Trigonometric Identities
Worksheet

I. Prove each identity

1) $\tan x \cos x = \sin x$

2) $\cot x \sec x = \csc x$

3) $\sin x \cot x = \cos x$

4) $\tan x \csc x = \sec x$

5) $\sin x = \dfrac{\tan x}{\sec x}$

6) $\dfrac{\cot x}{\csc x} = \cos x$

II. Prove each identity

1) $\csc x(1 + \sin x) = 1 + \csc x$

2) $\sin x(1 + \csc x) = 1 + \sin x$

3) $\cos cx(\sec x - 1) = 1 - \cos x$

4) $\sin x \sec x \cot x = 1$

5) $\dfrac{1 - \tan x}{1 - \cot x} = -\tan x$

6) $\cot x = \dfrac{1 + \cot x}{1 + \tan x}$

III. Prove each identity

1) $\sin x \tan x + \sec x = \dfrac{\sin^2 x + 1}{\cos x}$

2) $\dfrac{1 + \cos x}{1 - \cos x} = \dfrac{1 + \sec x}{\sec x - 1}$

3) $\dfrac{1 + \sin x}{1 - \sin x} = \dfrac{\csc x + 1}{\csc x - 1}$

4) $\cancel{\dfrac{1 + \tan x}{1 - \cot x} = \dfrac{1 - \tan x}{\cot x - 1}}$

5) $\dfrac{1 + \sin x}{1 + \csc x} = \sin x$

6) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$

Mathematics 536
Trigonometric Identities
Sheet I

Verify each of the following:

1. $\sec\vartheta - \sec\vartheta\sin^2\vartheta = \cos\vartheta$
2. $\sin\vartheta\sec\vartheta\cot\vartheta = 1$
3. $\sin^2\vartheta(1+\cot^2\vartheta) = 1$
4. $\sin^2\vartheta\sec^2\vartheta - \sec^2\vartheta = -1$
5. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$
6. $\tan^2 x\cos^2 x + \cot^2 x\sin^2 x = 1$
7. $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$
8. $\sin x\sec x = \tan x$
9. $(1-\sin^2 t)(1+\tan^2 t) = 1$
10. $(1-\cos A)(1+\sec A)\cot A = \sin A$
11. $\csc^2\theta(1-\cos^2\theta) = 1$
12. $\dfrac{\sin\vartheta}{\csc\vartheta} + \dfrac{\cos\vartheta}{\sec\vartheta} = 1$
13. $\dfrac{1-2\cos^2 x}{\sin x\cos x} = \tan x - \cot x$
14. $\tan^2 x\csc^2 x\cot^2 x\sin^2 x = 1$
15. $\sin A\cos A(\tan A + \cot A) = 1$
16. $1 - \dfrac{\cos^2 x}{1+\sin x} = \sin x$
17. $\dfrac{1}{\sec x + \tan x} = \sec x - \tan x$
18. $2\csc x = \dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$

9. $\dfrac{1}{1+\sin u}+\dfrac{1}{1-\sin u}\equiv 2\sec^2 u$

10. $\dfrac{1+\sec u}{\sec u}\equiv\dfrac{\sin^2 u}{1-\cos u}$

11. $\dfrac{1-\cos u}{\sin u}\equiv\dfrac{\sin u}{1+\cos u}$

12. $\dfrac{\sec u-1}{\sec u+1}\equiv\dfrac{1-\cos u}{1+\cos u}$

13. $\dfrac{\cos^2 u}{1-\sin u}\equiv\dfrac{\cos u}{\sec u-\tan u}$

14. $\cos u\cot u\equiv\dfrac{1}{\sin u}-\sin u$

15. $\tan A+\dfrac{1}{\tan A}\equiv\dfrac{\sec A}{\sin A}$

16. $\dfrac{\sin A+\tan A}{1+\sec A}\equiv\sin A$

17. $1+\dfrac{1}{\cos A}\equiv\dfrac{\tan^2 A}{\sec A-1}$

18. $\sin x+\cos x\equiv\dfrac{1+\tan x}{\sec x}$

19. $\tan u\equiv\dfrac{1+\sin u-\cos^2 u}{\cos u\,(1+\sin u)}$

20. $\cot u\equiv\dfrac{2+\csc u}{\sec u}-2\cos u$

21. $\dfrac{\tan A\sin A}{\tan A+\sin A}\equiv\dfrac{\tan A-\sin A}{\tan A\sin A}$

22. $\dfrac{\sec^4 u+\tan^4 u}{\sec^2 u\tan^2 u}\equiv\dfrac{\cos^4 u}{\sin^2 u}+2$

23. $\dfrac{\cot A+\csc A}{\sin A-\cot A-\csc A}\equiv-\sec A$

24. $\cot^2 A\sec^2 A\equiv 1+\cot^2 A$

25. $\dfrac{\cot A+\cot B}{\tan A+\tan B}+\dfrac{1-\cot A\cot B}{1-\tan A\tan B}\equiv 0$

26. $\dfrac{\sin A\cos A}{1-\cos A}\equiv\dfrac{\sin^2 A}{\tan A-\sin A}$

27. $\sin u\sec u\cot u\equiv 1$

28. $\cot^2 u-\cos^2 u\equiv\cos^2 u\cot^2 u$

29. $\dfrac{\cos u}{1+\sin u}+\dfrac{1+\sin u}{\cos u}\equiv 2\sec u$

30. $\dfrac{1-\tan^2 u}{1-\cot^2 u}\equiv 1-\sec^2 u$

31. $(\sec^2 u)(1-\cos^2 u)\equiv\tan^2 u$

32. $\dfrac{\sin^4 u-\cos^4 u}{1-\cot^4 u}\equiv\sin^4 u$

33. $\dfrac{1-2\sin u-3\sin^2 u}{\cos^2 u}\equiv\dfrac{1-3\sin u}{1-\sin u}$