Mathematics x-sheet 11

ANALYTICAL GEOMETRY

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KEY CONCEPTS:

- What is the Cartesian plane?
- Points on the Cartesian plane
- Distance between points
- Straight lines

  Gradient

  Equation of a straight line

  Parallel and perpendicular lines

  Mid-point on a line

TERMINOLOGY

Gradient of a line: the ratio of vertical change to horizontal change

A point on the Cartesian plane: A point is the simplest geometric object. We describe the position (location) of a point using an ordered pair of numbers written as (x; y)

Distance: A measure of the length between two points.

A straight line: A set of points with a constant gradient between any two of the points

X-PLANATION

What is the Cartesian plane?

In the 17th century by René Descartes revolutionised mathematics by providing a way to link between Euclidean geometry and algebra. He did this by using two number lines, called axes. The horizontal axis is called the x-axis and the vertical one the y-axis. These number lines provide a framework to describe an imaginary flat surface which we call this surface the Cartesian plane.

In Analytical Geometry, we use the Cartesian plane to explore the properties of lines and shapes.

Points on the Cartesian plane

The position of any point on the Cartesian plane can be described with reference to the horizontal axis (x- axis) and the vertical axis (y-axis). We say that each point has a set of co-ordinates. The value corresponding to its position relative to the (x-axis) is written first, followed by the value corresponding to its position relative to the (y-axis).

So a point A on the Cartesian plane has a pair of co-ordinates which we write as A
$(x_A; y_A)$

Distance between points

If AB is the line segment joining the points A(xA ; yA) and B(xB ; yB) , then the following formulas apply to line segment AB.

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

or

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

Straight lines

Straight lines on the Cartesian plane can be described algebraically as linear functions. In the Cartesian plane, we work with line segments between two points.

Gradient of a line segment

The gradient of a straight line or line segment refers to the steepness or slope of the line. The gradient of the line segment AB can be calculated using the formula:

Gradient of AB = $AB = \frac{y_B - y_A}{x_B - x_A}$

Equation of a straight line or line segment

The standard form of the straight line equation is y = mx + c.

The equation of a straight line can also be written as

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel and perpendicular lines

Parallel lines have equal gradients. If AB||CD then $m_{AB} = m_{CD}$

The product of the gradients of two perpendicular lines is -1. If AB $\perp$ CD, then $m_{AB} \times m_{CD} = -1$

Mid-point of a segment line

$M\left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right)$ where M is the midpoint of AB.

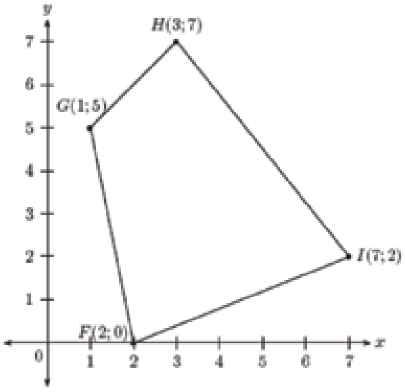
X-AMPLE QUESTIONS:

Question 1:

1. Represent the following figures in the Cartesian plane:
   (a) Triangle DEF with D(1; 2), E(3; 2) and F(2; 4)
   (b) Quadrilateral GHIJ with G(2;-1), H(0; 2), I(-2;-2) and J(1;-3)

Question 2

In the diagram below, the vertices of the quadrilateral are F(2; 0), G(1; 5), H(3; 7) and I(7; 2).



Calculate the lengths of the sides of FG.

Question 3

If the length of the line segment joining the points A(1; 2) and B(x ; 6) is equal to 5 units, determine the two possible values of x.

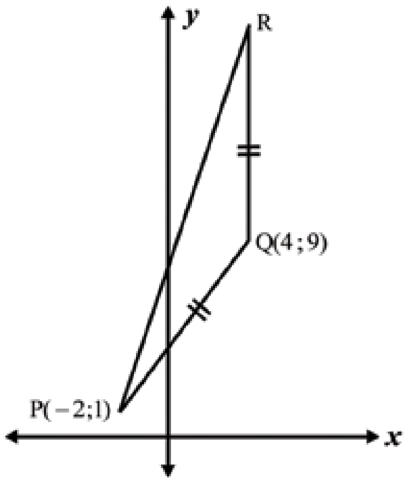
Question 4

PQRS is a quadrilateral with points P(0;-3), Q(-4; 5), R(2; 7) and S(3;-2) in the Cartesian plane.

(a) Find the gradient of PS.
(b) Calculate the equation of the line segment QR
(c) Show that X(-1;6) is a point on the line QR

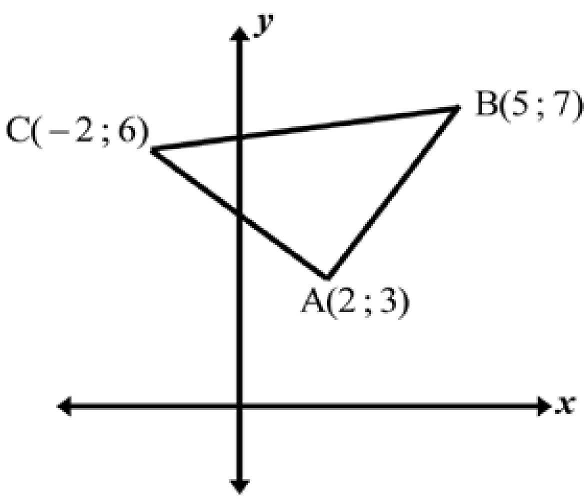
Question 5

In the diagram, PQR is an isosceles triangle with PQ = QR. The side QR is parallel to the y-axis.
(a) Calculate the length of PQ.
(b) Determine the coordinates of R.



Question 6

The coordinates of the vertices of ΔABC are A(2 ; 3) , B(5; 7) and C(-2 ; 6) .



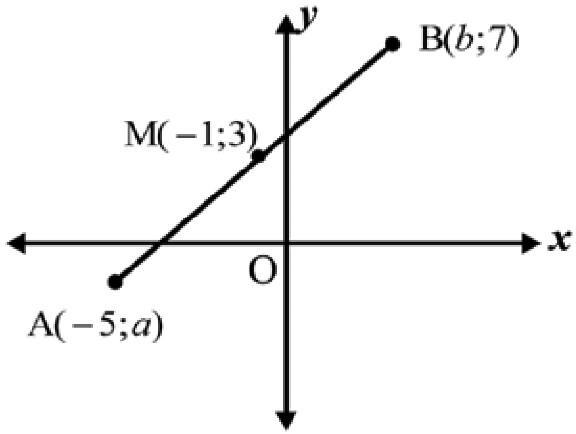
Show that Δ ABC is a right angled triangle.

Question 7

A(-2; 3) and B(2; 6) are points in the Cartesian plane. C(a; b) is the mid-point of AB. Find the values of a and b.

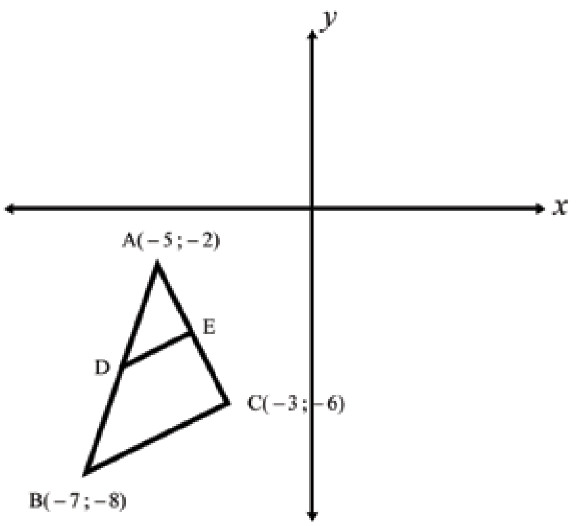
Question 8

In the diagram, M is the midpoint of AB. Calculate the value of a and b.



Question 9

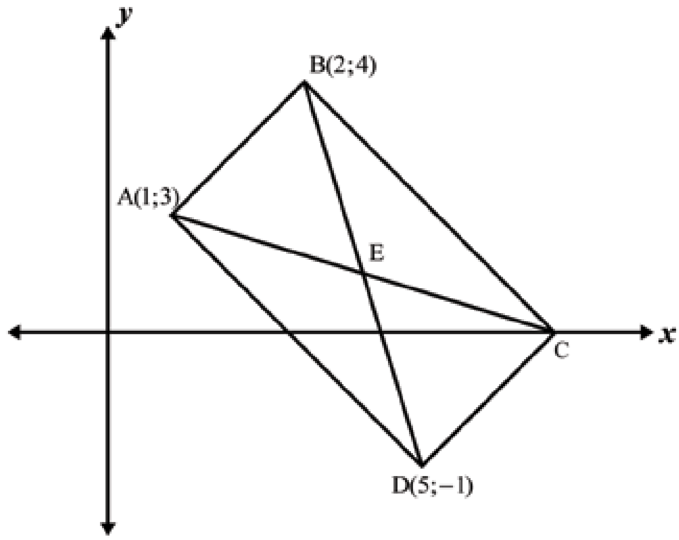
In the diagram ΔABC is drawn.
Answer the questions which follow.



(a) Calculate the coordinates of D, the midpoint of AB.
(b) Calculate the coordinates of E, the midpoint of AC.
(c) Without using a calculator, show that BC = 2DE.
(d) Calculate the gradient of DE and BC.
(e) What type of quadrilateral is DBCE? Give a reason for your answer.

Question 10

ABCD is a quadrilateral with vertices A(1;3), B(2;4), C and D(5;-1) . The diagonals bisect each other at E.



(a) Determine the coordinates of E.
(b) Determine the coordinates of C.
(c) Determine the gradient of AB and DC. What can you conclude?
(d) Determine the gradient of AD and BC. What can you conclude?
(e) Show that the interior angles of quadrilateral ABCD are 90^{o}
(f) Show that ABCD is a rectangle.