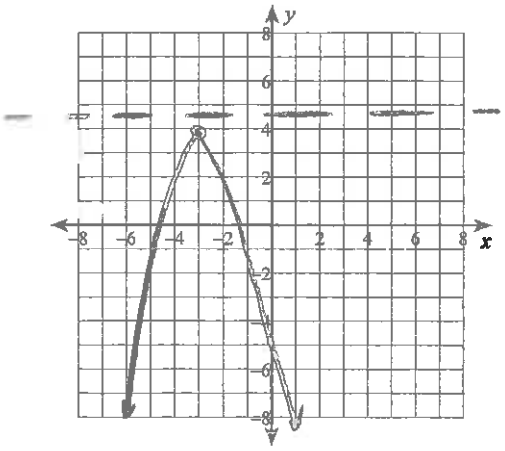
$\boxed{3x = 5(y-2)}$

$-\frac{3}{5} = \frac{y-2}{x}$

$\boxed{-3x = 5(y-2)}$

Identify the vertex and axis of symmetry of each. Then sketch the graph.

3) $-(y-4)=(x+3)^2$



$(x+3)^2 = -(y-4)$ ✓

$(x+3)^2 = -4\left(\frac{1}{4}\right)(y-4)$ ✓

$h=-3 \quad k=4 \quad c=\frac{1}{4}$ ✓

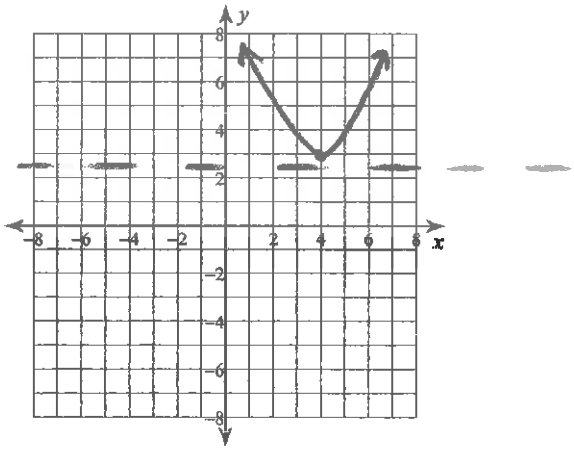
vertex $(-3,4)$

focus $\left(-3, \frac{15}{4}\right)$ ✓

directrix $y=\frac{17}{4}$ ✓

axis of symmetry $x=-3$

4) $2(y-3)=(x-4)^2$



$(x-4)^2 = 2(y-3)$ ✓

$(x-4)^2 = 4\left(\frac{1}{2}\right)(y-3)$ ✓

$h=4 \quad k=3 \quad c=\frac{1}{2}$ ✓

vertex $=(4,3)$ ✓

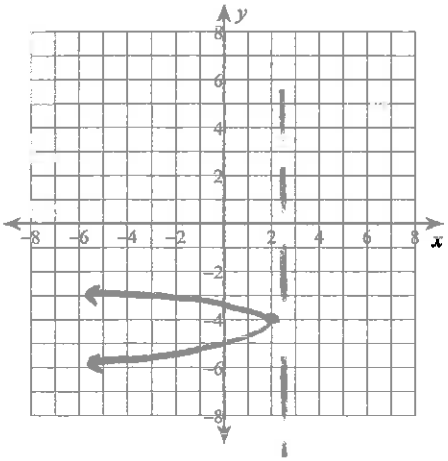
focus $\left(4, \frac{7}{2}\right)$ ✓

directrix $y=\frac{5}{2}$ ✓

axis of symmetry $x=4$ ✓

Identify the vertex, focus, axis of symmetry, and directrix of each. Then sketch the graph.

5) $7y^2 + x + 56y + 110 = 0$



$7y^2 + 56y = -x - 110$

$7(y^2 + 8y \quad) = -x - 110$

$7(y^2 + 8y + 16) = -x - 110 + 112$

$7(y+4)^2 = -x + 2$

$7(y+4)^2 = -(x-2)$

$7(y+4)^2 = -\frac{1}{7}(x-2)$

$7(y+4)^2 = -4\left(\frac{1}{28}\right)(x-2)$ ✓

$h = 2 \quad k = -4 \quad c = \frac{1}{28}$

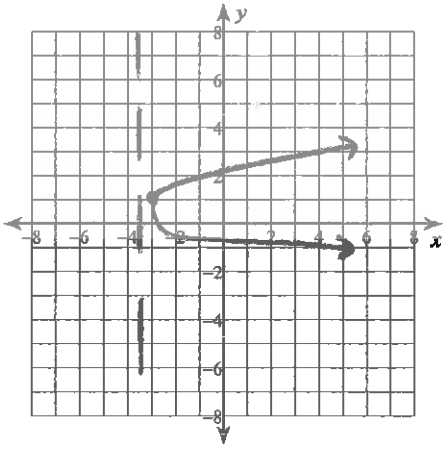
vertex $(2, -4)$ ✓

focus $\left(\frac{57}{28}, -4\right)$ ✓

directrix $x = \frac{57}{28}$ ✓

axis of symmetry $y = -4$ ✓

6) $-3y^2 + 4x + 6y + 9 = 0$



$-3y^2 + 6y = -4x - 9$

$-3(y^2 - 2y \quad) = -4x - 9$

$-3(y^2 - 2y + 1) = -4x - 9 - 3$

$-3(y-1)^2 = -4x - 12$

$-3(y-1)^2 = -4(x+3)$

$(y-1)^2 = \frac{4}{3}(x+3)$

$(y-1)^2 = 4\left(\frac{1}{3}\right)(x+3)$ ✓

$h = -3 \quad y = 1 \quad c = \frac{1}{3}$ ✓

vertex $(-3, 1)$ ✓

focus $\left(-\frac{8}{3}, 1\right)$ ✓

directrix $x = -\frac{10}{3}$ ✓

axis of symmetry $y = 1$ ✓

(1)

$\boxed{\cancel{46}}$ **Circle**
Centre: (13, 10) radius: 4 cm

End points of the diameter
(9, 10) and (17, 10)

Hyperbola
The vertices are (9, 10) and (17, 10) and therefore $a = 4$.

Half of the total length is 7 cm. So the foci are (6, 10) and (20, 10) and therefore $c = 7$.

Equation of hyperbola

$$c^2 = a^2 + b^2$$
$$7^2 = 4^2 + b^2$$
$$33 = b^2$$

$$\therefore \quad \frac{(x-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$

To find the height let $x = 6$ and find the y coordinate

$$\frac{(6-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$
$$33(-7)^2 - 16(y-10)^2 = 16(33)$$
$$1617 - 528 = 16(y-10)^2$$
$$1089 = 16(y-10)^2$$
$$68.0625 = (y-10)^2$$
$$\pm 8.25 = (y-10)$$

So $y = 10 + 8.25$ $= 18.25$ and $y = 10 - 8.25$ $= 1.75$

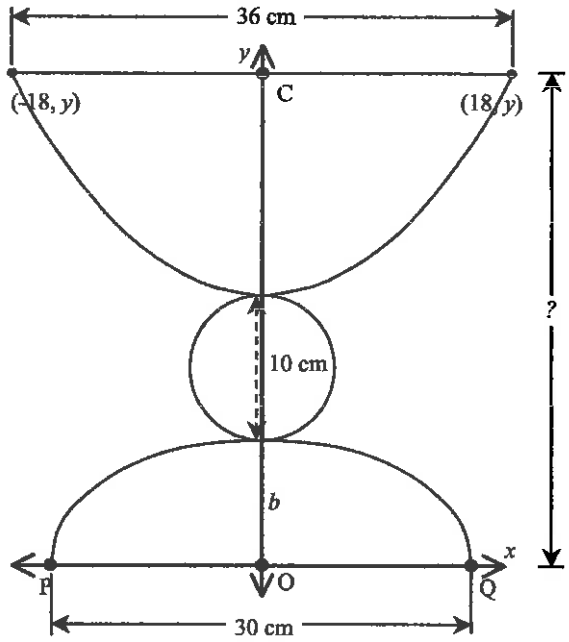
Answer: The height of the frame is **16.5** cm.

8

24

Calculate the length of the Semi-Minor Axis (b)

$c = 12$ (Given)



Length of Major Axis = 2 a

$$2a = 30$$
$$a = 15$$

$$a^2 = b^2 + c^2$$
$$b = \sqrt{a^2 - c^2}$$
$$b = \sqrt{(15)^2 - (12)^2}$$
$$b = \sqrt{225 - 144}$$
$$b = \sqrt{81}$$
$$b = 9$$

Find the equation of the parabola

Equation of Parabola: $(x - h)^2 = 4c(y - k)$ with Vertex V(h, k)

Note: Do not penalize students for having determined the rule in the form $y = a(x - h)^2$.

$h = 0$; $k = b + 10$
$k = (9) + 10$
$k = 19$

⇨ Equation of Parabola $(x - 0)^2 = 4c(y - 19)$
$x^2 = 4c(y - 19)$

With point O as the origin, the coordinates of the centre of the circle are (0, 14). Since the vertex of the parabola is V(0, 19), the vertical distance between these two points must be the value of c.

Hence $c = 19 - 14$
$c = 5$

Therefore the equation of the parabola is

$$x^2 = 4(5)(y - 19)$$
$$x^2 = 20(y - 19)$$

Find the height of the punchbowl

Substitute the point P(18, y) in the equation of the parabola

$$(18)^2 = 20(y - 19)$$
$$324 = 20(y - 19)$$
$$\frac{324}{20} + 19 = y$$
$$y = 35.2 \text{ cm}$$

Answer: The height of the punchbowl is **35.2 cm.**