

Trigonometric Identities

1. Simplify the trigonometric expression.

a) $\sin x \cot x$

c) $\frac{\sec^2 x - 1}{\sec^2 x}$

e) $\frac{\sec \theta - 1}{1 - \cos \theta}$

g) $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta$

b) $\sec x \cot x \sin x$

d) $\cot^2 x - \csc^2 x$

f) $\frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta}$

h) $\frac{\sin \theta}{1 - \tan \theta} + \frac{\cos \theta}{1 - \cot \theta}$

2. Verify the identity.

a) $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$

c) $\frac{1}{\cos x} - \frac{1}{\sec x} = \sec x - \cos x$

e) $\frac{1}{\tan x} + \tan x = \frac{\sec^2 x}{\tan x}$

g) $\sin^2 \phi - \sin^4 \phi = \cos^2 \phi - \cos^4 \phi$

i) $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$

k) $1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta$

m) $\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} = 4 \csc \theta \cot \theta$

o) $\frac{\tan \beta - \cot \beta}{\tan \beta + \cot \beta} = \sin^2 \beta - \cos^2 \beta$

q) $\frac{\cos \alpha - \cos \beta}{\sin \alpha + \sin \beta} + \frac{\sin \alpha - \sin \beta}{\cos \alpha + \cos \beta} = 0$

s) $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = 1 - \sin \theta \cos \theta$

u) $\frac{\sin^2 \psi - \cos^2 \psi}{1 - \cot^2 \psi} = \sin^2 \psi$

w) $\tan x + \frac{\cos x}{1 + \sin x} = \sec x$

b) $\sin \theta + \cos \theta \cot \theta = \csc \theta$

d) $\frac{\tan^2 \theta}{\sec \theta} = \sec \theta - \cos \theta$

f) $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

h) $\frac{\sin \theta \tan \theta}{1 - \cos \theta} - 1 = \sec \theta$

j) $\frac{\sec \theta - 1}{\sec \theta + 1} = \frac{1 - \cos \theta}{1 + \cos \theta}$

l) $\frac{\csc \theta - \sec \theta}{\csc \theta \sec \theta} = \cos \theta - \sin \theta$

n) $\sec \theta \csc \theta (\tan \theta + \cot \theta) = \sec^2 \theta + \csc^2 \theta$

p) $\frac{1}{\cos \phi + 1} + \frac{1}{\sec \phi + 1} = 1$

r) $\frac{1 - \cos \theta}{1 + \cos \theta} = (\csc \theta - \cot \theta)^2$

t) $\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$

v) $(\tan \theta + \cot \theta)^2 = \sec^2 \theta + \csc^2 \theta$

x) $\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$

$$y) \frac{(2\sin^2 \theta - 1)^2}{\sin^4 \theta - \cos^4 \theta} = 1 - 2\cos^2 \theta \quad z) \sec^4 \theta - \tan^4 \theta = 1 + 2\tan^2 \theta$$

$$aa) \frac{\cot \theta}{\csc \theta - 1} = \frac{\csc \theta + 1}{\cot \theta}$$

$$bb) (\tan \alpha + \tan \beta)(1 - \cot \alpha \cot \beta) + (\cot \alpha + \cot \beta)(1 - \tan \alpha \tan \beta) = 0$$

$$cc) (\sin x - \tan x)(\cos x - \cot x) = (\cos x - 1)(\sin x - 1)$$

$$dd) \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} = \sec \theta + \tan \theta$$

ANSWERS

1. a) $\sin x \cot x = \sin x \frac{\cos x}{\sin x} = \cos x$
- b) $\sec x \cot x \sin x = \frac{1}{\cos x} \frac{\cos x}{\sin x} \sin x = 1$
- c) $\frac{\sec^2 x - 1}{\sec^2 x} = \frac{\frac{1}{\cos^2 x} - 1}{\frac{1}{\cos^2 x}} = \frac{\frac{1-\cos^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{1-\cos^2 x}{\cos^2 x} \frac{\cos^2 x}{1} = 1 - \cos^2 x = \sin^2 x$
- d) $\cot^2 x - \csc^2 x = \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} = \frac{\cos^2 x - 1}{\sin^2 x} = \frac{-\sin^2 x}{\sin^2 x} = -1$
- e) $\frac{\sec \theta - 1}{1 - \cos \theta} = \frac{\frac{1}{\cos \theta} - 1}{1 - \cos \theta} = \frac{\frac{1 - \cos \theta}{\cos \theta}}{1 - \cos \theta} = \frac{1 - \cos \theta}{\cos \theta} = \sec \theta$
- f) $\frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta} = \cos^2 \theta + \sin^2 \theta = 1$
- g) $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \csc^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} \cos^2 \theta + \frac{\cos^2 \theta}{\sin^2 \theta} \sin^2 \theta = \sin^2 \theta + \cos^2 \theta = 1$
- h) $\frac{\sin \theta}{1 - \tan \theta} + \frac{\cos \theta}{1 - \cot \theta} = \frac{\cos \theta}{1 - \sin \theta} + \frac{\sin \theta}{1 - \cos \theta} = \frac{\sin \theta}{\cos \theta - \sin \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = \frac{\sin \theta \cos \theta}{\cos \theta - \sin \theta} - \frac{\sin \theta \cos \theta}{\cos \theta - \sin \theta} = 0$
2. a) $LS = (1 + \sin \theta)(1 - \sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta = RS$
- b) $LS = \sin \theta + \cos \theta \cot \theta = \sin \theta + \cos \theta \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} = \frac{1}{\sin \theta} = \csc \theta = RS$
- c) $LS = \frac{1}{\cos x} - \frac{1}{\sec x} = \sec x - \frac{1}{\frac{1}{\cos x}} = \sec x - \cos x = RS$
- d) $LS = \frac{\tan^2 \theta}{\sec \theta} = \frac{\sec^2 \theta - 1}{\sec \theta} = \frac{\sec^2 \theta}{\sec \theta} - \frac{1}{\sec \theta} = \sec \theta - \cos \theta = RS$
- e) $LS = \frac{1}{\tan x} + \tan x = \frac{1 + \tan^2 x}{\tan x} = \frac{\sec^2 x}{\tan x} = RS$
- f) $LS = (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = \sec^2 \theta - \tan^2 \theta = 1 + \tan^2 \theta - \tan^2 \theta = 1 = RS$
- g) $LS = \sin^2 \phi - \sin^4 \phi = \sin^2 \phi(1 - \sin^2 \phi) = (1 - \cos^2 \phi)\cos^2 \phi = \cos^2 \phi - \cos^4 \phi = RS$
- h) $LS = \frac{\sin \theta \tan \theta}{1 - \cos \theta} - 1 = \frac{\sin \theta \frac{\sin \theta}{\cos \theta}}{1 - \cos \theta} - 1 = \frac{\sin^2 \theta}{\cos \theta - \cos^2 \theta} - 1 = \frac{\sin^2 \theta - \cos \theta + \cos^2 \theta}{\cos \theta - \cos^2 \theta} = \frac{1 - \cos \theta}{\cos \theta(1 - \cos \theta)} = \frac{1}{\cos \theta} = \sec \theta = RS$
- i) $LS = \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{(1 + \sin \theta)^2 + \cos^2 \theta}{\cos \theta(1 + \sin \theta)} = \frac{1 + 2\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta(1 + \sin \theta)} = \frac{2 + 2\sin \theta}{\cos \theta(1 + \sin \theta)} = \frac{2}{\cos \theta} = 2 \sec \theta = RS$
- j) $LS = \frac{\sec \theta - 1}{\sec \theta + 1} = \frac{\frac{1}{\cos \theta} - 1}{\frac{1 + \cos \theta}{\cos \theta}} = \frac{1 - \cos \theta}{\cos \theta} \cdot \frac{\cos \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{1 + \cos \theta} = RS$
- k) $LS = 1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \frac{1 + \cos \theta - \sin^2 \theta}{1 + \cos \theta} = \frac{\cos^2 \theta + \cos \theta}{1 + \cos \theta} = \frac{\cos \theta(1 + \cos \theta)}{1 + \cos \theta} = \cos \theta = RS$
- l) $LS = \frac{\csc \theta - \sec \theta}{\csc \theta \sec \theta} = \frac{\frac{1}{\sin \theta} - \frac{1}{\cos \theta}}{\frac{1}{\sin \theta} \frac{1}{\cos \theta}} = \frac{\cos \theta - \sin \theta}{\sin \theta \cos \theta} \cdot \frac{\sin \theta \cos \theta}{1} = \cos \theta - \sin \theta = RS$
- m) $LS = \frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} = \frac{(1 + \cos \theta)^2 - (1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 + 2\cos \theta + \cos^2 \theta - 1 + 2\cos \theta - \cos^2 \theta}{1 - \cos^2 \theta} = \frac{4\cos \theta}{\sin^2 \theta} = 4 \frac{\cos \theta}{\sin \theta} \frac{1}{\sin \theta}$
 $= 4 \csc \theta \cot \theta = RS$
- n) $LS = \sec \theta \csc \theta (\tan \theta + \cot \theta) = \frac{1}{\cos \theta} \frac{1}{\sin \theta} \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) = \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} = \sec^2 \theta + \csc^2 \theta = RS$
- o) $LS = \frac{\tan \beta - \cot \beta}{\tan \beta + \cot \beta} = \frac{\frac{\sin \beta}{\cos \beta} - \frac{\cos \beta}{\sin \beta}}{\frac{\sin \beta}{\cos \beta} + \frac{\cos \beta}{\sin \beta}} = \frac{\frac{\sin^2 \beta - \cos^2 \beta}{\cos \beta \sin \beta}}{\frac{\sin^2 \beta + \cos^2 \beta}{\cos \beta \sin \beta}} = \frac{\sin^2 \beta - \cos^2 \beta}{\cos \beta \sin \beta} \frac{\cos \beta \sin \beta}{1} = \sin^2 \beta - \cos^2 \beta = RS$
- p) $LS = \frac{1}{\cos \phi + 1} + \frac{1}{\sec \phi + 1} = \frac{1}{\cos \phi + 1} + \frac{1}{\frac{1}{\cos \phi} + 1} = \frac{1}{\cos \phi + 1} + \frac{1}{\frac{1 + \cos \phi}{\cos \phi}} = \frac{1}{\cos \phi + 1} + \frac{\cos \phi}{1 + \cos \phi} = \frac{1 + \cos \phi}{\cos \phi + 1} = 1 = RS$
- q) $LS = \frac{\cos \alpha - \cos \beta}{\sin \alpha + \sin \beta} + \frac{\sin \alpha - \sin \beta}{\cos \alpha + \cos \beta} = \frac{\cos^2 \alpha - \cos^2 \beta + \sin^2 \beta - \sin^2 \alpha}{(\sin \alpha + \sin \beta)(\cos \alpha + \cos \beta)} = \frac{1 - 1}{(\sin \alpha + \sin \beta)(\cos \alpha + \cos \beta)} = 0 = RS$
- r) $RS = (\csc \theta - \cot \theta)^2 = \csc^2 \theta - 2 \csc \theta \cot \theta + \cot^2 \theta = \frac{1}{\sin^2 \theta} - 2 \frac{1}{\sin \theta} \frac{\cos \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}$
 $= \frac{1 - 2\cos \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} = LS$

$$s) LS = \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = \frac{\sin \theta (1 - \cos^2 \theta) + \cos \theta (1 - \sin^2 \theta)}{\sin \theta + \cos \theta} = \frac{\sin \theta - \sin \theta \cos^2 \theta + \cos \theta - \cos \theta \sin^2 \theta}{\sin \theta + \cos \theta} = \frac{\sin \theta + \cos \theta - \sin \theta \cos \theta (\cos \theta + \sin \theta)}{\sin \theta + \cos \theta} \\ = \frac{(\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta)}{\sin \theta + \cos \theta} = 1 - \sin \theta \cos \theta = RS$$

$$t) LS = \frac{\tan x + \tan y}{\cot x + \cot y} = \frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}} = \frac{\tan x + \tan y}{\frac{\tan y + \tan x}{\tan x \tan y}} = \frac{\tan x + \tan y}{1} \cdot \frac{\tan x \tan y}{\tan y + \tan x} = \tan x \tan y = RS$$

$$u) LS = \frac{\sin^2 \psi - \cos^2 \psi}{1 - \cot^2 \psi} = \frac{\sin^2 \psi - \cos^2 \psi}{1 - \frac{\cos^2 \psi}{\sin^2 \psi}} = \frac{\sin^2 \psi - \cos^2 \psi}{\frac{\sin^2 \psi - \cos^2 \psi}{\sin^2 \psi}} = \frac{\sin^2 \psi - \cos^2 \psi}{1} \cdot \frac{\sin^2 \psi}{\sin^2 \psi - \cos^2 \psi} = \sin^2 \psi = RS$$

$$v) LS = (\tan \theta + \cot \theta)^2 = \tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta = \tan^2 \theta + 2 \tan \theta \frac{1}{\tan \theta} + \cot^2 \theta \\ = \tan^2 \theta + 2 + \cot^2 \theta = \tan^2 \theta + 1 + 1 + \cot^2 \theta = \sec^2 \theta + \csc^2 \theta = RS$$

$$w) LS = \tan x + \frac{\cos x}{1 + \sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{\sin x(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)} = \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} = \frac{\sin x + 1}{\cos x(1 + \sin x)} = \frac{1}{\cos x} = \sec x = RS$$

$$x) LS = \frac{\cos \theta}{1 - \sin \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta} = \frac{\cos \theta(1 + \sin \theta)}{1 - \sin^2 \theta} = \frac{\cos \theta(1 + \sin \theta)}{\cos^2 \theta} = \frac{1 + \sin \theta}{\cos \theta} = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = \sec \theta + \tan \theta = RS$$

$$y) LS = \frac{(2 \sin^2 \theta - 1)^2}{\sin^4 \theta - \cos^4 \theta} = \frac{(\sin^2 \theta + \sin^2 \theta - 1)^2}{\sin^2 \theta(1 - \cos^2 \theta) - \cos^2 \theta(1 - \sin^2 \theta)} = \frac{(\sin^2 \theta - \cos^2 \theta)^2}{\sin^2 \theta - \sin^2 \theta \cos^2 \theta - \cos^2 \theta + \sin^2 \theta \cos^2 \theta} = \frac{(\sin^2 \theta - \cos^2 \theta)^2}{\sin^2 \theta - \cos^2 \theta}$$

$$= \sin^2 \theta - \cos^2 \theta = 1 - \cos^2 \theta - \cos^2 \theta = 1 - 2 \cos^2 \theta = RS$$

$$z) LS = \sec^4 \theta - \tan^4 \theta = \sec^2 \theta (1 + \tan^2 \theta) - \tan^2 \theta (\sec^2 - 1) \\ = \sec^2 \theta + \sec^2 \theta \tan^2 \theta - \tan^2 \theta \sec^2 \theta + \tan^2 \theta = 1 + \tan^2 \theta + \tan^2 \theta = 1 + 2 \tan^2 \theta = RS$$

$$aa) LS = \frac{\cot \theta}{\csc \theta - 1} \cdot \frac{\csc \theta + 1}{\csc \theta + 1} = \frac{\cot \theta (\csc \theta + 1)}{\csc^2 \theta - 1} = \frac{\cot \theta (\csc \theta + 1)}{\cot^2 \theta} = \frac{\csc \theta + 1}{\cot \theta} = RS$$

$$bb) LS = (\tan \alpha + \tan \beta)(1 - \cot \alpha \cot \beta) + (\cot \alpha + \cot \beta)(1 - \tan \alpha \tan \beta) \\ = (\tan \alpha + \tan \beta)\left(1 - \frac{1}{\tan \alpha \tan \beta}\right) + \left(\frac{1}{\tan \alpha} + \frac{1}{\tan \beta}\right)(1 - \tan \alpha \tan \beta) \\ = \tan \alpha + \tan \beta - \frac{1}{\tan \beta} - \frac{1}{\tan \alpha} + \frac{1}{\tan \alpha} + \frac{1}{\tan \beta} - \tan \beta - \tan \alpha = 0 = RS$$

$$cc) LS = (\sin x - \tan x)(\cos x - \cot x) = \left(\sin x - \frac{\sin x}{\cos x}\right)\left(\cos x - \frac{\cos x}{\sin x}\right) \\ = \sin x \cos x - \cos x - \sin x + 1$$

$$RS = (\cos x - 1)(\sin x - 1) = \sin x \cos x - \cos x - \sin x + 1 = LS$$

$$dd) LS = \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} \cdot \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta + \sin \theta} = \frac{1 + \cos \theta + \sin \theta + \cos \theta + \cos^2 \theta + \sin \theta \cos \theta + \sin \theta + \sin \theta \cos \theta + \sin^2 \theta}{1 + \cos \theta + \sin \theta + \cos \theta + \cos^2 \theta + \sin \theta \cos \theta - \sin \theta - \sin \theta \cos \theta - \sin^2 \theta} = \frac{2 + 2 \cos \theta + 2 \sin \theta + 2 \sin \theta \cos \theta}{1 + 2 \cos \theta + \cos^2 \theta - \sin^2 \theta} \\ = \frac{2(1 + \cos \theta) + 2 \sin \theta(1 + \cos \theta)}{1 + 2 \cos \theta + \cos^2 \theta + \cos^2 - 1} = \frac{2(1 + \cos \theta)(1 + \sin \theta)}{2 \cos \theta(1 + \cos \theta)} = \frac{1 + \sin \theta}{\cos \theta} = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = \sec \theta + \tan \theta = RS$$