

Chapter 5: Forces and Newton's Laws (Part II)

What now?

We will now look at situations where forces are being applied in **various directions** (not just parallel and perpendicular to the motion).

We will look at **horizontal and vertical forces separately**.

We will split forces into **components** that are either

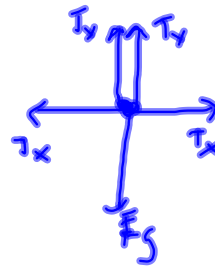
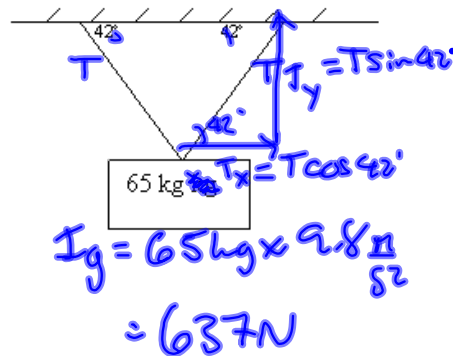
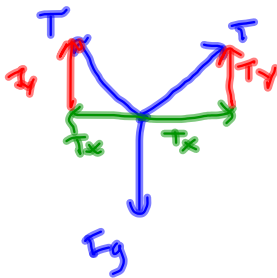
- **Parallel** to motion
- **Perpendicular** to motion

Note: When an object is at **rest**, $F_{\text{net}} = 0$ **horizontally and vertically**.

Case 1: Hanging Objects and Tension

Symmetrical situations:

Example: A sign is supported by two strings, as illustrated below. What is the tension in the strings?



① Horiz

$$T_x = T_x \quad (\text{not informative})$$

② Vert

$$F_g = T_y + T_y$$

$$F_g = 2T_y$$

$$F_g = 2T \sin 42^\circ$$

$$T = \frac{F_g}{2 \sin 42^\circ}$$

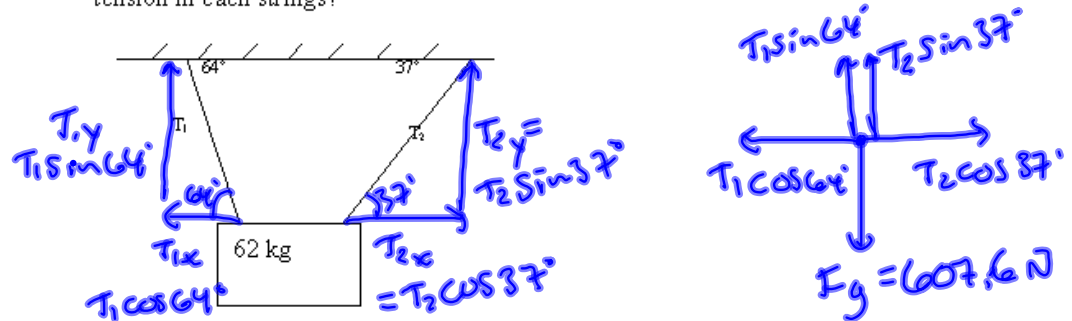
$$T = \frac{637 \text{ N}}{2 \sin 42^\circ}$$

$$T = \frac{637 \text{ N}}{2 \sin 42^\circ}$$

$$\boxed{T = 476 \text{ N}}$$

Non-Symmetrical situations:

Example: A sign is supported by two strings, as illustrated below. What is the tension in each string?



① Horizontal

$$T_1 \cos 64^\circ = T_2 \cos 37^\circ$$

$$T_1 = T_2 \frac{\cos 37^\circ}{\cos 64^\circ}$$

$$T_1 = 1.822 T_2$$

② Vertical

$$F_g = T_1 \sin 64^\circ + T_2 \sin 37^\circ$$

$$607.6 \text{ N} = (1.822 T_2) \sin 64^\circ + T_2 \sin 37^\circ$$

$$607.6 \text{ N} = 1.638 T_2 + 0.602 T_2$$

$$607.6 \text{ N} = 2.24 T_2$$

$$T_2 = \frac{607.6 \text{ N}}{2.24} = 271.25 \text{ N}$$

③ $T_1 = 1.822 T_2$
 $= 1.822 (271.25 \text{ N})$
 $= 494 \text{ N}$

ANS $T_1 = 494 \text{ N}$
 $T_2 = 271 \text{ N}$

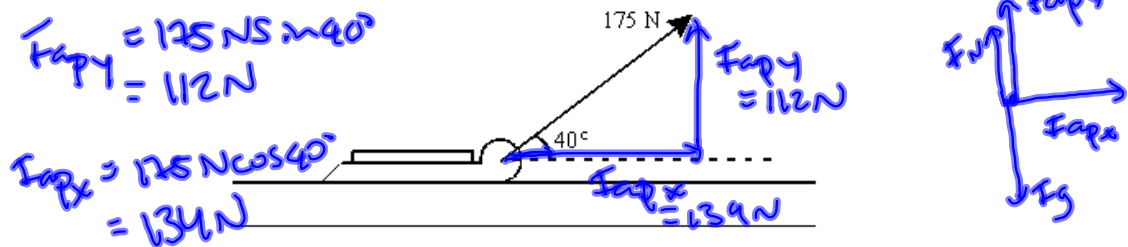
Case 2: Pulling at an angle

An object is being pulled or pushed, but the force is being applied at an angle.

Look at the horizontal (usually parallel to motion) and vertical (usually perpendicular to motion) components separately.

Remember that in this case, $F_N \neq F_g$

Example 1: A 25 kg sled is being pulled with a force of 175 N at an angle of 40° to the horizontal. What is the acceleration of the sled? (frictionless)



① Motion: horiz

$$F_{net} = F_{apx} \\ = 134\text{ N}$$

② $F_{net} = ma$

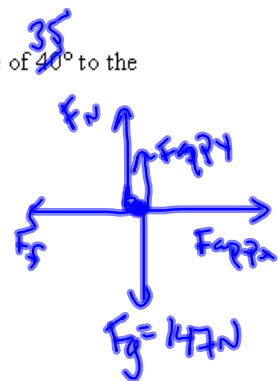
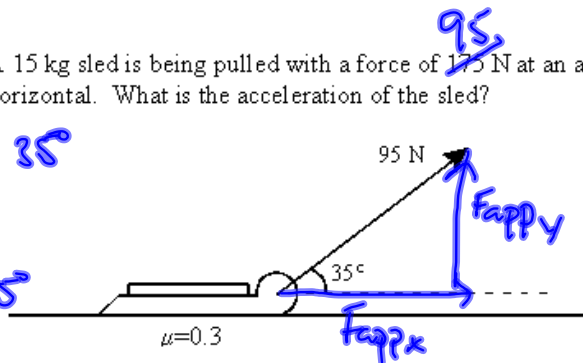
$$a = \frac{F_{net}}{m} \\ = \frac{134\text{ N}}{25\text{ kg}}$$

$$a = 5.4\text{ m/s}^2$$

Example 2: A 15 kg sled is being pulled with a force of 95 N at an angle of 35° to the horizontal. What is the acceleration of the sled?

$$F_{appx} = 95 \text{ N} \cos 35^\circ = 77.8 \text{ N}$$

$$F_{appy} = 95 \text{ N} \sin 35^\circ = 54.5 \text{ N}$$



① Vert. (equilibrium)

$$F_N + F_{appy} = F_g$$

$$F_N = F_g - F_{appy} = 147 \text{ N} - 54.5 \text{ N}$$

$$F_N = 92.5 \text{ N}$$

$$\begin{aligned} \text{② } F_f &= \mu F_N \\ &= (0.3)(92.5 \text{ N}) \\ &= 27.75 \text{ N} \end{aligned}$$

③ F_{net} (Horizontal)

$$\begin{aligned} F_{net} &= F_{appx} - F_f \\ &= 77.8 \text{ N} - 27.75 \text{ N} \\ &= 50.05 \text{ N} \end{aligned}$$

$$\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{1}{\text{kg}} = \frac{\text{m}}{\text{s}^2}$$

$$\text{④ } F_{net} = ma$$

$$a = \frac{F_{net}}{m}$$

$$= \frac{50.05 \text{ N}}{15 \text{ kg}} = \boxed{3.3 \text{ m/s}^2}$$

Case 3: Mass hanging off edge of table

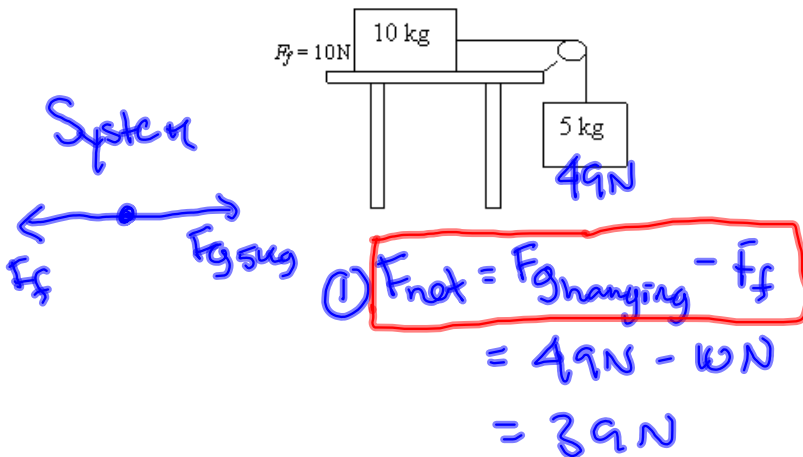
→ both have same a

Remember:

- Both masses (or more) **accelerate together** because they are tied together.
- We **add the masses** to find the acceleration of the system.

Examples:

- A 10 kg mass is tied to a 5 kg mass, as illustrated below. Friction exerts a force of 10 N. What is the acceleration of the 10 kg mass?



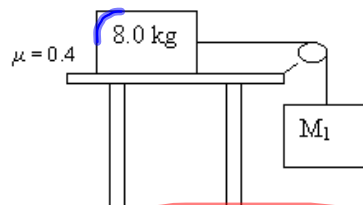
② $F_{net} = ma$
 ↑
 both masses together

$$a = \frac{F_{net}}{m}$$

$$= \frac{39\text{ N}}{5\text{ kg} + 10\text{ kg}}$$

$$= \boxed{2.6\text{ m/s}^2}$$

- The system below accelerates at 1.6 m/s^2 . The coefficient of kinetic friction between the table and the 8.0 kg box is 0.4. What is the mass of M_1 ?



$F_{net} = F_{g\text{hanging}} - F_f$

① $F_f = \mu F_N$

$$F_N = F_{g8}$$

$$= 9.8\text{ m/s}^2 (8.0\text{ kg})$$

$$F_N = 78.4\text{ N}$$

$$F_f = \mu F_N$$

$$= 0.4(78.4\text{ N})$$

$$= 31.36\text{ N}$$

$$(8.0\text{ kg} + M_1)(1.6\text{ m/s}^2) = M_1(9.8\text{ m/s}^2) - 31.36\text{ N}$$

$$12.8\text{ N} + 1.6\text{ m/s}^2 M_1 = 9.8\text{ m/s}^2 M_1 - 31.36\text{ N}$$

$$44.16\text{ N} = 8.2\text{ m/s}^2 M_1$$

$$M_1 = \frac{44.16\text{ N}}{8.2\text{ m/s}^2}$$

$$= 5.4\text{ kg}$$

② $F_{net} = ma$

$$F_{net} = (8\text{ kg} + M_1)(1.6\text{ m/s}^2)$$

③ $F_g = M(9.8\text{ m/s}^2)$

Forces II-5

Tension in between

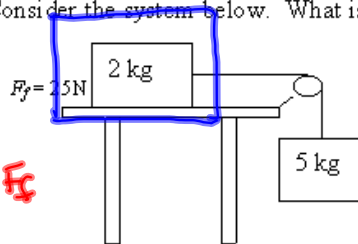
To find the tension in the string that connects to objects together:

- First we find the acceleration of the system.
- Then we isolate one of the masses, and apply Newton's second law.

* Same tension throughout same string

Examples:

3. Consider the system below. What is the tension in the string?



① Find a

$$F_{\text{net}} = F_g - F_f$$

hanging

$$= 49\text{ N} - 25\text{ N}$$

$$= 24\text{ N}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{24\text{ N}}{7\text{ kg}} = 3.4\frac{\text{m}}{\text{s}^2}$$

② $2\text{ kg} \rightarrow a = 3.4\frac{\text{m}}{\text{s}^2}$



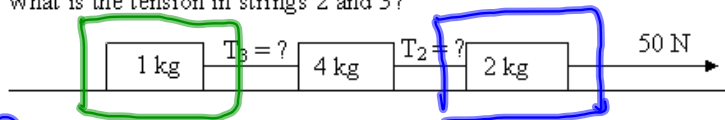
$$F_{\text{net}} = T - F_f$$

$$T = F_{\text{net}} + F_f$$

$$= (2\text{ kg} \times 3.4\frac{\text{m}}{\text{s}^2}) + 25\text{ N}$$

$$= 31.8\text{ N}$$

4. A series of masses are pulled along a frictionless surface, using a force of 50 N. What is the tension in strings 2 and 3?



① Find a ?

$$F_{\text{net}} = 50\text{ N}$$

$$a = \frac{F_{\text{net}}}{m}$$

$$1\text{ kg} + 4\text{ kg} + 2\text{ kg}$$

$$= 7.14\frac{\text{m}}{\text{s}^2}$$

② $2\text{ kg} \rightarrow a = 7.14\frac{\text{m}}{\text{s}^2}$

$$F_{\text{net}} = 2 \times 7.14\frac{\text{m}}{\text{s}^2}$$

$$= 14.28\text{ N}$$

$$F_{\text{net}} = 50\text{ N} - T_2$$

$$T_2 = 50\text{ N} - F_{\text{net}}$$

$$= 50\text{ N} - 14.28\text{ N}$$

$$T_2 = 35.72\text{ N}$$

③ $1\text{ kg} \rightarrow a = 7.14\frac{\text{m}}{\text{s}^2}$

$$F_{\text{net}} = 7.14\text{ N}$$

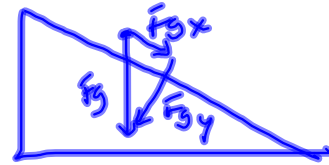
$$F_{\text{net}} = T_3$$

$$T_3 = 7.14\text{ N}$$

Case 4: Inclined Plane

When dealing with an inclined plane, we will call

- Direction parallel to motion: x
- Direction perpendicular to motion: y

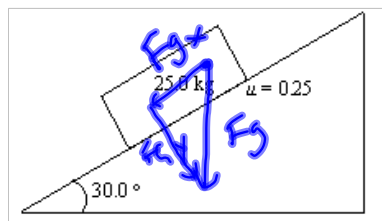


Note: when splitting F_g , remember that it is always the hypotenuse.

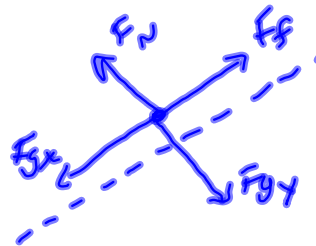
Examples:

1. What is the acceleration of the block down the incline?

$$F_g = 245 \text{ N}$$



$$\begin{aligned} F_{gx} &= F_g \sin 30^\circ \\ &= 245 \text{ N} \sin 30^\circ \\ &= 122.5 \text{ N} \end{aligned}$$



$$\begin{aligned} F_{gy} &= F_g \cos 30^\circ \\ &= 245 \text{ N} \cos 30^\circ \\ &= 212.2 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad F_N &= F_{gy} \\ F_N &= 212.2 \text{ N} \end{aligned}$$

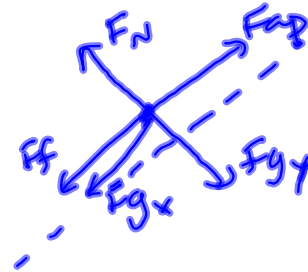
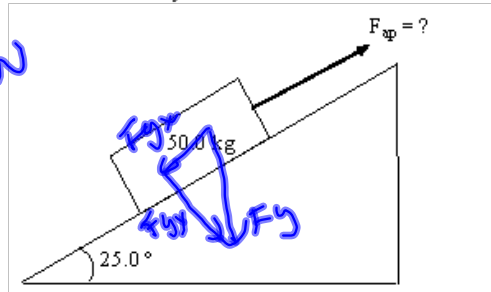
$$\begin{aligned} \textcircled{2} \quad F_f &= \mu F_N \\ &= 0.25(212.2 \text{ N}) \\ &= 53.05 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad F_{\text{net}} &= F_{gx} - F_f \\ &= 122.5 \text{ N} - 53.05 \text{ N} \\ &= 69.45 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad F_{\text{net}} &= ma \\ a &= \frac{F_{\text{net}}}{m} \\ &= \frac{69.45 \text{ N}}{25 \text{ kg}} \\ &= 2.8 \text{ m/s}^2 \end{aligned}$$

2. A girl pushes a 50.0 kg box up a ramp set at 25° . Friction exerts a force of 75 N. What is the magnitude of the force she must apply in order to slide the box at a constant velocity?

$$F_g = 490\text{ N}$$



$$\begin{aligned} F_{gx} &= F_g \sin 25^\circ \\ &= 490\text{ N} \sin 25^\circ \\ &= 207.08\text{ N} \end{aligned}$$

$$F_{\text{net}} = 0 \quad (\text{constant velocity})$$

$$F_{gx} + F_f = F_{\text{ap}}$$

$$\begin{aligned} F_{\text{ap}} &= 207.08\text{ N} + 75\text{ N} \\ &= \underline{\underline{282\text{ N}}} \end{aligned}$$