Square Root Inequality   Example

$$3 + \sqrt{5x-10} \leq 8$$

$$\sqrt{5x-10} \leq 5$$

Step 1: domain $5x-10 \geq 0$

let $5x-10 = 0$

$5x = 10$

$x = 2$ Critical point $x = 2$

Step 2: $\sqrt{5x-10} = 5$

$5x-10 = 25$

$5x = 35$

$x = 7$ Critical point $x = 7$

Step 3:   number line + test point

Step 4:   Solution

$$2 \leq x \leq 7$$

or $[2, 7]$

Square Root Inequalities - problems

① $\sqrt{b-7} + 6 \le 12$

② $\sqrt{3a+3} - 1 \le 2$

③ $\sqrt{3x+6} + 2 \le 5$

④ $6 - \sqrt{2y+1} < 3$

Solution

Solve each inequality.

① 16. $\sqrt{b-7}+6 \le 12$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $b-7 \ge 0$.

$$b-7 \ge 0$$
$$b \ge 7$$

Solve $\sqrt{b-7}+6 \le 12$.

$$\sqrt{b-7}+6 \le 12$$
$$\sqrt{b-7} \le 6$$
$$\left(\sqrt{b-7}\right)^2 \le 6^2$$
$$b-7 \le 36$$
$$b \le 43$$

The solution region is $7 \le b \le 43$.

② 18. $\sqrt{3a+3}-1 \le 2$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $3a+3 \ge 0$.

$$3a+3 \ge 0$$
$$3a \ge -3$$
$$a \ge -1$$

Solve $\sqrt{3a+3}-1 \le 2$.

$$\sqrt{3a+3}-1 \le 2$$
$$\sqrt{3a+3} \le 3$$
$$\left(\sqrt{3a+3}\right)^2 \le 3^2$$
$$3a+3 \le 9$$
$$3a \le 6$$
$$a \le 2$$

The solution region is $-1 \le a \le 2$.

③

20. $\sqrt{3x+6}+2\le 5$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $3x+6\ge 0$.

$$3x+6\ge 0$$
$$3x\ge -6$$
$$x\ge -2$$

Solve $\sqrt{3x+6}+2\le 5$.

$$\sqrt{3x+6}+2\le 5$$
$$\sqrt{3x+6}\le 3$$
$$\left(\sqrt{3x+6}\right)^2\le 3^2$$
$$3x+6\le 9$$
$$3x\le 3$$
$$x\le 1$$

The solution region is $-2\le x\le 1$.

④ 22. $6-\sqrt{2y+1}<3$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $2y+1\geq0$.

$2y+1\geq0$

$\quad 2y\geq-1$

$\quad\quad y\geq-\dfrac{1}{2}$

Solve $6-\sqrt{2y+1}<3$.

$6-\sqrt{2y+1}<3$

$\quad\sqrt{2y+1}>3$

$\left(\sqrt{2y+1}\right)^2>3^2$

$\quad\quad 2y+1>9$

$\quad\quad\quad 2y>8$

$\quad\quad\quad\quad y>4$

The solution region is $y>4$.