Grade Ten Math
Algebra Review Package

Notes:

- no memory aide or other resources
- scientific calculator allowed

Section One - Exponent Laws

- exponent laws
  - $(x^m)^n = x^{mn}$
  - $x^m x^n = x^{m+n}$
  - $\dfrac{x^m}{x^n} = x^{m-n}$
  - $x^0 = 1$
  - $x^{-1} = \dfrac{1}{x}$
  - $\left(\dfrac{x}{y}\right)^{-n} = \left(\dfrac{y}{x}\right)^n$

1. Simplify the following, writing with positive exponents:

a) $\dfrac{(2x^3)^5}{2x^{-3}}$

b) $(a^2b^3)(2a^4b^2)^3$

c) $2(x^3y^2)^2(3x^{-1}y)^4$

d) $(3x^2y^{-3})^5$

e) $(-2x^{-4}y^{-3})^{-2}$

f) $\left(\dfrac{2x^2y^3}{5xy^{-3}}\right)^{-3}$

g) $(x^{\frac{1}{2}}y^{\frac{2}{3}})^6$

h) $(x^{\frac{-5}{3}}y^{\frac{4}{3}})(x^{\frac{2}{3}}y^{\frac{-1}{3}})$

2. Simplify the following:

a) $\dfrac{(a^x)(a^{x-2})}{a^{-x+4}}$

b) $\dfrac{(x^m)^3\left(\dfrac{1}{x^2}\right)^m(x^3)}{x^{m+2}}$

c) $\dfrac{(x^{a-b})(x^{2a+b})}{(x^2)^{a+2b}}$

d) $\dfrac{(x^2)^{m+n}(y)^{2n-m}}{(x)^{2m+n}(y^3)^{2m-n}}$

3. Simplify the following:

a) $\dfrac{(\sqrt{9})(3^5)(9^2)}{(27)^{1/3}(3^2)^3(27)^{-3}}$

b) $\dfrac{(2^5)^3\left(\frac{1}{4}\right)^{-10}(4^{10})}{\left(\frac{1}{8}\right)^{-3}(16)^4}$

c) $\dfrac{(3^x)(27^x)^{-2}}{\left(\frac{1}{3}\right)^{2x}(9)^{x-2}}$

d) $(2^a)\left(\frac{1}{2}\right)^{-4a}(16)^{2a+1}$

e) $\dfrac{(2^{x+y})(4^{x-y})}{8^{3x-y}}$

f) $\dfrac{(25)^{a+b}\left(\frac{1}{5}\right)^{a-2b}}{(125^a)(5)^{-b}}$

<u>Section Two - Radicals</u>

- rational exponents, and simplifying, with or without negative exponents:
- ex) $\sqrt{x} = x^{\frac{1}{2}}$, $\sqrt[3]{x} = x^{\frac{1}{3}}$, $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$, $(16)^{\frac{3}{4}} = (\sqrt[4]{16})^3 = (2)^3 = 8$, $64^{\frac{5}{6}} = 32$, $8^{\frac{-2}{3}} = \dfrac{1}{4}$
- multiply and divide radicals: $(\sqrt{a})(\sqrt{b}) = \sqrt{ab}$, $\dfrac{(\sqrt{a})}{(\sqrt{b})} = \sqrt{\dfrac{a}{b}}$
- reduce with number and variables
  - ex) $\sqrt{40x^2y^5} = 2xy^2\sqrt{10y}$ and $\sqrt[3]{16x^3y^5} = 2xy\sqrt[3]{2y^2}$
- add and subtract radical expressions, and simplify first when needed:
  - ex) $\sqrt{12} + 2\sqrt{2} - 3\sqrt{2} + 2\sqrt{27} = 2\sqrt{3} + 2\sqrt{2} - 3\sqrt{2} + 6\sqrt{3} = 8\sqrt{3} - \sqrt{2}$
- be able to expand binomials: ex) $(\sqrt{2}+1)(\sqrt{3}-4) = \sqrt{6} + \sqrt{3} - 4\sqrt{2} - 4$
- all denominators must be **rationalized.**
- multiply by the conjugate if the denominator is a binomial.

1. Simplify the following:

a) $8^{\frac{-2}{3}}$

b) $(27^{\frac{5}{3}})(81^{\frac{-3}{4}})$

c) $\sqrt{36x^2y^5}$

d) $\sqrt{125x^3y^{10}z^7}$

e) $\sqrt[3]{27x^6y^9}$

f) $\sqrt[3]{54x^4y^9z^{11}}$

g) $\sqrt[4]{16x^8y^{20}}$

h) $\sqrt[4]{32x^7y^{17}}$

2. Simplify the following:

a) $\sqrt{2} + 3\sqrt{2} - 5\sqrt{3} + 8\sqrt{3}$

b) $\sqrt{8} + 3\sqrt{18} - 3\sqrt{32}$

c) $4\sqrt{27} - 5\sqrt{12} - 3\sqrt{80} - 2\sqrt{45}$

d) $2\sqrt{90} + 5\sqrt{40} - 3\sqrt{75} + 2\sqrt{48}$

3. Simplify the following:

a) $(3\sqrt{2} - 4)(\sqrt{3} + 2\sqrt{2})$

b) $(2\sqrt{3} - \sqrt{2})^2$

c) $(5\sqrt{6} - \sqrt{3})(5\sqrt{6} + \sqrt{3})$

d) $(2 + 4\sqrt{5})(\sqrt{3} - \sqrt{15})$

4. Simplify the following: (by rationalizing the denominator)

a) $\dfrac{3}{2\sqrt{2}}$

b) $\dfrac{3\sqrt{2} - 1}{\sqrt{3}}$

c) $\dfrac{\sqrt{3}}{1 + 3\sqrt{2}}$

d) $\dfrac{\sqrt{6}}{3\sqrt{5} - 1}$

e) $\dfrac{4\sqrt{3} - 1}{2\sqrt{5} - \sqrt{3}}$

f) $\dfrac{1 + \sqrt{6}}{2\sqrt{5} + 3\sqrt{7}}$

5. Simplify the following:

a) $(\sqrt{x} + 3)(\sqrt{x} - 1)$

b) $(\sqrt{x} - 4)^2$

c) $(2\sqrt{x} - 1)(\sqrt{x} - 3)$

d) $(3\sqrt{x + 2} - 1)^2$

e) $(3\sqrt{x - 1} - 5)(3\sqrt{x - 1} + 5)$

Section Three - Factoring

- trinomials, common factoring, difference of squares
- $(a+b)^2 = a^2 + 2ab + b^2$

1. Factor the following completely, if possible:

a) $x^2 + 7x + 12$

b) $x^2 - 100$

c) $x^2 + 7x - 30$

d) $x^2 - 15x + 44$

2. Factor the following completely, if possible:

a) $2x^2 + 7x + 3$

b) $6x^2 - 7x - 3$

c) $3x^2 - 7x + 6$

d) $15x^2 - 8x - 16$

3. Factor the following completely, if possible:

a) $4x^2 + 20x + 25$

b) $9x^2 - 4y^2$

c) $25x^2 - 40xy + 16y^2$

d) $(3x+2y)^2 - (x-y)^2$

e) $x^2(x+1) - 4x(x+1) + 4(x+1)$

f) $x^4 - 13x^2 - 48$

Section Four - Quadratic Equations

- solve by factoring, or quadratic formula
- Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Express all answers **EXACTLY** in lowest terms, eg.) $\frac{3}{7}$, $1 + 2\sqrt{2}$, $\frac{1}{2} - \frac{\sqrt{3}}{2}$

1. Solve the following quadratic equations:

a) $(x - 1)(x + 2) = 0$
b) $(2x - 1)(3x + 4) = 0$
c) $(2x)(7x - 5) = 0$

2. Solve the following quadratic equations:

a) $x^2 - 23x + 126 = 0$
b) $x^2 + 8x + 16 = 0$
c) $x^2 + 3x = 40$

3. Solve the following quadratic equations:

a) $2x^2 - 5x = 12$
b) $10x^2 = 7x + 12$
c) $6x^2 + 27x + 12 = 0$
d) $5x^2 + 21x - 54 = 0$

4. Solve the following quadratic equations:

a) $x^2 - 2x - 4 = 0$
b) $x^2 - x - 5 = 0$
c) $2x^2 + 8x - 3 = 0$
d) $6x + 6 = 15x^2$
e) $1 = 8x + 3x^2$

5. Solve the following quadratic equations:

a) $3x(x - 2) - x(x + 1) + 5 = 0$
b) $x^2 + (x + 1)^2 = 13$
c) $3(x - 1)(x + 4) - 2(2x + 1)^2 = -18$

6. Solve the following quadratic equations:

a) $2(1-x^2)-3x(1-x) = 7$

b) $2(x-2)(x+1)-(x+3) = 0$

c) $(3x-2)(x-3) = (x-4)(x-1)$

Section Five - Long Division

- be sure to add in any terms that are missing.
- write answer in proper form and state restriction(s).

1. Simplify by performing long division:

a) $(x^3-2x^2+2x-15) \div (x-3)$

b) $(x^3+3x^2-9x-20) \div (x+4)$

c) $(24+6x-7x^2+x^3) \div (x-5)$

d) $(x^4+4x^3+2x^2-3x+2) \div (x+2)$

2. Simplify by performing long division:

a) $(6x^3+13x^2-9) \div (2x+3)$

b) $(x^3+9x+10) \div (x+2)$

3. Simplify by performing long division:

a) $(x^4+x^3-13x^2-25x-12) \div (x^2+2x+1)$

b) $(x^4-17x^2-36x-20) \div (x^2-3x-10)$

c) $(x^4-3x^3+2x^2-x+2) \div (x^2+2x-1)$

Section Six - Rational Expressions

1. Simplify the following rational expressions. **DO NOT** state restrictions.

a) $\dfrac{x^2-y^2}{x^2-2xy+y^2}$

b) $\dfrac{6x^2+11x+3}{6x^2-7x-3}$

c) $\dfrac{x^2-y^2}{y-x}$

d) $\dfrac{-xm-ym}{x^2+2xy+y^2}$

e) $\dfrac{4(x^2-y^2)}{(x-y)^2}$

f) $\left(\frac{x^2-9y^2}{x^2+4xy}\right)\cdot\left(\frac{x^2+9xy+20y^2}{x^2+10xy+21y^2}\right)\div(x^2+2xy-15y^2)$

2. Simplify the following rational expressions and state restrictions.

a) $\frac{2x-10}{x^2+2x-35}$

b) $\frac{x^2-5x+6}{x^2-5x-6}$

c) $\frac{x-4}{4-x}$

d) $\frac{x^2+3x-28}{12+x-x^2}$

e) $\left(\frac{x+1}{x^2-x-6}\right)\cdot\left(\frac{x^2-7x+12}{x^2-16}\right)$

f) $\left(\frac{x^2+3x+2}{4-x^2}\right)\cdot\left(\frac{10x}{5x+5}\right)$

g) $\left(\frac{x^2-4x+3}{x^2-16}\right)\div\left(\frac{x^2+3x-4}{x^2+x-20}\right)$

h) $\left(\frac{x+7}{x^2-5x-36}\right)\cdot\left(\frac{x^2-15x+54}{x^2-36}\right)\div\left(\frac{x^2-2x-63}{x+4}\right)$

i) $\left(\frac{x^2-49}{x^2+3x-88}\right)\cdot\left(\frac{x^2+6x-55}{x^2-11x+28}\right)\div\left(\frac{x^2+2x-35}{x^2-12x+32}\right)$

3. Simplify the following rational expression and state restrictions.

a) $\frac{5}{3x^2}-\frac{1}{2x}+\frac{3}{5x^3}$

b) $\frac{3x}{x+2}+\frac{x+5}{x+2}-\frac{x}{x+2}$

c) $\frac{3}{x+1}+\frac{4}{x-3}$

d) $\frac{6}{x-4}-\frac{3}{x-2}$

e) $\frac{3}{x-2}-\frac{4}{2-x}$

f) $\frac{2}{x^2-x-12}+\frac{5}{x^2+8x+15}$

g) $\frac{2x}{x^2-6x+5}-\frac{3x}{x^2-11x+30}$

h) $\frac{x+6}{x^2+9x+18}+\frac{x-3}{x^2-2x-3}$

i) $\dfrac{x^2+x-6}{x^2-3x+2} - \dfrac{x^2+3x+2}{x^2-x-2}$

j) $\dfrac{x+2}{x^2-x-6} - \dfrac{x-4}{x^2-x-12} + \dfrac{x+3}{x^2+2x-3}$

k) $\dfrac{3x-12}{x^2-x-12} - \dfrac{2}{x^2+6x+9} - \dfrac{1}{x^2-4x-21}$

Section Seven - Solving Equations with Rational Expressions

1. Solve the following equations:

a $\dfrac{3}{x+1} + \dfrac{4}{x+2} = 2$

b) $\dfrac{2x}{x-2} - 3x + 8 = 0$

c) $\dfrac{4}{x+2} - 4 = \dfrac{3}{x-3}$

d) $\dfrac{5}{x-1} + \dfrac{6}{x+1} - 7 = 0$

2. Solve the following equations: (These may need quadratic formula)

a) $\dfrac{2}{x} - \dfrac{3}{x+1} = 1$

b) $\dfrac{3}{x+2} + \dfrac{2}{x-3} = 2$

c) $\dfrac{x+3}{2x-1} = \dfrac{x-3}{x+4}$

d) $\dfrac{2x}{x+1} - \dfrac{3}{x-1} = \dfrac{4}{x^2-1}$

e) $\dfrac{4}{x-2} + \dfrac{2}{x+2} = 3$

f) $\dfrac{3}{x-1} + \dfrac{-2}{x+2} = -2$

Section Eight - Equivalent Shapes

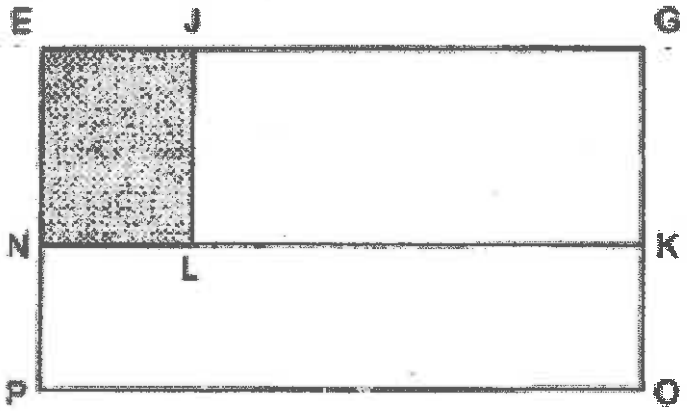
1.

Rectangles JGKL and NKOP were drawn in rectangle EGOP shown below.

Rectangles JGKL and NKOP are equivalent.

In addition:

- $m\overline{NL} = (x-1)$ units
- $m\overline{LK} = (2x+4)$ units
- $m\overline{KO} = (x-1)$ units
- $m\overline{JL} = (x+1)$ units

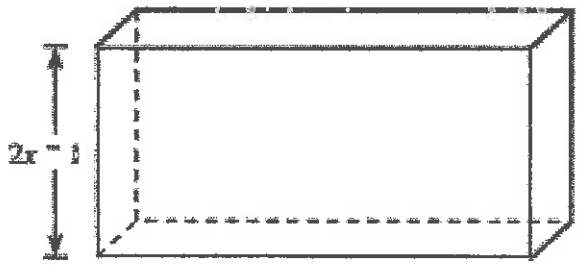


What is the numerical area of rectangle EJLN?

2.

The polynomial $4x^3 + 16x^2 + 11x - 10$ represents the volume of a right prism with a rectangular base.

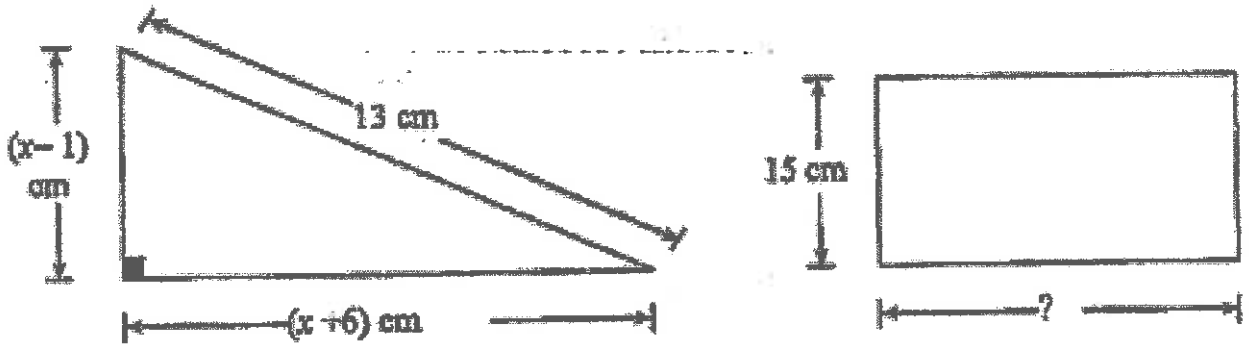
The height of this prism is represented by the binomial $2x - 1$.



What binomials represent the length and the width of the base of this prism?

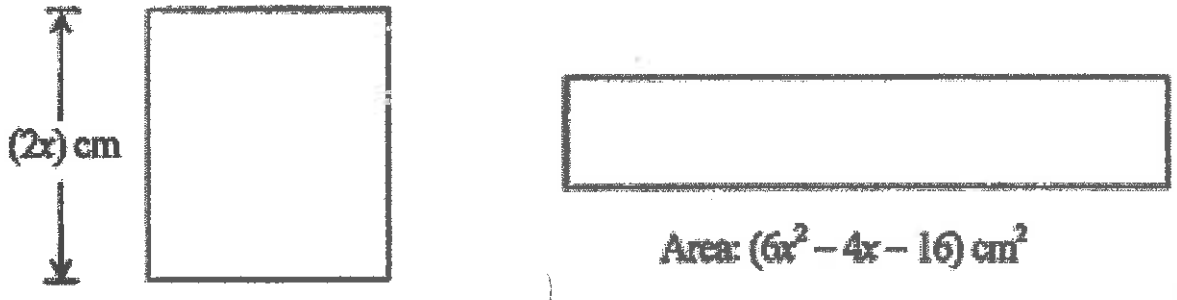
3.

The right triangle and the rectangle below are equivalent. (equal areas)
The hypotenuse of the triangle measures 13 cm. The sides of the triangle are (x - 1) cm and (x + 6) cm. The height of the rectangle is 15 cm.
What is the numerical length of the base of the rectangle?



4.

The square and the rectangle shown below are equivalent figures (ie. Have the same area). Each side of the square measures $(2x)$ cm. The area of the rectangle is $(6x^2 - 4x - 16)$ cm^2.



What is the perimeter of the rectangle? (There are two possible answers)
Your final answer must be a number. Show all your work.

Answers

Section One - Exponent Laws

1.a) $16x^{18}$ b) $8a^{14}b^{9}$ c) $162x^{2}y^{8}$ d) $\frac{243x^{10}}{y^{15}}$

e) $\frac{1}{4}x^{8}y^{6}$ f) $\frac{125}{4x^{3}y^{18}}$ g) $x^{3}y^{4}$ h) $\frac{y}{x}$

2. a) a^{3x-6} b) x c) x^{a-4b} d) $x^{n}y^{-7m+5n}$

3. a) 3^{12} b) 2^{30} c) 3^{-5x+4} d) 2^{13a+4}

e) 2^{-6x+2y} f) 5^{-2a+5b}

Section Two - Radicals

1. a) $\frac{1}{4}$ b) 9 c) $6xy^{2}\sqrt{y}$ d) $5xy^{5}z^{3}\sqrt{5xz}$

e) $3x^{2}y^{3}$ f) $2xy^{3}z^{3}\sqrt[3]{2xz^{2}}$ g) $2x^{2}y^{5}$

h) $2xy^{4}\sqrt[4]{2x^{3}y}$

2. a) $4\sqrt{2}+3\sqrt{3}$ b) $-\sqrt{2}$

c) $2\sqrt{3}-18\sqrt{5}$ d) $16\sqrt{10}-7\sqrt{3}$

3. a) $3\sqrt{6}-4\sqrt{3}-8\sqrt{12}+12$ b) $14-4\sqrt{6}$

c) 147 d) $-18\sqrt{3}+2\sqrt{15}$

4. a) $\frac{3\sqrt{2}}{4}$ b) $\frac{3\sqrt{6}-\sqrt{3}}{3}$

c) $\frac{\sqrt{3}-3\sqrt{6}}{-17}$ or $-\frac{\sqrt{3}-3\sqrt{6}}{17}$ d) $\frac{3\sqrt{30}+\sqrt{6}}{44}$

e) $\frac{8\sqrt{15}+12-2\sqrt{5}-\sqrt{3}}{17}$

f) $\frac{2\sqrt{5}-3\sqrt{7}+2\sqrt{30}-3\sqrt{42}}{-43}$ or $-\frac{2\sqrt{5}-3\sqrt{7}+2\sqrt{30}-3\sqrt{42}}{43}$

5. a) $x+2\sqrt{x}-3$ b) $x-8\sqrt{x}+16$

c) $2x-7\sqrt{x}+3$ d) $9x+19-6\sqrt{x+2}$

e) $9x-34$

Section Three - Factoring

1. a) $(x+4)(x+3)$ b) $(x-10)(x+10)$
c) $(x+10)(x-3)$ d) $(x-11)(x-4)$

2. a) $(2x+1)(x+3)$ b) $(2x-3)(3x+1)$
c) not factorable d) $(5x+4)(3x-4)$

3. a) $(2x+5)^2$ b) $(3x+2y)(3x-2y)$
c) $(5x-4y)^2$ d) $(4x+y)(2x+3y)$
e) $(x-2)^2(x+1)$ f) $(x-4)(x+4)(x^2+3)$

Section Four - Quadratic Equations

1. a) $x \in \{1, -2\}$ b) $x \in \{\frac{1}{2}, -\frac{4}{3}\}$ c) $x \in \{0, \frac{5}{7}\}$

2. a) $x \in \{9, 14\}$ b) $x \in \{-4\}$ c) $x \in \{5, -8\}$

3. a) $x \in \{-\frac{3}{2}, 4\}$ b) $x \in \{-\frac{4}{5}, \frac{3}{2}\}$

c) $x \in \{-\frac{1}{2}, -4\}$ d) $x \in \{\frac{9}{5}, -6\}$

4. a) $x \in \{ 1+\sqrt{5}, 1-\sqrt{5} \}$

b) $x \in \{ \frac{1+\sqrt{21}}{2}, \frac{1-\sqrt{21}}{2} \}$

c) $x \in \{ \frac{-4+\sqrt{22}}{2}, \frac{-4-\sqrt{22}}{2} \}$

d) $x \in \{ \frac{1+\sqrt{11}}{5}, \frac{1-\sqrt{11}}{5} \}$

e) $x \in \{ \frac{-4+\sqrt{19}}{3}, \frac{-4-\sqrt{19}}{3} \}$

5. a) $x \in \{ \frac{5}{2}, 1 \}$

b) $x \in \{ -3, 2 \}$

c) $x \in \{ -\frac{4}{5}, 1 \}$

6. a) $x \in \{ \frac{3+\sqrt{29}}{2}, \frac{3-\sqrt{29}}{2} \}$

b) $x \in \{ \frac{3+\sqrt{65}}{4}, \frac{3-\sqrt{65}}{4} \}$

c) $x \in \{ \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2} \}$

Section Five - Long Division

1. a) $x^2 + x + 5$, $x \neq 3$

b) $x^2 - x - 5$, $x \neq -4$

c) $x^2 - 2x - 4$ Remainder 4, or $x^2 - 2x - 4 + \frac{4}{x-5}$, $x \neq 5$

d) $x^3 + 2x^2 - x - 1$, $x \neq -2$

2. a) $3x^2 + 2x - 3$, $x \neq -\frac{3}{2}$

b) $x^2 - 2x + 13$ Remainder -16 or $x^2 - 2x + 13 - \frac{16}{x+2}$, $x \neq -2$

3. a) $x^2 - x - 12$, $x \neq -1$

b) $x^2 + 3x + 2$, $x \neq -2$ and $x \neq 5$

c) $x^2 - 5x + 13$ Remainder $-32x + 15$ or $x^2 - 5x + 13 + \frac{-32x+15}{x^2+2x-1}$,

$x \neq -1 - \sqrt{2}$

Section Six - Rational Expressions

1. a) $\frac{x+y}{x-y}$ b) $\frac{2x+3}{2x-3}$ c) $-x-y$

d) $-\frac{m}{x+y}$ e) $\frac{4(x+y)}{x-y}$ f) $\frac{1}{x(x+7y)}$

2. a) $\frac{2}{x+7}$, $x \neq 5, -7$

b) $\frac{(x-3)(x-2)}{(x-6)(x+1)}$, $x \neq 6, -1$

c) -1, $x \neq 4$

d) $\frac{-x-7}{x+3}$. $x \neq 4, -3$

e) $\frac{x+1}{(x+2)(x+4)}$, $x \neq 3, -2, 4, -4$

f) $\frac{-2x}{x-2}$, $x \neq 2, -2$

g) $\frac{(x-3)(x+5)}{(x+4)^2}$, $x \neq 4, -4, -5, 1$

h) $\frac{1}{(x-9)(x+6)}$, $x \neq 9, -4, 6, -6, -7$

i) 1, $x \neq 11, -8, 7, 4, -7, 5, 8$

3. a) $\frac{50x-15x^2+18}{30x^2}$, $x \neq 0$

b) $\frac{3x+5}{x+2}$. $x \neq -2$

c) $\frac{7x-5}{(x+1)(x-3)}$, $x \neq -1, 3$

d) $\frac{3x}{(x-4)(x-2)}$, $x \neq 4, 2$

e) $\frac{7}{x-2}$, $x \neq 2$

f) $\frac{7x-10}{(x-4)(x+3)(x+5)}$, $x \neq 4, -3, -5$

g) $\frac{-x(x+9)}{(x-1)(x-5)(x-6)}$, $x \neq 1, 5, 6$

h) $\frac{2(x+2)}{(x+3)(x+1)}$, $x \neq -6, -3, 3, -1$

i) $\dfrac{-4}{(x-2)(x-1)}$, $x \neq 1, 2, -1$

j) $\dfrac{x^2+6x-15}{(x-3)(x+3)(x-1)}$, $x \neq 3, -2, 4, -3, 1$

k) $\dfrac{3x^2-15x-52}{(x+3)^2(x-7)}$, $x \neq 4, -3, 7$

Section Seven - Solving Equations with Rational Expressions

1. a) $x \in \{-\frac{3}{2}, 2\}$

b) $x \in \{\frac{4}{3}, 4\}$

c) $x \in \{-\frac{3}{4}, 2\}$

d) $x \in \{-\frac{3}{7}, 2\}$

2. a) $x \in \{-1+\sqrt{3}, -1-\sqrt{3}\}$

b) $x \in \{\frac{7+\sqrt{105}}{4}, \frac{7-\sqrt{105}}{4}\}$

c) $x \in \{7+\sqrt{58}, 7-\sqrt{58}\}$

d) $x \in \{\frac{7}{2}\}$ Note: $x = -1$ is a restriction and not valid.

e) $x \in \{\frac{3+\sqrt{57}}{3}, \frac{3-\sqrt{57}}{3}\}$

f) no solution

Section Eight - Equivalent Shapes

1. $x = 7$, EJ = 6, JL = 8, area = 48 units^2
2. Divide by (2x - 1) to get $2x^2 + 9x + 10$
   Factor to get length = 2x + 5 and width = x + 2
3. Use Pythagoras to get x = 6. Rectangle height = 5 cm, base = 12 cm.
   Area of triangle = 30 = area of rectangle. Base of rectangle = 2 cm.
4. x = 4
   TWO ANSWERS, it can factor 2 ways:
   1. $(6x+8)(x-2)$ Dimensions = 32 x 2, perimeter = 68 cm
   2. $(3x+4)(2x-4)$ Dimensions 16 x 4, perimeter = 40 cm