Chapter 4: 2-D Kinematics (Projectile Motion)

Trajectory

The trajectory of an object is its apparent path. The apparent path depends and the location and motion of the observer (i.e. it's all relative!)

Examples: -frame of reference

1. A soccer ball is kicked up from the ground. The player kicks the ball straight ahead of him.

    a. What is the apparent motion of the ball according to the goalie who stands directly in front of the player.

    Straighup, Straight down

    b. What is the apparent motion of the ball according to a stationary bird (let's pretend) above the field

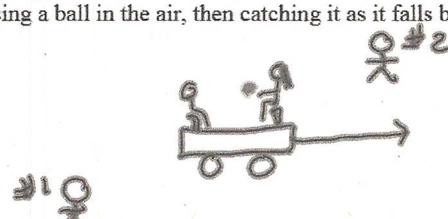
    

    (straight side ways)

    c. What is the apparent motion of a ball according to a fan standing on the sideline?

2. A girl is sitting on a train that moves at a constant velocity as is illustrated below. She is tossing a ball in the air, then catching it as it falls back down.



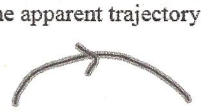
    a. What is the apparent trajectory of the ball for the girl?

    straight up and down

    b. What is the apparent trajectory of the ball for boy who sits across from her on the train?

    Straight up + down

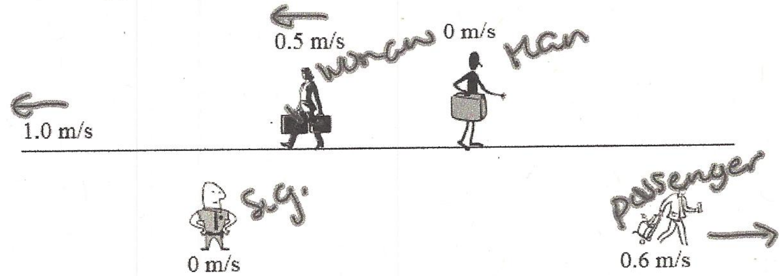
    c. What is the apparent trajectory of the ball for observer 1?

    d. What is the apparent trajectory of the ball for observer 2?

3. At the airport, a man and a woman are on a moving sidewalk. The moving sidewalk moves at a speed of 1.0 m/s. The man is stationary while the woman walks at a speed of 0.5 m/s. An airport security guard is standing on the side of the moving sidewalk. A passenger is walking at a speed of 0.6 m/s, in the direction opposite to the moving sidewalk.



a. Relative to the stationary man, what is the speed of the woman?

0.5 m/s

b. Relative to the security guard, what is the speed of the man?

1.0 m/s

c. Relative to the security guard, what is the speed of the woman?

$1.0\frac{m}{s} + 0.5\frac{m}{s} = 1.5\frac{m}{s}$

d. Relative to the passenger, what is the speed of the woman?

$1.0\frac{m}{s} + 0.5\frac{m}{s} + 0.6\frac{m}{s} = 2.1\frac{m}{s}$

e. Relative to the man, what is the speed of the passenger?

$1.0\frac{m}{s} + 0.6\frac{m}{s} = 1.6\frac{m}{s}$

f. Relative to the man, what is the speed of the security guard?

$1.0\frac{m}{s}$

So, what is projectile motion?

- The study of the motion of projectiles
- 2-D kinematics deals with objects that move vertically and horizontally at the same time.

Independence of motion

When we study the motion of projectlie, we look at the vertical and horizonatl motions seprately.

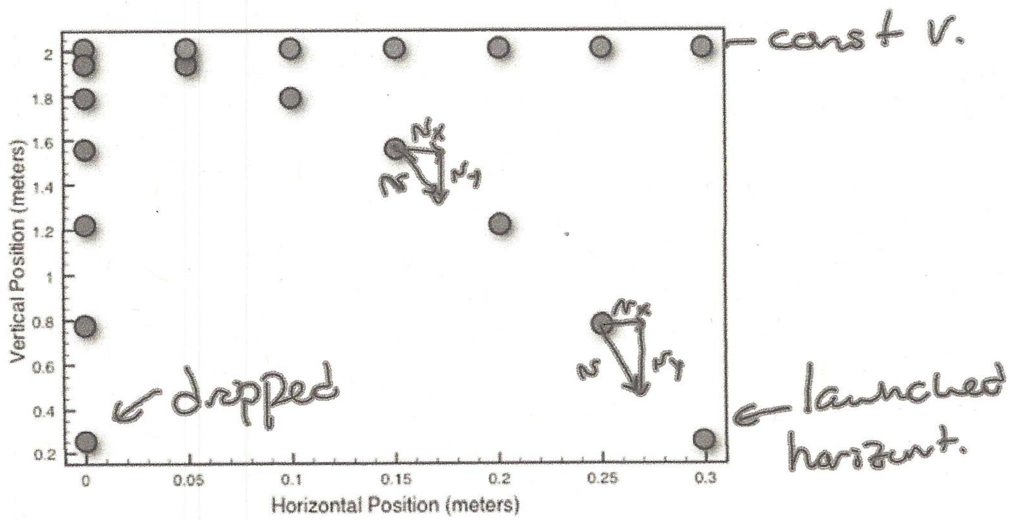
The horizontal and vertical motions do not affect each other; i.e. they are independent.

The link between the horizontal and vertical motions is TIME. Because the projectile travels the horizontal and vertical distances AT THE SAME TIME, Δt is the same for both the horizontal and vertical motions.

In order to solve these problems, we will use the same equations we just learn in the previous chapter.

Case 1: Objects Launched Horizontally

The diagram below illustrates the motion of a projectile launched horizontally. It is also compared to the motion of an object dropped, and to constant horizontal motion.



Horizontal Motion: Constant velocity $\Delta d = v \Delta t$

Vertical Motion: acceleration due to gravity

• 5 eq

* $a_y = 9.8 \frac{m}{s^2}$ (down)

* $v_{iy} = 0$

Examples:

1. A car drives off the edge of a cliff at a speed of 15.0 m/s. The car hits the bottom of the cliff 45.0 m from the edge. How high is the cliff?

Vert

$a = -9.8 \frac{m}{s^2}$

$v_{iy} = 0$

$\Delta d_y = ?$

$\Delta t = 3.00 s$

$\Delta d_y = \cancel{v_{iy} \Delta t}^{\,0} + \frac{1}{2} a (\Delta t)^2$

$= \frac{1}{2} (-9.8 \frac{m}{s^2})(3.00 s)^2$

$= -44.1 m$

Ans: 44.1 m

Hor

$v_x = 15.0 \frac{m}{s}$

$\Delta d_x = 45.0 m$

$\Delta t = ?$

$\Delta d = v_x \Delta t$

$\Delta t = \frac{\Delta d}{v} = \frac{45.0 m}{15.0 m/s}$

$= 3.00 s$

2. A marble rolls off the edge of a table 1.5 m high. It hits the ground 2.0 m from the edge of the table. With what speed did the marble roll off the table?

Vert

$\Delta d_y = -1.5 m$

$a = -9.8 m/s^2$

$v_{iy} = 0$

$\Delta t = ?$

$\cancel{m} \times \frac{s^2}{\cancel{m}}$

$\Delta d_y = \cancel{v_{iy} \Delta t}^{\,0} + \frac{1}{2} (a)(\Delta t)^2$

$\Delta d_y = \frac{1}{2} a (\Delta t)^2$

$(\Delta t)^2 = \frac{2 \Delta d_y}{a_y}$

$= \frac{2(-1.5 m)}{-9.8 \frac{m}{s^2}}$

$(\Delta t)^2 = 0.306... s^2$

$\Delta t = 0.55 s$

Hor

$\Delta d_x = 2.0 m$

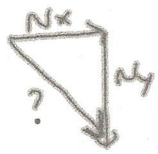
$\Delta t = 0.55 s$

$v_x = ?$

$v_x = \frac{\Delta d_x}{\Delta t}$

$= \frac{2.0 m}{0.55 s}$

$= 3.6 \frac{m}{s}$



3. A cat runs off a 2.0 m high balcony while running at a speed of 4.0 m/s. Luckily, the cat lands on its paws and is not injured. What is the velocity of the cat when it hits the ground?

$v_x = 4.0\,m/s$

$mag = \sqrt{(4.0\,m/s)^2 + (6.26\,m/s)^2}$

$= 7.4\,m/s$

$\theta = \tan^{-1}\left(\frac{6.26}{4.0}\right)$

$= 57°$

$\boxed{7.4\,m/s,\ -57°}$

Vert

$v_i = 0$

$a = -9.8\,m/s^2$

$\Delta d = -2.0\,m$

$v_f = ?$

$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta d$

$= 2(-9.8\,\frac{m}{s^2})(-2.0\,m)$

$v_f^2 = 39.2\,\frac{m^2}{s^2}$

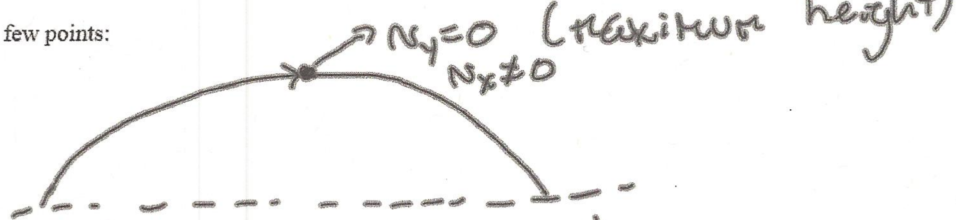
$v_y = \pm 6.26\,\frac{m}{s}$

Case 2: Objects Launched at an Angle

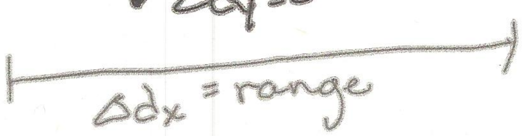
Similar to "objects launched at an angle", except $v_{iy} \neq 0$.

When objects are launched at an angle, the initial velocity has BOTH vertical and horizontal components.

A few points:

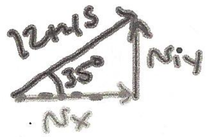


* from the ground to ground
- $v_{iy} = -v_{fy}$
- $\Delta d_y = 0$



Splitting the initial velocity:

Ex: A soccer ball is kicked at a speed of 12 m/s at an angle of 35° above the horizontal. Find the horizontal and ~~horizontal~~ vertical components of the initial velocity?



$$\cos 35° = \frac{v_x}{12\,m/s}$$

$$v_x = 12\,m/s \cos 35° = 9.83\,\frac{m}{s}$$

$$\sin 35° = \frac{v_{iy}}{12\,m/s}$$

$$v_{iy} = 12\,\frac{m}{s} \sin 35° = 6.88\,\frac{m}{s}$$

Examples:

*Always keep at least 1 extra sig fig for intermediate steps

1. Julia kicks a soccer ball, giving it an initial velocity of 15 m/s at an angle of 25 ° above the ground. How far from where she kicked it will the ball hit the ground?

$$v_x = 15\,\frac{m}{s}\cos 25° = 13.6\,m/s$$

$$v_{iy} = 15\,\frac{m}{s}\sin 25° = 6.34\,m/s$$

Hori

$\Delta d = ?$

$v_x = 13.6\,m/s$

$\Delta t = 1.29\,s$

$\Delta d = v_x \Delta t$

$= (13.6\,\frac{m}{s})(1.29\,s)$

$= \underline{\underline{18\,m}}$

Vert

$v_{iy} = 6.34\,m/s$

$a = -9.8\,m/s^2$

$\Delta t = ?$

$\Delta d_y = 0$

(or $v_f = -6.34\,m/s$)

$\cancel{\Delta d}^{\,0} = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$

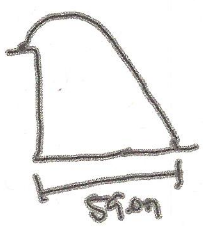
$-v_i \cancel{\Delta t} = \frac{1}{2}a(\Delta t)^{\cancel{2}}$

$\Delta t = -\frac{2v_i}{a}$

$= \frac{-2(6.34\,m/s)}{-9.8\,m/s^2}$ $\quad \frac{\cancel{m}}{\cancel{s}} \times \frac{s^{\cancel{2}}}{\cancel{m}}$

$= 1.29\,s$

2. A ball is kicked from the roof of a building. The ball leaves the kicker's foot with a velocity of 22 m/s at an angle of 40 ° above the horizontal. The ball hits the ground 59.0 m away from the edge of the building. How tall is the building?



$v_x = 22\frac{m}{s}\cos 40°$ $= 16.9\frac{m}{s}$

$v_{iy} = 22\frac{m}{s}\sin 40°$ $= 14.1\frac{m}{s}$

Hori

$v_x = 16.9\,m/s$

$\Delta d_x = 59.0m$

$\Delta t = ?$

$\Delta d_x = v_x \Delta t$

$\Delta t = \frac{\Delta d_x}{v_x}$

$= \frac{59.0m}{16.9\,m/s}$ $\cancel{m} \times \frac{s}{\cancel{m}}$

$= 3.49s$

Vert

$\Delta t = 3.49\,s$

$v_{iy} = 14.1\,m/s$

$a = -9.8\,m/s^2$

$\Delta d_y = ?$

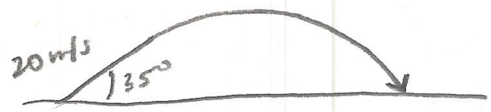
$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a(\Delta t)^2$

$= (14.1\frac{m}{s})(3.49)$

$+ \frac{1}{2}(-9.8\frac{m}{s^2})(3.49s)^2$

$\Delta d_y = -10.473m$

Ans: 10.m



3. A ball is kicked from the ground with an initial speed of 20.0 m/s at an angle of 35 ° above the horizontal.

   a. What is the maximum height reached by the projectile?

$V_{iy} = 20 \sin 35°$
$= 11.47 \, m/s$
$V_{ix} = 20 \cos 35°$
$= 16.38 \, m/s$

y: $V_{iy} = 11.47 \, m/s$
$V_{fy} = 0 \, m/s$
$a = -9.8 \, m/s^2$
$\Delta d_y = ?$

$V_f^2 = V_i^2 + 2a \Delta d$

$\Delta d = \dfrac{V_f^2 - V_i^2}{2a}$

$= \dfrac{0 - (11.47)^2}{2(-9.8)} = 6.71 \, m$

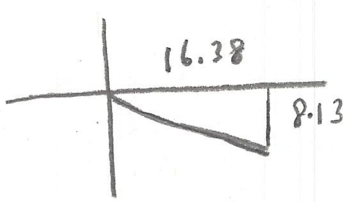
   b. What is the velocity of the projectile, 2.0 s after it was kicked?

$V_{fx} = 16.38 \, m/s$
(velocity is constant in horizontal direction)

$V_{iy} = 11.47 \, m/s$
$\Delta t = 2.0 \, s$
$a = -9.8 \, m/s^2$
$V_{fy} = ?$

$V_f = V_i + a \Delta t$

$V_f = 11.47 + (-9.8)(2.0)$
$= -8.13 \, m/s$



$mag = \sqrt{(16.38)^2 + (8.13)^2}$
$= 18.28$

$\theta = \tan^{-1}\left(\dfrac{8.13}{16.38}\right) = 26.4°$

$V = 18.3 \, \dfrac{m}{s}, \ -26.4°$