Math 5SN

Answer Key

EXAMINATION #1

Part A	
Questions 1 to 10	***4 marks or 0 marks***

1	D	**6**	B
2	C	**7**	D
3	C	**8**	A
4	B	**9**	D
5	D	**10**	B

Part B	
Questions 11 to 15	***4 marks each***

11 The ordered pairs are **(1, 10), (5, 8)** as well as the ordered pair whose coordinates are **(3, 9)**. *2 marks* *2 marks* /4

12 Rounded to the nearest hundredth, the value of x is **0.73**. /4

Note: Do not penalize students who did not round their answer to the nearest hundredth.

13 To the nearest degree, the angle measure is **38°**. /4

Note: Do not penalize students who did not round their answer.

14 The exact values of x are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. /4

15 The number of hours elapsed is **5**. /4

Part C

Questions 16 to 25 ***4 marks each***

No marks are to be given if work is not shown. Examples of correct solutions are given. However, other acceptable solutions are possible.

16 Example of an appropriate method /4

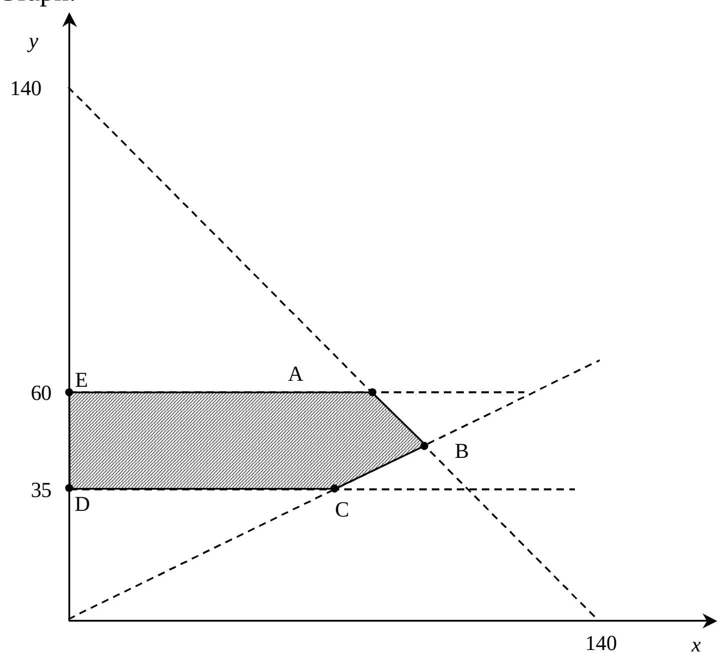
Let x: number of lobsters
y: number of crabs

Constraints:

$$x \geq 0 \quad y \geq 0$$
$$y \geq 35$$
$$y \leq 60$$
$$x \leq 2y$$
$$x + y \leq 140$$

Objective Function: $R = 8.70x + 9.60y$

Graph:



Vertex	$R = 8.70x + 9.60y$
A(80, 60)	1272 ← max
B($93.\overline{3}$, $46.\overline{6}$)	1259
C(70, 35)	945
D(0, 35)	336
E(0, 60)	576

Answer: The maximum revenue this fisherman can expect to make is **$1272**.

17 Example of an appropriate method /4

Find the rate

$$y = a \bullet b^x$$
$$y = 110 \bullet b^x$$
$$835 = 110 \bullet b^x$$
$$7.59 = b^5$$
$$1.5 = b$$

Find time that elapsed when 2000 victims have been infected

$$110 \times 1.5^t = 2000$$
$$1.5^t = 18.\overline{18}$$
$$t = \frac{\log 18.\overline{18}}{\log 1.5} \approx 7.15$$

Find the year the vaccine will be offered

$$1996 + 7.15 = 2003.15$$

Answer: The population will be offered the vaccine in the year **2003**.

Note: Students who have determined the rate have shown a partial understanding of the problem.

18 Example of an appropriate method /4

$$\left.\begin{array}{c} \frac{1}{\sin x} - \sin x \\ \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \\ \frac{1 - \sin^2 x}{\sin x} \end{array}\right\} \text{1 mark}$$

$$\frac{\cos^2 x}{\sin x} \qquad \textit{1 mark}$$

$$\frac{\cos x}{\sin x} \bullet \frac{\cos x}{1} \qquad \textit{1 mark}$$

$$\cot x \bullet \cos x \qquad \textit{1 mark}$$

19 Example of an appropriate method /4

$a = 10$ $b = 8$

Find the equation of the semi-ellipse centre (0, 0) (Students may use other centres.)

$$\frac{x^2}{100} + \frac{y^2}{64} = 1$$

Find the value of y at $(7, y)$

$$\frac{49}{100} + \frac{y^2}{64} = 1$$

$$\frac{y^2}{64} = 1 - \frac{49}{100}$$

$$y^2 = 32.64$$

$$y \approx +5.71 \text{ (-5.71 is rejected)}$$

Answer: The cameras are **5.71 m** from the ground.

20 Equation of the circle in standard form /4

$$x^2 + 6x + y^2 - 2y = 26$$

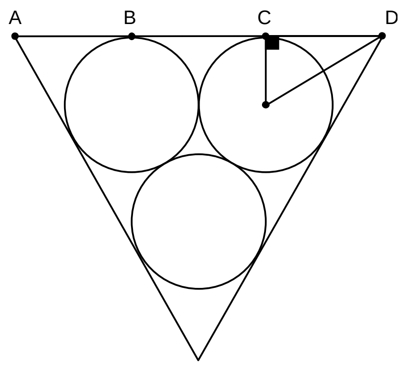
$$(x-3)^2 + (y-1)^2 = 26 + 9 + 1$$

$$(x-3)^2 + (y-1)^2 = 36$$

Each radius measures 6 units

Since the three circles are congruent, the border forms an equilateral triangle.

According to the diagram on the right,



Note: Adjustments will have to be made for different labelling.

Δ COD is a right triangle, m $\overline{OC}$ = 6 units and m $\angle$ CDO = 30°

$$\tan 30° = \frac{m\ \overline{OC}}{m\ \overline{CD}} = \frac{6}{m\ \overline{CD}}$$

$$m\ \overline{CD} = 10.39$$

$$m\ \overline{AD} = m\ \overline{AB} + m\ \overline{BC} + m\ \overline{CD}$$

$$= 10.39 + 12 + 10.39$$

$$32.78$$

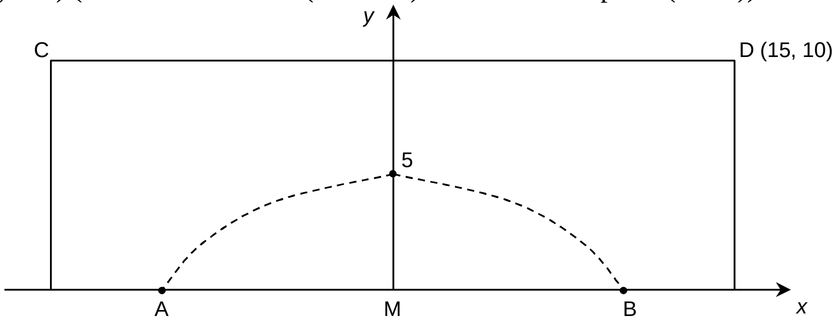
$$\text{Perimeter: } P = 3 \bullet 32.78$$

$$= 98.34$$

Answer: To the nearest hundredth of a unit, the border measures **98.34**.

21 It is a parabola in the form of /4

$x^2 = 4p(y - k)$ (distance to the line (directrix) = distance to a point (focus))



Focus: M(0, 0) Vertex (0, 5) p = 5

Equation : $x^2 = -4p\ (y - 5)$

$x^2 = -20\ (y - 5)$

a and b are the zeros of the function:

$x^2 = -20\ (0 - 5)$

$x^2 = 100$

$x = \pm\ 10$

Answer: Points A and B are **20 m** apart.

EXAMINATION #2

Section A

Questions 1 to 13 ***4 marks or 0 marks***

1 A

2 D

3 D

4 B

5 C

6 A

7 C

8 A

Section B

Questions 13 to 25 ***4 marks each***

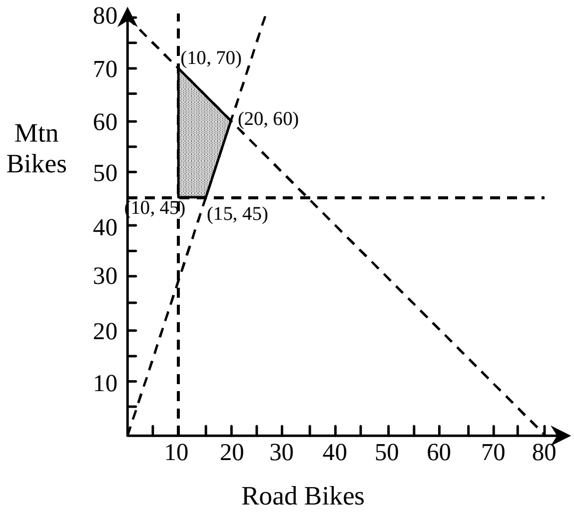
No marks are to be given if work is not shown. Examples of correct solutions are given. However, other acceptable solutions are possible.

13 Example of an appropriate method /4

x = number of road bikes
y = number of mountain bikes

$x \geq 0$
$y \geq 0$
$x \geq 10$
$y \geq 45$
$x + y \leq 80$
$y \geq 3x$

Objective Function
Max. Profit = $250x + 175y$



Points (x, y)		Calculation	Profit
1.	(10, 45)	250(10) + 175(45)	\$10 375
2.	(15, 45)	250(15) + 175(45)	\$11 625
3.	(20, 60)	250(20) + 175(60)	\$15 500
4.	(10, 70)	250(10) + 175(70)	\$14 750

Answer The maximum weekly profit is \$15 500.

14 Example of an appropriate method /4

$y = a|x - b| + c$
$b = (2 + 8) \div 2 = 5$
$a = -\left(\frac{\Delta y}{\Delta x}\right) = \frac{(-4 - 0)}{(0 - 2)} = -2$
$(c - (-4)) \div (5 - 0) = 2$
$c = 6$

/4

Answer The rule of correspondence is $y = -2|x - 5| + 6$.

15 Example of an appropriate method /4

$\sin A \cot A + 2\cos^2 A = 1$

$\sin A \frac{\cos A}{\sin A} + 2\cos^2 A = 1$
$\cos A + 2\cos^2 A = 1$
$2\cos^2 A + \cos A - 1 = 0$
$(2\cos A - 1)(\cos A + 1) = 0$
$\cos A = \frac{1}{2}$ or $\cos A = -1$

$A = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ or $A = \pi$

Answer $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

16 Example of an appropriate method /4

Let x = width of fenced-in plot in metres
$25 - 2x$ = length of fenced-in plot in metres

Area of plot = length $\times$ width = $x(25 - 2x)$

$x(25 - 2x) \geq 50$
$25x - 2x^2 \geq 50$
$-2x^2 + 25x - 50 \geq 0$
$2x^2 - 25x + 50 \leq 0$
$(2x - 5)(x - 10) \leq 0$

Zeros
$2x - 5 = 0$ or $x - 10 = 0$
$x = 2.5$ $x = 10$

Zeros are 2.5 and 10

Width of plot 2.5 m

Answer The smallest value of dimension x is 2.5 m.

17 Example of an appropriate method /4

$V(t) = -25|t - 8| + 65$
$V(t) \geq 35$

$-25|t - 8| + 65 \geq 35$
$-25|t - 8| \geq 35 - 65$
$|t - 8| \leq \frac{-30}{-25}$
$|t - 8| \leq 1.2$

If $t - 8 \geq 0$, then $|t - 8| = t - 8$;
$t - 8 \leq 1.2$
$t \leq 9.2$

if $t - 8 < 0$, then $|t - 8| = -(t - 8)$
$-(t - 8) \leq 1.2$
$-t + 8 \leq 1.2$
$-t \leq -6.8$
$t \geq 6.8$

$9.2 - 6.8 = 2.4$

Answer The police officer should be on duty for 2.4 hours.
(Accept 2 hours and 24 minutes)

18 Example of an appropriate method /4

$\log_5 (x - 1) + \log_5 (x + 3) - 1 = 0$

$\log_5 (x - 1)(x + 3) = 1$ Sum of logs = log of the product

$(x - 1)(x + 3) = 5^1$

$x^2 + 2x - 3 = 5$

$x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$x - 2 = 0$ or $x + 4 = 0$

$x = 2$ or -4

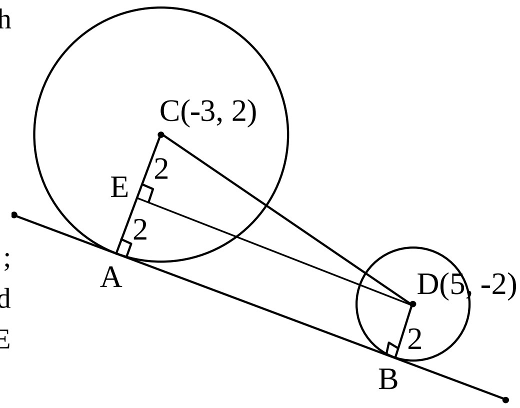
-4 is an extraneous root

Answer The solution of the equation is 2.

19 Example of an appropriate method /4

Step 1: Determine centre and radius for each circle.

	Centre	Radius
LG	C(-3, 2)	4 units
SM	D(5, -2)	2 units



Step 2: Draw radii $\overline{CA}$ and $\overline{DB}$; draw $\overline{CD}$; Draw $\overline{DE}$, perpendicular to $\overline{CA}$ and parallel to $\overline{AB}$, forming rectangle ABDE and right ΔCED.

Step 3: In right ΔCED,

$$m\,\overline{CD} = \sqrt{(5-(-3))^2 + (-2-2)^2} = \sqrt{8^2 + (-4)^2} = \sqrt{64+16} = \sqrt{80}$$

$$m\,\overline{CE} = m\,\overline{CA} - m\,\overline{EA} = 4 - 2 = 2 \text{ units}$$
$$\left(m\,\overline{EA} = m\,\overline{DB} = 2 \text{ units}\right)$$

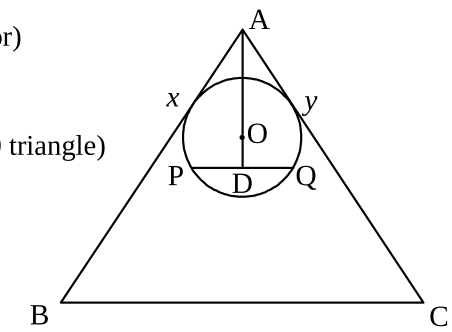
$$m\,\overline{DE} = \sqrt{80-4} = \sqrt{76}$$

Step 4: $m\,\overline{AB} = m\,\overline{DE}\ (\text{rect. ABDE}) = \sqrt{76} \approx 8.72$ uits

Answer The distance from point A to point B is approximately 8.72 units.

20 Example of an appropriate method /4

Since ΔABC is equilateral, $m\,\angle DAC = \frac{1}{2}\,60° = 30°$ ($\overline{AD}$ is an angle bisector)



$m\,\overline{OY} = 5$ units	(Given)
$\therefore\ m\,\overline{AO} = 10$ units	(Hypotenuse = twice length of short side of 30-60-90 triangle)
$m\,\overline{DQ} = 4$	(A segment passing through the centre of a circle drawn perpendicular to a chord bisects the chord)
$m\,\overline{OQ} = m\,\overline{OY} = 5$	(Radii of circle are congruent)

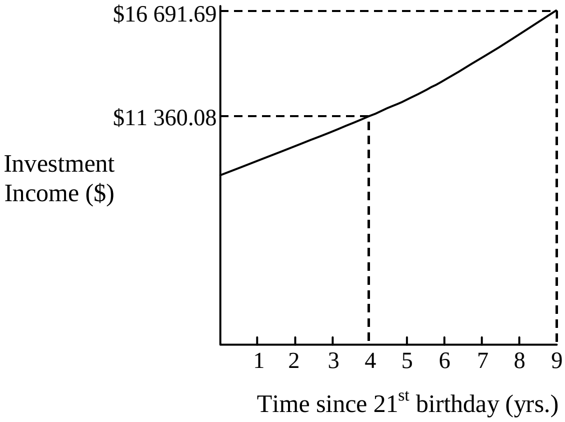
In right triangle DOQ, $m\,\overline{OD} = \sqrt{(m\,\overline{OQ})^2 - (m\,\overline{DQ})^2} = \sqrt{5^2 - 4^2} = 3$ (Pythagorean Theorem)

$\therefore\ m\,\overline{AD} = m\,\overline{AO} + m\,\overline{OD} = 10 + 3 = 13$ units

Answer The measure of $\overline{AD}$ is 13 units.

21 Example of an appropriate method /4

Use exponential model:



$$y = 11\,360.08(1+i)^{9-4}$$

where i = annual interest rate
x = time, in years, since 21st birthday
y = accumulated investment ($)

Substituting (9, 16 691.69), we get

$$16\,691.69 = 11\,360.08(1+i)^{9-4}$$

$$\Rightarrow \frac{16\,691.69}{11\,360.08} = (1+i)^5 \qquad \Rightarrow \sqrt[5]{\frac{16\,691.69}{11\,360.08}} = 1+i \qquad \Rightarrow \sqrt[5]{\frac{16\,691.69}{11\,360.08}} - 1 = i$$

$$\Rightarrow 1.08 - 1 \approx i \qquad \Rightarrow i \approx 0.08 \quad \text{or} \quad 8\%$$

Alternate solution let $y = 11\,360.08(1+i)^x$
where x = number of years since 25th birthday

Substituting (5, 16 691.69), we get

$$16\,691.69 = 11\,360.08(1+i)^x$$

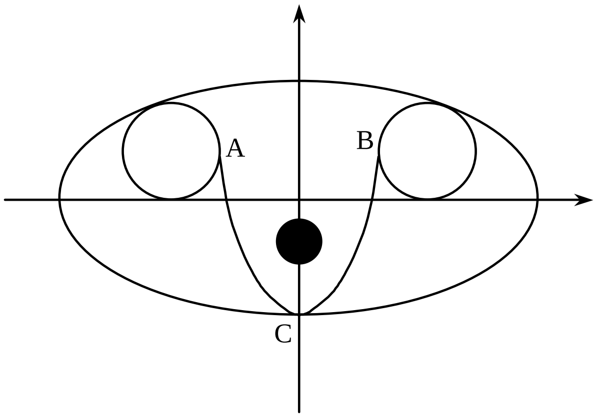
$$\Rightarrow i = \sqrt[5]{\frac{16\,691.69}{11\,360.08}} - 1 \approx 0.08 \quad \text{or} \quad 8\%$$

Answer The annual fixed interest rate is approximately 8%.
(Accept 8.0% as well as 8%)

22 Example of an appropriate method /4

Determine location of foci of the ellipse:

$c^2 = a^2 - b^2$
$c^2 = 100 - 36$
$c^2 = 64$
$c = \pm 8$



Coordinates of foci at (±8, 0)

Since circles have a radius of 2 units, coordinates of centres are (-8, 2) and (8, 2)

Coordinates of points A and B are (-8 + 2, 2) and (8 – 2, 2) or (-6, 2) and (6, 2)

Coordinates of point C are (0, -6)

Equation of parabola in canonic form

$(x - h)^2 = 4c(y - k)$

Substituting coordinates of points B and C into the equation to obtain value of c

B(x, y) = (6, 2) C(h, k) = (0, -6)

$6^2 = 4c(2 - -6)$
$36 = 4c(8)$
$36 = 32c$
$c = \frac{36}{32} = \frac{9}{8}$ or 1.125

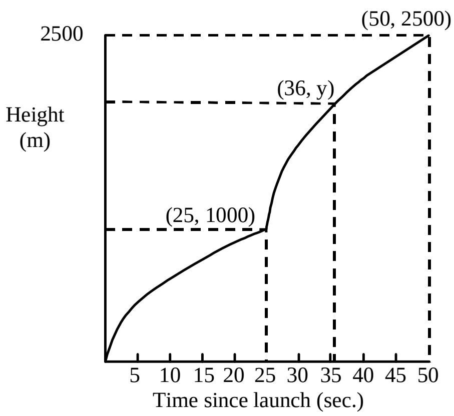
Therefore coordinates of centre of dark circle = $\left(0, -6 + \frac{9}{8}\right) = \left(0, -4\frac{7}{8}\right)$ or (0, -4.875)

Answer The coordinates of the centre of the dark circle are $\left(0, -4\frac{7}{8}\right)$ or (0, -4.875).

23 /4

Step 1: Find the height of the rocket at 25 seconds

$H(25) = 200\sqrt{25} = 200(5) = 1000$ m



Step 2: Find the equation of the 2nd stage

$H_2(t) = a\sqrt{x - h} + k$

Using (h, k) = (25, 1000), we get

$H_2(t) = a\sqrt{x - 25} + 1000$

Substituting (50, 2500), we get

$2500 = a\sqrt{50 - 25} + 1000$
$2500 = a\sqrt{25} + 1000$
$2500 = 5a + 1000$
$5a = 1500$
$a = 300$

Therefore, $H_2(t) = 300\sqrt{x - 25} + 1000$

11 seconds after the firing of the second stage (at 25 sec.) is 36 sec.

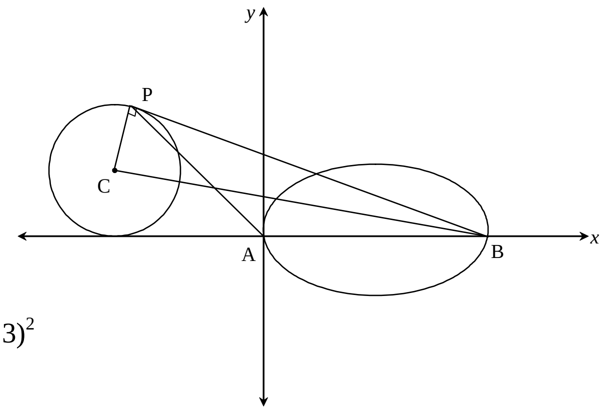
Step 3: Find the image of 36 in H_2

$$H_2(36) = 300\sqrt{36 - 25} + 1000 = 300\sqrt{11} + 1000 \approx 1994.99 \text{ m}$$

Answer Rounded to the nearest metre, the height of the rocket 11 seconds after firing of the 2nd stage was 1995 m.

25

The equation of the circle is $(x + 5)^2 + (y - 3)^2 = 9$, and the equation of the ellipse is $\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$. /4



Solution Total length of pipes

$= \text{Perimeter of } \Delta PAB = m\ \overline{PA} + m\ \overline{AB} + m\ \overline{BP}$

Step 1: Ellipse: centre is (5,0).

Since a = 5, the vertices are (0,0) and (10,0)

$\therefore m\ \overline{AB} = 10$

Step 2: Circle: centre is (-5, 3) and radius is 3

Step 3: $PB^2 + PC^2 = CB^2$ $\therefore PB^2 + 9 = (10 - -5)^2 + (0 - 3)^2$

$\therefore PB^2 + 9 = 225 + 9$

$\therefore PB = 15$

Step 4: In ΔPCB, $\tan \angle PBC = \frac{3}{15}$ $\therefore m\angle PBC = 11.31°$

$m\ \overline{QB} = m\ \overline{AB} + m\ \overline{QA} = 15$

In ΔQCB, $\tan \angle QBC = \frac{3}{15}$ $\therefore m\angle QBC = 11.31°$

$\therefore$ In ΔPAB, $m\angle PBA = 22.62°$

Step 5: $PA^2 = 10^2 + 15^2 - 2(10)(15)\cos 22.62°$

$\therefore m\ \overline{PA} = 6.93$

Step 6: Perimeter = 10 + 15 + 6.93 = 31.93

Answer Rounded to the nearest tenth, the total length of the irrigation pipes is 31.9 m.

EXAMINATION #3

Section A

Questions 1 to 13 *4 marks or 0 marks*

1	C	**8**	D
2	A	**9**	B
		10	C
4	D		
5	A		
6	C		
7	B		

Section B

14 Rounded to the nearest tenth $\left\| \vec{u} + \vec{v} \right\|$ is 16.6. *4 marks or 0 marks*

Section C

16 /4

cot P + tan P = cosec P sec P

$\dfrac{\cos P}{\sin P} + \dfrac{\sin P}{\cos P}$

$\dfrac{\cos^2 P + \sin^2 P}{\sin P \cos P}$

$\dfrac{\cos^2 P + \sin^2 P}{\sin P \cos P}$

cosec P sec P

17 /4

$2\cos^2 x - 3\sin x - 3 = 0$

$2(1 - \sin^2 x) - 3\sin x - 3 = 0$

$2 - 2\sin^2 x - 3\sin x - 3 = 0$

$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ $\quad$ $\sin x + 1 = 0$

$2\sin x = -1$ $\quad$ $\sin x = -1$

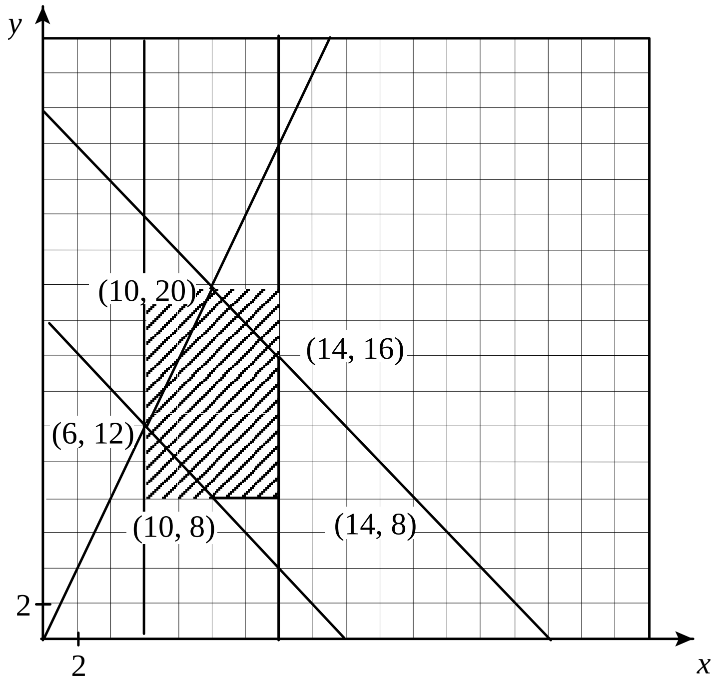
$\sin x = \dfrac{-1}{2}$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ $\quad$ $x = \dfrac{3\pi}{2}$

Answer The values of x in the domain are $\dfrac{11\pi}{6}$ and $\dfrac{3\pi}{2}$.

18 /4

x: number of supervisors
y: number of staff workers



Constraints : $x \geq 0, y \geq 0$
$x + y \leq 30$
$x + y \geq 18$
$x \geq 6$
$x \leq 14$
$y \geq 8$
$y \leq 2x$

Vertices of polygon of constraints:

(10, 8)	⇒	10(3500) + 8(1500) =	\$47 000
(6, 12)	⇒	6(3500) + 12(1500) =	\$39 000
(10, 20)	⇒	10(3500) + 20(1500) =	\$65 000
(14, 16)	⇒	14(3500) + 16(1500) =	\$73 000
(14, 8)	⇒	14(3500) + 8(1500) =	\$61 000

The minimum cost is \$39 000.
Answer The town should hire 6 supervisors and 12 staff workers in order to minimize its costs.

19 /4

Let t: time after 1995 (years)
V(t): value of the car (\$)

$V(t) = 17\,500(r)^t$ where r is the rate at which the value declines and

$$10\,000 = 17\,500(r)^3$$
$$0.5714285 = (r)^3$$
$$r \approx \sqrt[3]{0.571\,428}$$
$$r \approx 0.83$$

When $V(t) = 5000$

$$5000 \approx 17\,500(0.83)^t$$
$$\frac{5000}{17\,500} \approx (0.83)^t$$
$$t \approx \frac{\ln\left(\frac{5000}{17\,500}\right)}{\ln(0.83)}$$
$$t \approx 6.72$$

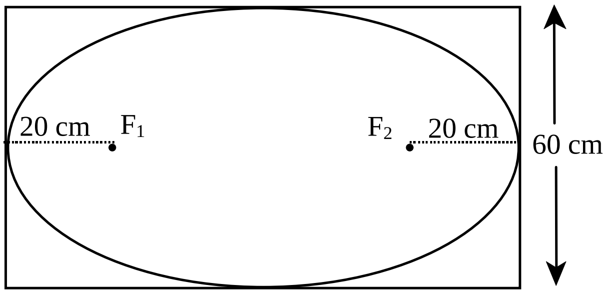
Answer The value of the car falls below \$5000 when it is 6.72 years ≈ 6 years 9 months.

20 Example of an appropriate solution

/4

Since width of rectangle is 60 cm,
minor axis measures 60 cm. So $b = 30$ cm.

If c represents distance between centre and focal point, then $c + 20$ equals the length of the semi-major axis, a.



Solving for c:

$$\begin{aligned} a^2 &= b^2 + c^2 \\ (c + 20)^2 &= 30^2 + c^2 \\ c^2 + 40c + 400 &= 900 + c^2 \\ 40c &= 500 \\ c &= 12.5 \end{aligned}$$

Therefore length of plywood is $2(c + 20) = 2(12.5 + 20) = 65$

Area of plywood: $65 \times 60 = 3900$

Answer Rounded to the nearest cm^2, the area of the plywood is 3900 cm^2.

21 Example of an appropriate solution

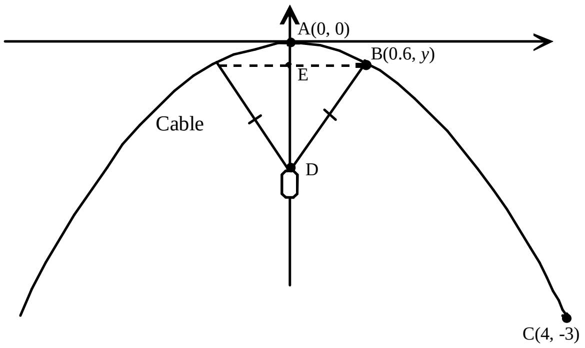
/4

$x^2 = 4py$, sub in (4, -3)

$p = \text{m } \overline{AD} = \dfrac{-4}{3}$

$x^2 = \dfrac{-16}{3} y$, let $x = 0.6$

$y = -0.0675$

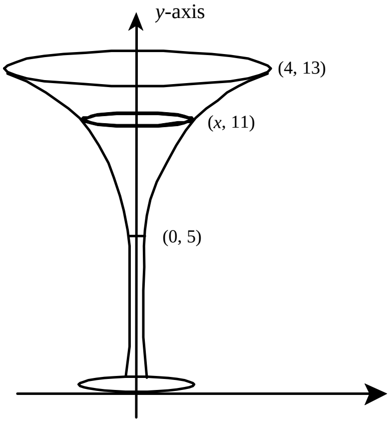


Therefore, $\text{m } \overline{ED} = \left| \dfrac{-4}{3} - (-0.0675) \right| = 1.2658\overline{3}$

Triangle DEB is right angled. Therefore, $\text{m } \overline{DB} = \sqrt{0.6^2 + (1.2658\overline{3})^2} = 1.4$ m

Answer Each cable is 1.4 m in length.

22 Example of an appropriate solution /4



The square root function must be in the form: $y = a\sqrt{x} + k$

Substituting (0, 5) we get: $5 = a\sqrt{0} + k$ So $k = 5$

Substituting (4, 13) we get: $13 = a\sqrt{4} + 5$ So $a = 4$

So the function is: $y = 4\sqrt{x} + 4$

At the ring, $y = 11$ $11 = 4\sqrt{x} + 5$ So $x = 2.25$

But 2.25 represents the radius of the gold ring in centimetres.

So the circumference is: $C = 2\pi\,(2.25) \approx 14.13$ cm

Therefore the gold ring will cost: $14.13 \times 2 = 28.26$ cents.

Answer Rounded to the nearest cent, the cost of the gold ring is 28 cents.

EXAMINATION #4

Section A

Questions 1 to 13 *4 marks or 0 marks*

1	B
2	D
3	C
4	D
5	D
6	A
7	A
8	B
9	C
10	B

Section B
Questions 14 to 16

14 The values of x belonging to $\left[0, \frac{3\pi}{2}\right]$ and that satisfy the equation are $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$ and $\frac{7\pi}{6}$. ___/4

Note *Give 3 marks if the student identifies 4 of the 5 values.*
Give 2 marks if the student identifies 3 of the 5 values.
Deduct 1 mark if the student expresses the values in degrees.

15 The scalar product of vectors u and v is 16. *4 marks or 0 marks* ___/4

Section C
Questions 17 to 25

17 Example of an appropriate method ___/4

Height at which the rockets explode

$h_1(6) = -12.5(6 - 4)^2 + 200 = 150$ metres

Value of parameter d

Since the two rockets explode at the same time and at the same height,

$h_2(6) = 150$

$$25\sqrt{\frac{6}{d}} + 50 = 150$$

$$25\sqrt{\frac{6}{d}} = 100$$

$$\sqrt{\frac{6}{d}} = 4$$

$$\frac{6}{d} = 16$$

$$d = \frac{6}{16} = \frac{3}{8}$$

Answer The value of parameter d is $\frac{3}{8}$.

18 Example of an appropriate method /4

Start-up time

$$-3\,|x-6| + 36 = 21$$
$$-3\,|x-6| = -15$$
$$|x-6| = 5$$
$$x - 6 = 5 \quad \text{or} \quad x - 6 = -5$$
$$x = 11 \quad \text{or} \quad x = 1$$

Since the start-up time is located to the left of the vertex on the graph, we must use the value of x that is less than 6.

The air conditioning system starts up 1 hour after sunrise.

Shutdown time

$$-3\,|x-6| + 36 = 20$$
$$-3\,|x-6| = -16$$
$$|x-6| = \frac{16}{3}$$
$$x - 6 = \frac{16}{3} \quad \text{or} \quad x - 6 = \frac{-16}{3}$$
$$x = \frac{34}{3} \quad \text{or} \quad x = \frac{2}{3}$$

Since the shutdown time is located to the right of the vertex on the graph, we must use the value of x that is greater than 6.

The air conditioning system stops $\frac{34}{3}$ hours after sunrise.

Time during which air conditioning system is in operation

$$\frac{34}{3} - 1 = 10.\overline{3} \text{ hours}$$

Answer On that day, the air conditioning system was in operation for 10 hours and 20 minutes or approximately 10.3 hours.

19 Example of an appropriate method /4

Value of parameter i

$C_o = 2000$
$C(t) = C_o(1 + i)^t$
$2662 = 2000(1 + i)^3$
$1.331 = (1 + i)^3$
$1.1 = 1 + i$
$0.1 = i$
$C(t) = 2000(1.1)^t$

Value after 10 years

$C(10) = 2000(1.1)^{10} \approx 5187.4849$

Answer Emily's investment will be worth \$5187.48 after 10 years.

20 Example of an appropriate method /4

Form of the equation of the hyperbola

$$-\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Parameters h and k

The coordinates of A are A(0, 0).
The coordinates of C are C(12, 0), because the circle is 12 units in diameter.
The coordinates of point B are B(6, 0), because it is the midpoint of $\overline{AC}$.
Since point B is the centre of the hyperbola, h = 6 and k = 0.

Parameter a and b

$2b = m\,\overline{ED} = 12$ therefore, $b = 6$

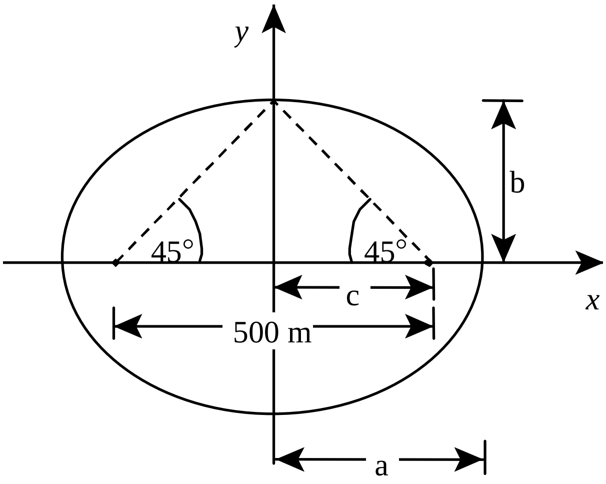
The distance 2c between the foci is 20 units; therefore, c = 10.

$a^2 + b^2 = c^2$
$a^2 + 6^2 = 10^2$
$a = 8$

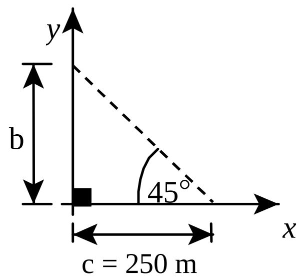
Answer The equation of the hyperbola with vertices D and E is $-\frac{(x-6)^2}{64} + \frac{y^2}{36} = 1$.

21 Example of an appropriate method /4

Diagram of the situation



Value of b



The distance c between each focus and the centre of the ellipse is 250 m.

The triangle shown on the right is an isosceles triangle. Hence, b = 250 m.

Value of a

$a^2 = b^2 + c^2$

$a^2 = 250^2 + 250^2$

$a = \sqrt{125\ 000} \approx 353.553$ m

Distance between Luke and the track

$a - c = \sqrt{125\ 000} - 250 \approx 103.553$ metres

Answer Rounded to the nearest tenth, the shortest distance between Luke and the track is 103.6 metres.

22 Example of an appropriate method /4

$$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$$

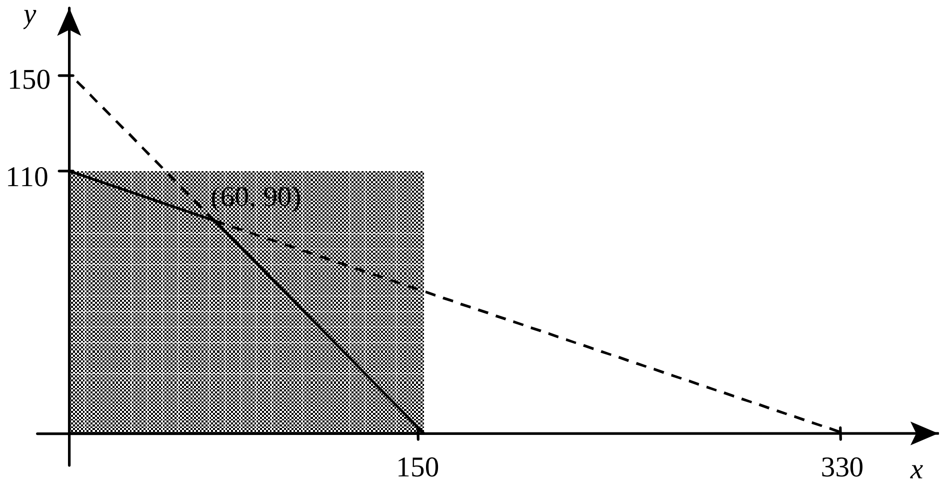
$$\overrightarrow{PO} = \overrightarrow{PD} + \overrightarrow{DO} = \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$$

Vectors MN and PO are therefore parallel and of equal length.

Quadrilateral MNOP is therefore a parallelogram.

23 Example of an appropriate method /4

Polygon of constraints



Function to be maximized

Profit = cx + 2cy where c represents the cost of renting a campsite for a tent.

Vertex	Profit = cx + 2cy
(0, 0)	0
(150, 0)	150c
(60, 90)	240c ← maximum
(0, 110)	220c

Value of c

Maximum profit 240c = $4800; therefore c = $20

Answer It cost $20 per day to rent a campsite for a tent.
It cost $40 per day to rent a campsite for a trailer.