Dr. Rossy

Chapter 3: 1-D Kinematics (Equations of Motion)

We can study motion even if we do not have a graph. In fact we often don't have a graph. In that case, we are going to use equations to study motion.

U A R M

In this chapter, we will study uniformly accelerated rectilinear motion. It is motion in a straight line, with 2 possible directions (ex: forward/backward, up/down) with constant acceleration.

The equations we will use in this chapter are only valid for **uniformly accelerated motion** (i.e. constant acceleration).

The five quantities

When studying motion over a certain period of time, we work with 5 physical quantities (5 variables). They are:

Δt : change in time or time interval (seconds)

Δd : displacement (m)

v_i : initial velocity (beginning of time) (m/s)

v_f : final velocity (end of time) (m/s)

a : acceleration (m/s²)

Note: We will not be using vector notation here, but we are still going to use "+" and "-" to distinguish between the 2 different possible directions.

The 5 equations:

We will derive the equations relating the above quantities (4 at a time).

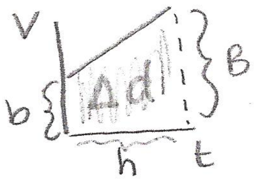
First equation: The definition of acceleration.

$$a = \frac{\Delta v}{\Delta t} \qquad a = \frac{v_f - v_i}{\Delta t}$$

$$a\Delta t = v_f - v_i$$

$$\boxed{v_f = a\Delta t + v_i} \longrightarrow \text{No } \Delta d$$

Second Equation: ...



$$\Delta d = \text{area} = \frac{(B+b)h}{2} = \frac{(v_f + v_i)\Delta t}{2}$$

Examples:

$$\Delta d = \frac{(v_f + v_i)\Delta t}{2}$$

1. A car is going at a constant speed of 12 m/s. It then accelerates for 5.0 s at a rate of 3.0 m/s^2. What is the final velocity of the car?

$v_i = 12$ m/s
$a = 3.0$ m/s^2
$\Delta t = 5.0$ s
$v_f = ?$

$v_f = v_i + a\Delta t$
$v_f = 12 \text{ m/s} + 3.0 \text{ m/s}^2 (5.0 \cancel{s})$
$v_f = 27$ m/s^2

2. A cyclist decelerated from 15.0 m/s to 5.0 m/s at a rate of 2.0 m/s^2. How long did this take?

$a = -2.0$ m/s^2
$v_f = 5.0$ m/s
$v_i = 15.0$ m/s
$\Delta t = ?$

$v_f = v_i + a\Delta t$

$\frac{v_f - v_i}{a} = \Delta t \rightarrow \Delta t = \frac{v_f - v_i}{a}$

$\frac{\text{m/s}}{\text{m/s}^2} = \frac{\cancel{m}}{\cancel{s}} \times \frac{s^{\cancel{2}}}{\cancel{m}}$

$\Delta t = \frac{5.0 \text{ m/s} - 15.0 \text{ m/s}}{2.0 \text{ m/s}^2}$

$\Delta t = 5.0$ s

3. A bus accelerates from rest over a distance of 125 m. What is the velocity of the bus after the acceleration if this took10.0 s?

$a = \emptyset$
$v_f = ?$
$v_i = 0$ m/s
$\Delta t = 10.0$ s
$\Delta d = 125$ m

$\Delta d = \frac{(v_f - v_i)\Delta t}{2}$

$\frac{2\Delta d}{\Delta t} = v_f - v_i$

$\frac{2\Delta d}{\Delta t} + v_i = v_f$

$\frac{2(125\text{m})}{10\text{s}} + 0 \text{ m/s} = v_f$

$25 \text{ m/s} = v_f$

Third Equation: Finding an equation that doesn't have v_f.

In the equation $\Delta d = \frac{(v_i+v_f)\Delta t}{2}$, replace v_f by $v_f = v_i + a\Delta t$

$$\Delta d = \frac{(V_i + V_i + a\Delta t)\Delta t}{2}$$

$$\Delta d = \frac{(2V_i + a\Delta t)\Delta t}{2}$$

$$\Delta d = \frac{2V_i\Delta t}{2} + \frac{a\Delta t^2}{2}$$

$$\Delta d = V_i\Delta t + \frac{a\Delta t^2}{2}$$

$$\Delta d = V_i\Delta t + \frac{1}{2}a\Delta t^2$$

Fourth Equation: Finding an equation that doesn't have v_i.

In the equation $\Delta d = \frac{(v_i+v_f)\Delta t}{2}$, replace v_i by $v_i = v_f - a\Delta t$.

$$\Delta d = \frac{(V_f - a\Delta t + V_f)\Delta t}{2}$$

$$\Delta d = \frac{(2V_f - a\Delta t)\Delta t}{2}$$

$$\Delta d = \frac{2V_f\Delta t}{2} - \frac{a\Delta t^2}{2}$$

$$\Delta d = V_f\Delta t - \frac{a\Delta t^2}{2}$$

$$\Delta d = V_f\Delta t - \frac{1}{2}a\Delta t^2$$

$$\Delta d = \frac{(V_i + V_f)\Delta t}{2}$$

Replace Δt with $\Delta t = \frac{V_f - V_i}{a}$ → from equation 1

$$\Delta d = \frac{(V_i + V_f)}{2}\frac{(V_f - V_i)}{a}$$

difference of squares.

$$2a\Delta d = (V_i + V_f)(V_f - V_i) \rightarrow \cancel{V_i V_f} - V_i^2 + V_f^2 - \cancel{V_i V_f}$$

$$2a\Delta d = V_f^2 - V_i^2$$

$$V_f^2 = V_i^2 + 2a\Delta d$$

<u>Examples:</u>

1. A cyclist is traveling at constant speed of 4.0 m/s. He then accelerates at a rate of 0.40 m/s^2 for 5.0 s. What distance does he cover while he accelerates?

$V_i = 4.0$ m/s

$a = 0.40$ m/s^2

$\Delta t = 5.0$ s

$\Delta d = ?$

$$\Delta d = V_i \Delta t + \frac{1}{2}a\Delta t^2 \qquad \frac{m}{\cancel{s^2}} \cdot \cancel{s^2}$$

$$\Delta d = (4.0\,m/\cancel{s})(5.0\,\cancel{s}) + \frac{1}{2}(0.40\,m/s^2)(5.0\,s)^2$$

$\Delta d =$ 25 m

2. A driver going 25 m/s sees a stalled car 125 m in front of him. At what rate must the car decelerate if he wants to avoid a crash? STOP

0 m/s

$V_i = 25$ m/s

$\Delta d = 125$ m

$V_f = 0$ m/s

$a = ?$

Eq. 5

no Δt

$\cancel{V_f^2} = V_i^2 + 2a\Delta d$

$\frac{-V_i^2}{2\Delta d} = \frac{2a\Delta d}{2\Delta d}$

$\frac{-V_i^2}{2\Delta d} = a$

$\frac{-(25\frac{m}{s})^2}{2(125m)} = a$

$\frac{-625}{250} = a$

$-2.5 = a$
m/s²

3. A bus starts from rest and accelerates at rate of 1.5 m/s^2. How long does it take this bus to cover 55 m?

$V_i = 0$ m/s ✓

$a = 1.5$ m/s² ✓

$\Delta d = 55$m ✓

$\Delta t = ?$ ✓

no V_f

$\Delta d = \cancel{V_i \Delta t} + \frac{1}{2}a\Delta t^2$

$\frac{2\Delta d}{a} = \frac{\frac{1}{2}\cancel{a}\Delta t^2}{\cancel{a}}$

$\frac{2\Delta d}{a} = \Delta t^2$

$\sqrt{\frac{2\Delta d}{a}} = \sqrt{\Delta t^2}$

$\sqrt{\frac{2\Delta d}{a}} = \Delta t$

$\Delta t = \sqrt{\frac{2(55m)}{1.5 \text{ m/s}^2}}$

$\Delta t = 8.6$s

$m \times \frac{s^2}{m}$

4. How long does it take a runner to cover 23.5 m ... m/s and accelerates at a rate of 1.0 m/s^2?

$\Delta d = 23.5 m$

$V_i = 2.2 m/s$

$a = 1.0 m/s^2$

$V_f = $ —

$\Delta t = ?$

$\Delta d = V_i \Delta t + \frac{1}{2} a \Delta t^2$

$Ax^2 + Bx + C = 0$

$\frac{1}{2} a \Delta t^2 + V_i \Delta t + \Delta d = 0$

$\underbrace{\frac{1}{2}(1.0 m/s)\Delta t^2}_{A} + \underbrace{2.2 \frac{m}{s}(\Delta t)}_{B} \underbrace{-23.5m}_{C} = 0$

$0.5 \frac{m}{s^2}(\Delta t)^2 + 2.2 \frac{m}{s}(\Delta t) - 23.5 m = 0$

ON THE FORMULA SHEET.

$* \ \Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta t = \frac{-2.2 \pm \sqrt{(2.2)^2 - 4(0.5)(23.5)}}{2(0.5)}$

$\Delta t = \frac{-2.2 \pm \sqrt{51.84}}{2(0.5)}$

$\Delta t = \frac{-2.2 \pm 7.2}{2(0.5)}$

$\frac{-2.2 + 7.2}{2(0.5)} = 5 sec$ $\qquad$ $\frac{-2.2 - 7.2}{2(0.5)} = \cancel{-9.4 \text{ sec}}$

Summary of equations

$$v_f = v_i + a\Delta t$$

$$\Delta d = \left(\frac{v_i + v_f}{2}\right)\Delta t$$

$$\Delta d = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$\Delta d = v_f\Delta t - \frac{1}{2}a(\Delta t)^2$$

$$v_f^2 = v_i^2 + 2a\Delta d$$

FORMULA SHEET

Special Case 1: Constant Velocity

When the velocity is constant, $a = 0$.

Equations 1 and 5 simplify to $v_f = v_i$

Equations 2, 3, 4 simplify to $\Delta d = v\Delta t$ (since $v = v_i = v_f$)

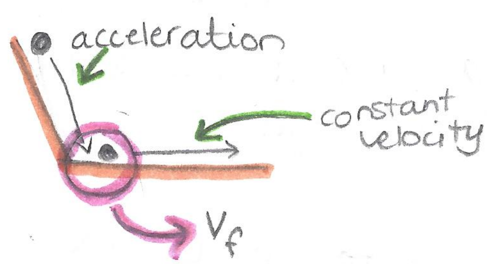
$\Delta d = v\Delta t$

This only works when theres constant velocity.

So when dealing with constant velocity, we only need one equation:

$$\Delta d = v\Delta t$$

Example: A marble accelerates from rest at a rate of 0.25 m/s^2 down an incline that is 1.5 m long. When it reaches the bottom of the incline, the marble continues on a flat, frictionless surface for 6.0 s. What distance does the marble cover on the flat surface?



Incline

$\Delta d = 1.5\,m$

$a = 0.25\,m/s^2$

$V_i = 0\,m/s$

$V_f = ?$

$V_f^2 = V_i^2 + 2a\Delta d$

$V_f^2 = 0 + 2(0.25\,m/s)(0.5)$

$\sqrt{V_f^2} = \sqrt{0.75\,\frac{m^2}{s^2}}$

$V_f = 0.87\,m/s$

Flat Surface

$\Delta t = 6.0s$

$V_i = 0.87\,m/s$

$\Delta d = ?$

$\Delta d = v\Delta t$

$\Delta d = (0.87\,\frac{m}{s})(6.0s)$

$\boxed{\Delta d = 5.22\,m}$

Special Case 2: Acceleration due to Gravity (Vertical motion)

Free fall: Objects are in freefall when gravity is the only force acting on the object.

Objects in free fall have a constant acceleration. This acceleration is due to gravity.

Acceleration due to gravity is a vector.

Its **magnitude** is 9.8 m/s^2.

Its **direction** is DOWN (toward the center of the Earth).

Note: We can choose + to be up or down. We have to make sure we are always consistent with our signs (i.e. all values going "up" have the same sign, all values going "down" have the same sign).

Different possible (useful) situations

An object is dropped

$v_i = 0$ m/s
no change in direction
choose down to be +ve or -ve

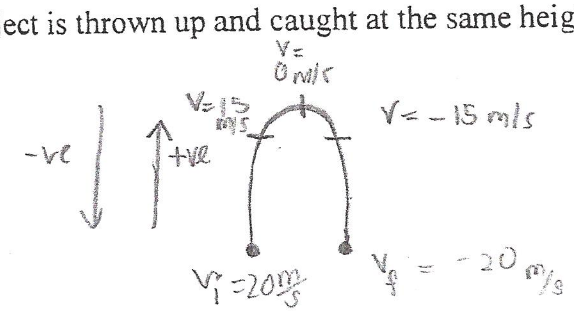
An object is thrown down

v_i = NOT ZERO
no change in direction
v_i + a are in the same direction
∴ same sign.

An object is thrown up

$v_i \neq 0$ m/s → up direction
a = acting down
↑ +ve , ↓ -ve

An object is thrown up and caught at the same height.



d_i

$\downarrow$

d_f

Examples:

1. You drop a rock from a window 6.00 m above the ground. How long does it take for the rock to hit the ground?

$\uparrow \oplus$

$\downarrow \ominus$

$\Delta d = (-6.00 m)$

$a = (-9.8 m/s^2)$

$V_i = 0 m/s$

$\Delta t = ?$

$\Delta d = \cancel{V_i \Delta t} + \frac{1}{2} a (\Delta t)^2$

$\Delta d = \frac{1}{2} a (\Delta t)^2$

$\frac{2 \Delta d}{a} = \Delta t^2$

$\frac{2(-6.00 m)}{-9.8 \frac{m}{s^2}} = \Delta t^2$

$\cancel{m} \times \frac{s^2}{\cancel{m}}$

$1.11 s = \Delta t$

2. A rock is launched directly upward from the ground at a speed of 12.0 m/s. What is the maximum height reached by the rock?

$\uparrow +$

$\downarrow -$

$a = (-9.8 m/s^2)$

$V_i = 12.0 m/s$

$V_f = 0 m/s$

$\Delta d = ?$

$\cancel{V_f^2} = V_i^2 + 2 a \Delta d$

$-2 a \Delta d = V_i^2$

$\Delta d = \frac{V_i^2}{-2a}$

$\frac{m^{\cancel{2}}}{\cancel{s^2}} \times \frac{\cancel{s^2}}{\cancel{m}}$

$\Delta d = \frac{(12.0 m/s)^2}{-2(-9.8 m/s^2)}$

$\Delta d = 7.35 m$

3. You toss a ball up in the air, giving it an initial speed of 8.00 m/s ... ball at the same height as you tossed it. How long does the ball stay in the air?

↑ +

↓ −

$V_i = 8.00\ m/s$

$a = -9.8\ m/s^2$

$\Delta d = 0\ m$

$\Delta t = ?$

$\Delta d = V_i \Delta t + \frac{1}{2} a \Delta t^2$

$\frac{1}{2} a \Delta t^2 + V_i \Delta t + \Delta d = 0$

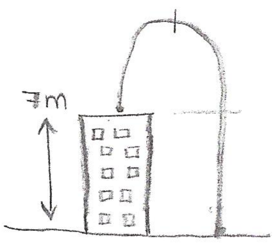
$\frac{1}{2}(-9.8\ m/s^2)(\Delta t^2) + 8(\Delta t) = 0$

$-4.9 \Delta t^2 + 8 \Delta t = 0$

$\Delta t = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

$\cancel{\frac{-8+8}{-9.8} = 0\ s}$ $\qquad \frac{-8-8}{-9.8} = \underline{1.63\ s}$ $\qquad \frac{-8 \pm \sqrt{(8^2)+4(-4.9)(0)}}{2(-4.9)} = \frac{8 \pm 8}{-9.8}$

4. Standing on the edge of the roof of a building 7.00 m tall, you toss a ball straight up at a speed of 5.00 m/s. How fast will the ball be moving when it hits the ground?



↑ +

↓ −

$V_i = 5\ m/s$

$a = -9.8\ m/s^2$

$\Delta d = -7\ m$

$V_f = ?$

$V_f^2 = V_i^2 + 2a\Delta d$

$V_f^2 = (5\ \frac{m}{s})^2 + 2(-9.8\frac{m}{s^2})(-7m)$

$V_f^2 = 25 + 137.2)$

$V_f^2 = 162.2\ m^2/s^2$

$V_f = \pm 12.7\ m/s$

$V_f = -\ 12.7\ m/s$

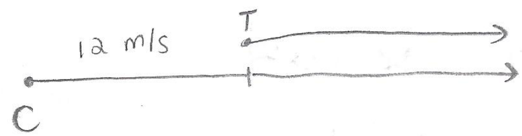
Catch-up/Meeting Problems

There is no single way of soving these problems. However, here are a few starting points:

- Use x and y for your unknowns (for both bodies in motion).
- Ask yourself what is the same for both bodies $\longrightarrow$?

Examples:

1) A car moving at 12 m/s passes a truck that is stationary. At that moment, the truck begins to accelerate at 2.0 m/s^2. How long does it take for the truck to catch-up to the car?



(1) Car

$v = 12\ m/s$
$\Delta d = x$
$\Delta t = y$

formula

Truck

$a = 2.0\ m/s^2$
$v_i = 0\ m/s$
$\Delta d = x$
$\Delta t = y$

(2) $\Delta d = vt$

$x = (12\ m/s)(y)$

System of Equations

(2) $\Delta d = \cancel{v_i \Delta t} + \frac{1}{2} a \Delta t^2$

$x = \frac{1}{2}\left(2.0\ \frac{m}{s^2}\right) y^2$

$x = x$

$12\ \frac{m}{s}\ y = \frac{1}{2}\left(2.0\ \frac{m}{s^2}\right) y^2$

$\frac{12\ \frac{m}{s}\ \cancel{y}}{\cancel{y}} = \frac{1\ \frac{m}{s^2}\ y^{\cancel{2}}}{y}$

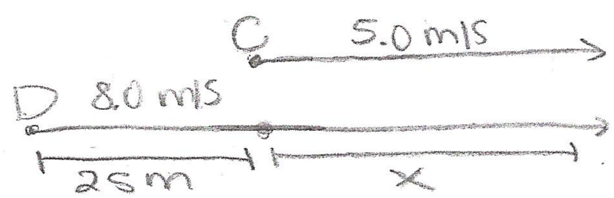
$\frac{12\ \frac{m}{s}}{1\ m/s^2} = \frac{1\ \frac{m}{s^2}\ y}{1\ m/s^2}$

units

$\frac{\cancel{m}}{\cancel{s}} \div \frac{s^{\cancel{2}}}{\cancel{m}}$

$y = 12\ s$

The cat has a 25 m head start. How far will the dog run before



Cat

$V = 5.0 m/s$

$\Delta d = x$

$\Delta t = y$

Dog

$V_i = 8.0 m/s$

$\Delta d = x + 25$

$\Delta t = y$

$\Delta d = vt$

$x = 5 \frac{m}{s} y$

$\Delta d = vt$

$x + 25 = 8 \frac{m}{s} y$

$$5 \frac{m}{s} y + 25 = 8 \frac{m}{s} y$$

$$\frac{25 m}{30 m/s} = \frac{\cancel{3.0} \cancel{\frac{m}{s}} y}{\cancel{3.0} \cancel{m/s}}$$

$$8.33 sec = y$$

Dog

$$\Delta d = (8.0 \frac{m}{s})(8.33 sec)$$

$$\Delta d = 67 m$$