Chapter 3

Real functions

CHALLENGE 3

EVALUATION 3

CHALLENGE 3

1. Determine the domain and range of the following functions.

a) $f(x) = -2|x - 3| + 1$ **b)** $f(x) = -\sqrt{-x+1} + 1$ **c)** $f(x) = \frac{2}{3(x-1)} - 1$

a) ***dom f* = ℝ**; ***ran f* =]–∞, 1]**

b) ***dom f* =]–∞, 1]**; ***ran f* =]–∞, 1]**

c) ***dom f* = ℝ \ {1}**; ***ran f* = ℝ \ {–1}**

2. Consider the functions $f(x) = 2x - 1$ and $g(x) = 3x^2 - 2x + 1$. Find the rule of

a) $g \circ f$

$g \circ f(x) = 3(2x-1)^2 - 2(2x-1) + 1$
$= 12x^2 - 16x + 6$

b) $f \circ g$

$f \circ g(x) = 2(3x^2 - 2x + 1) - 1$
$= 6x^2 - 4x + 1$

3. Determine the zeros of the following functions.

a) $f(x) = -2|x - 1| + 6$ **b)** $f(x) = -2\sqrt{x-3} + 6$ **c)** $f(x) = \frac{-3}{2(x+1)} + 1$

a) ***–2 and 4***

b) ***12***

c) $\frac{1}{2}$

4. What are the equations of the asymptotes of the function $f(x) = \frac{2}{5(x-1)} - 4$?

The lines defined by the equations x = 1 and y = –4.

5. Study the sign of the following functions.

a) $f(x) = 4\left|-\frac{1}{2}(x-1)\right| - 4$ $f(x) \leq 0$ if $x \in [-1, 3]$; $f(x) \geq 0$ if $x \in]-\infty, -1] \cup [3, +\infty[$

b) $f(x) = -2\sqrt{x+3} + 4$ $f(x) \leq 0$ if $x \in [1, +\infty[$; $f(x) \geq 0$ if $x \in [-3, 1]$

c) $f(x) = \frac{4}{x-3} + 2$ $f(x) \leq 0$ if $x \in [1, 3[$; $f(x) \geq 0$ if $x \in]-\infty, 1] \cup]3, +\infty[$

6. Describe the variation of the following functions.

a) $f(x) = -\frac{2}{3}|x - 2| + 4$ $f \nearrow$ over $]-\infty, 2]$; $f \searrow$ over $[2, +\infty[$

b) $f(x) = -\frac{1}{2}\sqrt{-2(x-1)} + 1$ $f \nearrow$ over $]-\infty, 1]$

c) $f(x) = \frac{2}{x-1} + 1$ $f \searrow$ over $\mathbb{R} \setminus \{1\}$

7. Find the rule of

a) an absolute value function whose graph has a vertex at V(–2, 6) and passes through the point A(1, –3). $y = -3|x + 2| + 6$

b) a rational function passing through the point A(3, 4) with asymptotes defined by the lines $x = 1$ and $y = 2$.

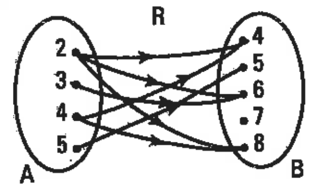
$y = \frac{4}{x-1} + 2$

c) a square root function whose graph has a vertex at V(–4, –2) and passes through the point A(5, 4). $y = 2\sqrt{x+4} - 2$

3.1 Function

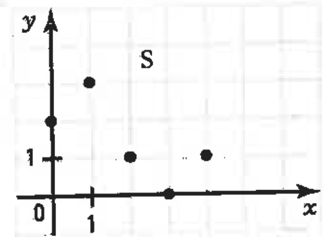
Activity 1 Recognizing a function

a) Consider the mapping diagram of the relation R represented on the right.



1. What is the source set? ***A = {2, 3, 4, 5}***
2. What is the target set? ***B = {4, 5, 6, 7, 8}***
3. Complete: An element x from set A is in relation with an element y from set B if x is ***a divisor*** of y.
4. Is there an element from the source set that is in relation with more than one element from the target set? ***Yes***
5. Is this relation a function? Justify your answer.
   No, 2 is in relation with three elements and 4 with two elements.

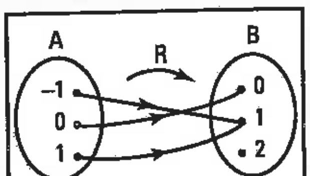
b) Consider the Cartesian graph of the relation S represented on the right. The point (1, 3) means that the element 1 from the source set is in relation with the element 3 from the target set.



1. What is the image of 4? ***1***
2. What is the antecedent of 2? ***0***
3. Is there an element from the source set that is in relation with more than one element from the target set? ***No***
4. Is this relation a function? Justify your answer.
   Yes, since each element from the source set is in relation to at most one element from the target set.

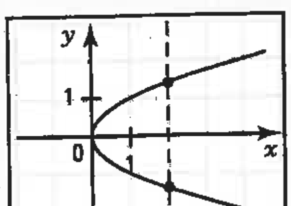
DEFINITION OF A FUNCTION

- A relation given by a source set A to a target set B is a **function** if each element from A is associated with at **most** one element from B.
- **Mapping diagram**
  Given the mapping diagram of a relation, this relation is a function if, from each element of the source set, at most one arrow is drawn.



R is a function.

- **Cartesian graph**
  Given the Cartesian graph of a relation, this relation is a function if any vertical line intersects the graph of this relation in at most one point.



R is not a function.

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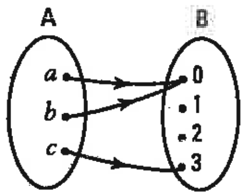
- **Set of ordered pairs**
  Given a relation's set of ordered pairs, this relation is a function if the first coordinate of each pair verifying the relation appears only once.

$G_R = \{(a, 0), (b, 1), (a, 2)\}$

R is not a function.

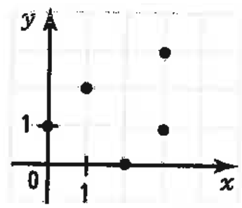
1. In each of the following cases, indicate if the relation is a function.

a)



Yes

b)

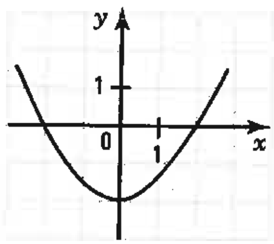


No

c) $G = \{(0, 0), (1, -1), (1, 1)\}$

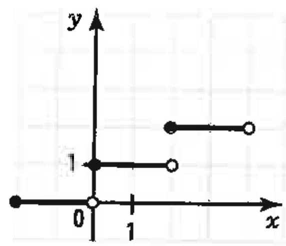
No

d)



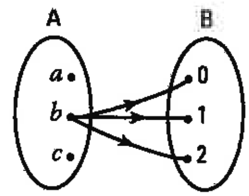
Yes

e)



Yes

f)

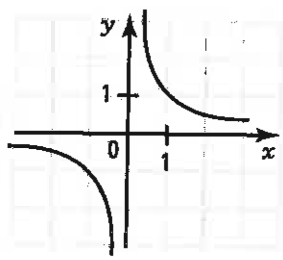


No

g) $G = \{(4, 3), (5, 3), (6, 3)\}$

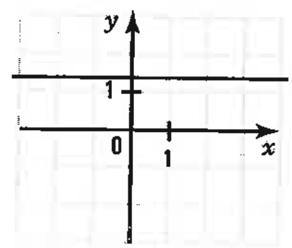
Yes

h)



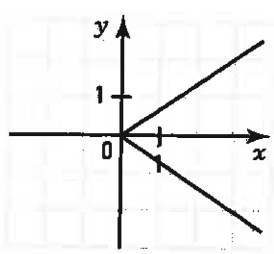
Yes

i)



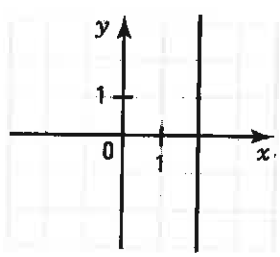
Yes

j)



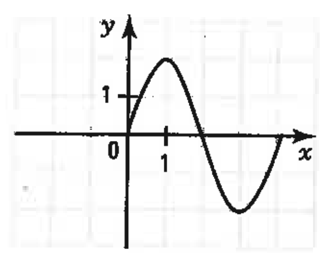
No

k)



No

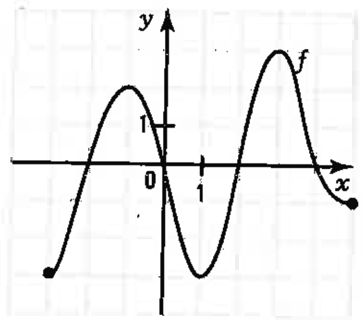
l)



Yes

ACTIVITY 2 Properties of functions

Consider the function f represented on the right.



a) What is the domain of f? **[–3, 5]**

b) What is the range of f? **[–3, 3]**

c) What are the zeros of f? **–2, 0, 2 and 4**

d) What is the initial value of f? **0**

e) Over what interval is the function f

1. positive? **$[-2, 0] \cup [2, 4]$** 2. negative? **$[-3, -2] \cup [0, 2] \cup [4, 5]$**

f) Over what interval is the function f

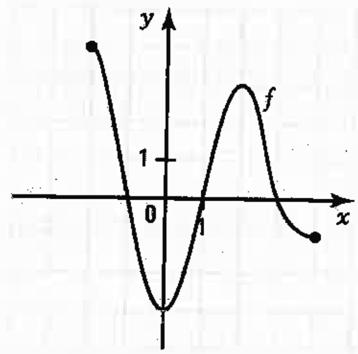
1. increasing? **$[-3, -1] \cup [1, 3]$** 2. decreasing? **$[-1, 1] \cup [3, 5]$**

g) What is, for function f, its

1. absolute maximum? **3** 2. absolute minimum? **–3**

PROPERTIES OF FUNCTIONS

Consider the function f represented on the right.



- The **domain** of a function f is the subset of the elements of the source set which have an image in f.
  dom $f = [-2, 4]$
- The **range** of a function f is the subset of the elements of the target set which are images by f.
  ran $f = [-3, 4]$
- The **zeros** of the function f are the values of x for which the function is equal to zero. The zeros of f are: –1, 1 and 3.
- The **initial value** of the function f is the value of y when $x = 0$. The initial value of f is –3.
- Studying the **sign** of a function consists of finding the values of x for which the function is positive or those for which the function is negative.
  $f(x) \geq 0$ if $x \in [-2, -1] \cup [1, 3]$.
  $f(x) \leq 0$ if $x \in [-1, 1] \cup [3, 4]$.
- Studying the **variation** of a function consists of finding the values of x for which the function is increasing or those for which the function is decreasing.
  f is increasing if $x \in [0, 2]$.
  f is decreasing if $x \in [-2, 0] \cup [2, 4]$.
- The **absolute maximum** (or **minimum**) of a function is the highest image (or the lowest image) when it exists.
  max $f = 4$, min $f = -3$

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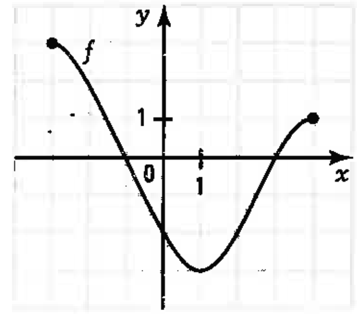
2. Consider the function f represented on the right. Determine



a) 1. dom $f =$ [−3, 4] 2. ran $f =$ [−3, 3]

b) 1. the zeros of f: −1 and 3

2. the initial value: −2

c) the values of x for which the function f is

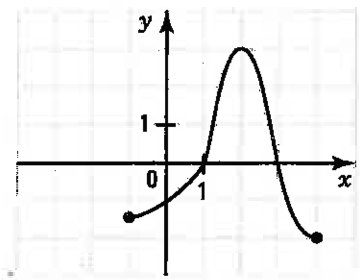
1. positive: [−3, −1] ∪ [3, 4] 2. negative: [−1, 3]

d) the values of x for which the function f is

1. increasing: [1, 4] 2. decreasing: [−3, 1]

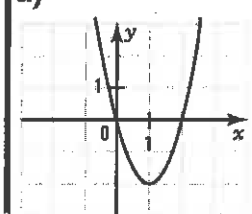
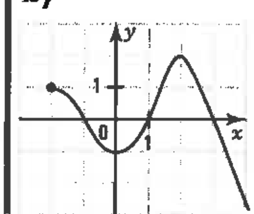
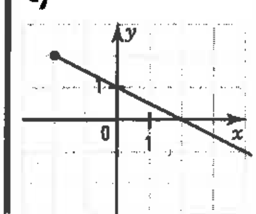
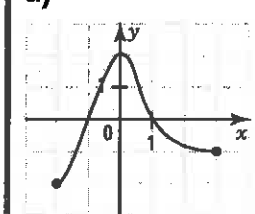
e) 1. the maximum of f: 3 2. the minimum of f: −3

3. Draw the graph of a function satisfying the following conditions.



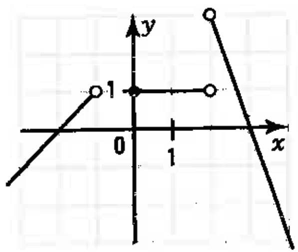
1. dom $f = [-1, 4]$.
2. ran $f = [-2, 3]$.
3. The zeros of f are: 1 and 3.
4. The initial value is −1.
5. The function is negative when $x \in [-1, 1] \cup [3, 4]$.
6. The function is increasing when $x \in [-1, 2]$ and decreasing when $x \in [2, 4]$.
7. max $f = 3$ and min $f = -2$.

4. Study the following functions by completing the table below.

	a) 	b) 	c) 	d) 
domain	ℝ	[−2, +∞[	[−2, +∞[	[−2, 3]
range	[−2, +∞[	]−∞, 2]	]−∞, 2]	[−2, 2]
zeros	0 and 2	−1, 1 and 3	2	−1 and 1
initial value	0	−1	1	2
$f(x) \geq 0$ if $x \in$	]−∞, 0] ∪ [2, +∞[	[−2, −1] ∪ [1, 3]	[−2, 2]	[−1, 1]
$f(x) \leq 0$ if $x \in$	[0, 2]	[−1, 1] ∪ [3, +∞[	[2, +∞[	[−2, −1] ∪ [1, 3]
$f \nearrow$ if $x \in$	[1, +∞[	[0, 2]	never	[−2, 0]
$f \searrow$ if $x \in$	]−∞, 1]	[−2, 0] ∪ [2, +∞[	[−2, +∞[	[0, 3]
extrema	min $f = -2$	max $f = 2$	max $f = 2$	max $f = 2$, min $f = -2$

5. Determine the domain and range of the following functions.

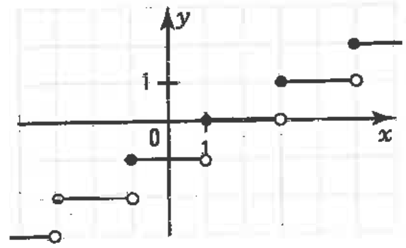
a)



dom =]−∞, −1[∪ [0, 2[∪]2, +∞[

ran =]−∞, 2[

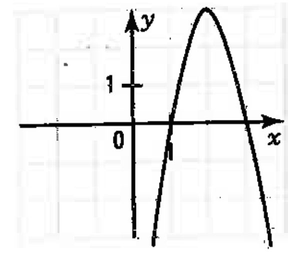
b)



dom = ℝ

ran = ℤ

c)



dom = ℝ

ran =]−∞, 3]

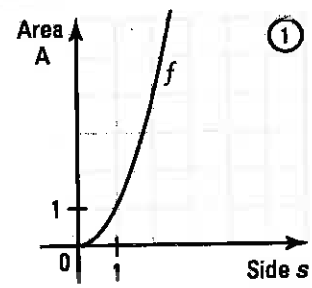
ACTIVITY 3 Inverse of a function

Let s represent the side of a square and A represent its area.

a) 1. What is the rule of the function f that associates, to the square's side s, its area? *$A = s^2$*

2. Complete the table of values below and represent the function f in the Cartesian plane ①.

Side s	0	0.5	1	1.5	2
Area A	*0*	*0.25*	*1*	*2.25*	*4*

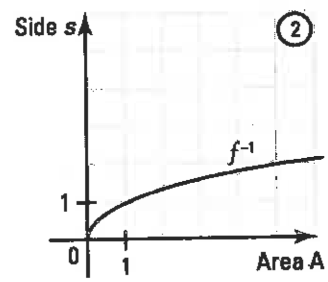


b) 1. What is the rule of the inverse f^{-1} that associates, to the square's area A, its side length s?

$s = \sqrt{A}$

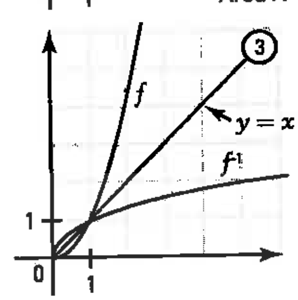
2. Complete the table of values below and represent the function f^{-1} in the Cartesian plane ②.

Area A	0	0.25	1	2.25	4
Side s	*0*	*0.5*	*1*	*1.5*	*2*



3. Explain why the inverse f^{-1} is a function.

Any vertical line only intersects the curve at a maximum of one point.



c) 1. Reproduce the two graphs in the same Cartesian plane ③ where the axes are not labeled.

2. Verify that the graphs of f and f^{-1} are symmetrical about the bisector of the 1st quadrant.

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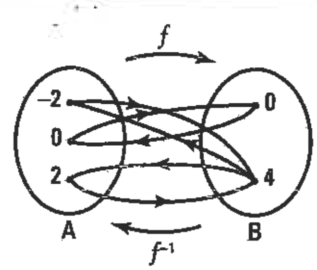
ACTIVITY 4 Functions whose inverse is not a function

a) Consider the sets A and B on the right, and the function f of A toward B with the rule $f(x) = x^2$.



1. Use a mapping diagram to represent function f.
2. Deduce the mapping diagram of the inverse f^{-1}.
3. Explain why f^{-1} is not a function.

4 is in relation with two elements –2 and 2 by f^{-1}. Therefore, f^{-1} is not a function.

b) Consider the table of values on the right of a function f.

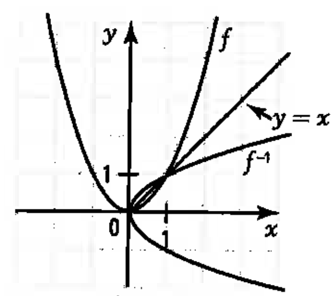
x	–2	–1	0	1	2
$f(x)$	2	1	0	1	2

1. Deduce a table of values for f^{-1}.
2. Explain why f^{-1} is not a function.

x	*2*	*1*	*0*	*1*	*2*
$f^{-1}(x)$	*-2*	*-1*	*0*	*1*	*2*

1 is in relation with two elements –1 and 1.

c) The function f on the right has the rule $f(x) = x^2$.



1. Deduce, by symmetry about the bisector of the 1st quadrant, the graph of the inverse f^{-1}.
2. Explain why the inverse f^{-1} is not a function.

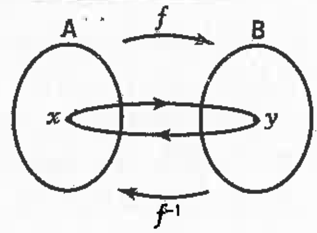
There is a vertical line that intersects the graph of f^{-1} at 2 points.

3. True or false?

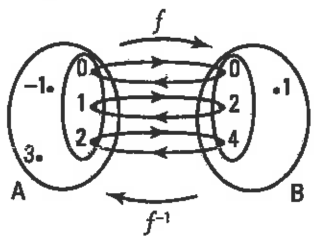
The inverse of f is not a function when a horizontal line can be drawn to intersect the graph of f at more than one point. *True*

INVERSE OF A FUNCTION

- If f is the function of a source set A toward a target set B, the **inverse of** f, written f^{-1}, has the source set B and the target set A.
- The inverse of a function is not necessarily a function.

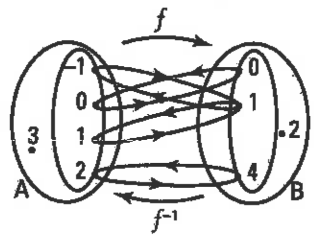


Ex.: f: A → B
$x \mapsto y = 2x$



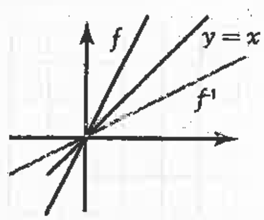
f^{-1} is a function.
$\text{dom} f = \text{ran} f^{-1} = \{0, 1, 2\}$
$\text{ran} f = \text{dom} f^{-1} = \{0, 2, 4\}$

Ex.: f: A → B
$x \mapsto y = x^2$

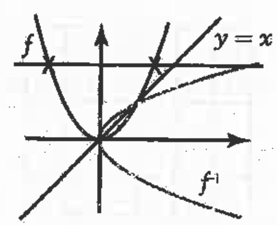


f^{-1} is not a function.
$\text{dom} f = \text{ran} f^{-1} = \{-1, 0, 1, 2\}$
$\text{ran} f = \text{dom} f^{-1} = \{0, 1, 4\}$

- For any function f, we have: $\boxed{\text{dom} f = \text{ran} f^{-1}}$ and $\boxed{\text{ran} f = \text{dom} f^{-1}}$
- The Cartesian graphs of a function and its inverse are **symmetrical** about the line with the equation $y = x$.



f^{-1} is a function.



f^{-1} is not a function.

- The inverse of a function f is not a function when a **horizontal line** can be drawn to intersect the graph of f at more than one point.

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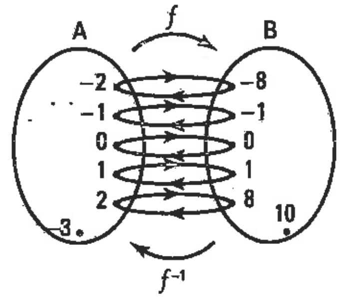
6. Consider the mapping diagram of a function f.



a) Deduce the mapping diagram of f^{-1}.

b) Explain why f^{-1} is a function.

There is at most one arrow from each element of the source set B.

c) Determine

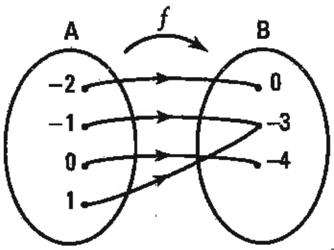
1. dom f. ***{-2, -1, 0, 1, 2}***
2. ran f. ***{-8,-1, 0, 1, 8}***
3. dom f^{-1}. ***{-8,-1, 0, 1, 8}***
4. ran f^{-1}. ***{-2, -1, 0, 1, 2}***

d) Verify that

1. dom f = ran f^{-1}.
2. ran f = dom f^{-1}.

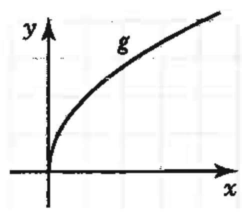
7. Indicate which of the following functions have an inverse that is also a function.

a)



No

b)

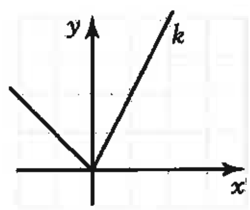


Yes

c) $h = \{(-2,-4), (-1,-2), (0,0), (1,2), (2,4)\}$

Yes

d)

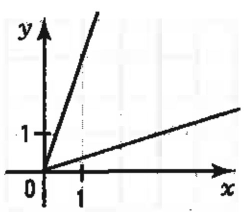


No

8. For each of the following functions,

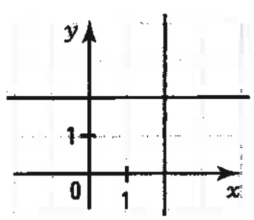
1. deduce the graph of the inverse.
2. indicate if the inverse is a function.

a)



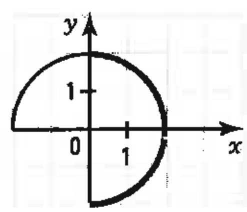
Yes

b)



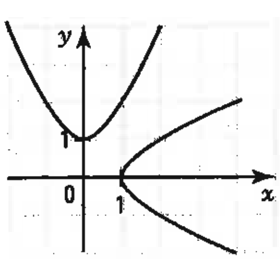
No

c)



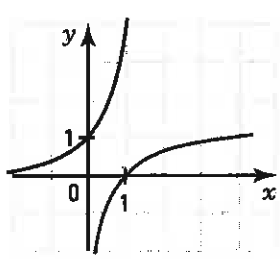
No

d)



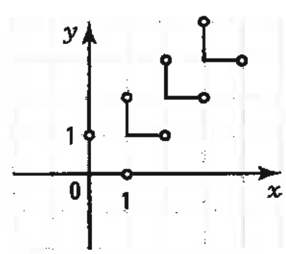
No

e)



Yes

f)



No

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ACTIVITY 5 Rule of the inverse

A salesman in a store receives a weekly base salary of \$250 and a sales commission of \$10 per item sold for the week.

a) Let a represent the number of items sold for the week, and s represent the total weekly salary. Determine the rule of

1. the function f which gives the total salary s as a function of the number of items sold a. $s = 250 + 10a$
2. the function f^{-1} which associates, to a given salary s, the number of items sold a. $a = \frac{s - 250}{10}$

b) Complete the table of values on the right for the functions f and f^{-1}.

a	0	5	10	15	20
s	250	300	350	400	450

RULE OF THE INVERSE

Given the function f with the rule: $y = 2x + 6$. To determine the rule of the inverse f^{-1},

1. we isolate x in the rule of f.

$$y = 2x + 6$$
$$2x = y - 6$$
$$x = \frac{1}{2}y - 3$$

2. we switch the letters x and y.

$$y = \frac{1}{2}x - 3$$

f^{-1} therefore has the rule: $y = \frac{1}{2}x - 3$.

We interchange the letters x and y to respect the convention of function notation which assigns x as elements of the source set and y as elements of the target set.

9. For each of the following rules of functions, find the rule of its inverse.

a) $y = 5x$
$y = \frac{x}{5}$

b) $y = 3x - 6$
$y = \frac{x}{3} + 2$

c) $y = -2x + 10$
$y = \frac{-x}{2} + 5$

d) $y = 0.1x + 100$
$y = 10x - 1000$

e) $y = \frac{2}{3}x - 6$
$y = \frac{3}{2}x + 9$

f) $y = -\frac{3}{4}x + 12$
$y = \frac{-4}{3}x + 16$

10. A capital of \$1000 is invested on January 1st, 2009 at an annual interest rate of 10%. Find the rule which associates

a) a given number of elapsed years t since the beginning, to the accumulated capital C.
$C = 1000 + 100t$

b) a given accumulated capital C, to the number of elapsed years t. $t = 0.01C - 10$

11. A car's gas tank initially contains 60 litres of gas. This car consumes on average 12 litres/100 km. Find the rule of the function which associates,

a) a given distance traveled d (in km) to the quantity q of gas remaining in the tank.
$q = -0.12d + 60$

b) a given quantity q of gas remaining in the tank, to the distance traveled d (in km).
$d = -\frac{25}{3}q + 500$

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ACTIVITY 6 Composition of functions

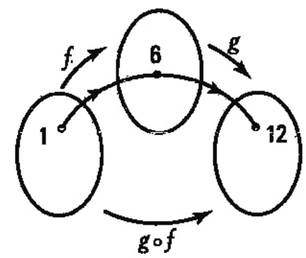
Consider the function f defined by $f(x) = x + 5$ and the function g defined by the rule $g(x) = 2x$.

a) Determine

1. $f(1)$ 6 2. $g(f(1))$ 12

b) The composition of f by g, written $g \circ f$ is defined by $g \circ f(x) = g(f(x))$.

1. Calculate $g \circ f(1)$ 12
2. Determine the rule of $g \circ f$. $g \circ f(x) = g(f(x)) = g(x + 5) = 2x + 10$



c) Determine

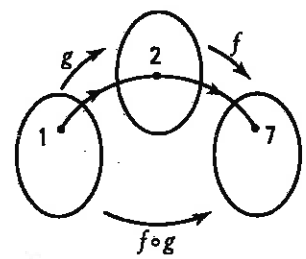
1. $g(1)$ 2 2. $f(g(1))$ 7

d) The composition of g by f, written $f \circ g$, is defined by $f \circ g(x) = f(g(x))$.

1. Calculate $f \circ g(1)$ 7
2. Determine the rule of $f \circ g$. $f \circ g(x) = f(g(x)) = f(2x) = 2x + 5$

e) Compare the rules of $g \circ f$ and $f \circ g$.

$g \circ f(x) \neq f \circ g(x)$



COMPOSITION OF FUNCTIONS

- Given two functions f and g,
  - the composition of f by g, written $g \circ f$, is defined by the rule:

$$\boxed{g \circ f(x) = g(f(x))}$$

  - the composition of g by f, written $f \circ g$, is defined by the rule:

$$\boxed{f \circ g(x) = f(g(x))}$$

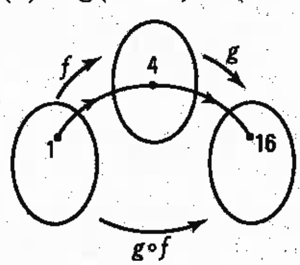
Ex.: Given $f(x) = x + 3$ and $g(x) = x^2$, we have:

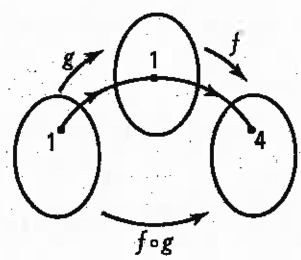
$g \circ f(1) = g(f(1)) = g(4) = 16;$

$g \circ f(x) = g(x + 3) = (x + 3)^2;$

$f \circ g(1) = f(g(1)) = f(1) = 4$

$f \circ g(x) = f(x^2) = x^2 + 3$





Note that, in general, $g \circ f(x) \neq f \circ g(x)$.

12. Consider the functions $f(x) = 3x - 5$ and $g(x) = -2x + 8$. Determine

a) $g \circ f(2) =$ 6 **b)** $f \circ g(-1) =$ 25 **c)** $f \circ g(4) =$ –5

d) $g \circ f(0) =$ 18 **e)** $g \circ g(7) =$ 20 **f)** $f \circ g(-5) =$ 49

13. Consider the functions $f(x) = -2x + 5$ and $g(x) = 4x - 3$.
Determine the rules of the following functions.

a) $f \circ g(x) =$ $f(g(x)) = f(4x - 3) = -2(4x - 3) + 5 = -8x + 11$

b) $g \circ f(x) =$ $g(f(x)) = g(-2x + 5) = 4(-2x + 5) - 3 = -8x + 17$

c) $f \circ f(x) =$ $f(f(x)) = f(-2x + 5) = -2(-2x + 5) + 5 = 4x - 5$

d) $g \circ g(x) =$ $g(g(x)) = g(4x - 3) = 4(4x - 3) - 3 = 16x - 15$

14. Consider the functions $f(x) = 2x + 3$ and $g(x) = 3x - 2$.

a) Determine the rule of

1. $g \circ f$. $g \circ f(x) = 6x + 7$
2. $f \circ g$. $f \circ g(x) = 6x - 1$

b) Verify that $g \circ f(x) \neq f \circ g(x)$.

15. Consider $f(x) = x + 5$ and $g(x) = x - 2$. Verify that, $g \circ f(x) = f \circ g(x)$.
$g \circ f(x) = x + 3$, $f \circ g(x) = x + 3$

16. Consider the function $f(x) = 2x + 8$.

a) Determine the rule of the inverse f^{-1}. $f^{-1}(x) = \frac{1}{2}x - 4$

b) 1. Determine the rule of the composite $f^{-1} \circ f$.
$f^{-1} \circ f(x) = f^{-1}(f(x)) = f^{-1}(2x + 8) = \frac{1}{2}(2x + 8) - 4 = x$

2. Determine the rule of the composite $f \circ f^{-1}$.
$f \circ f^{-1}(x) = f(f^{-1}(x)) = f\left(\frac{1}{2}x - 4\right) = 2\left(\frac{1}{2}x - 4\right) + 8 = x$

3. Verify that $f^{-1} \circ f(x) = f \circ f^{-1}(x) = x$.

c) Repeat this exercise with the function $f(x) = -5x + 10$.
$f^{-1}(x) = -\frac{1}{5}x + 2$; $f^{-1} \circ f(x) = x$; $f \circ f^{-1}(x) = x$

17. Consider the functions $f(x) = x + 5$ and $f(x) = 3x + 4$.

a) Determine the rule of the functions f^{-1} and g^{-1}.
$f^{-1}(x) = x - 5$ $\quad g^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$

b) Determine

1. $f \circ f^{-1}(x) =$ $f(f^{-1}(x)) = f(x - 5) = x - 5 + 5 = x$
2. $g \circ g^{-1}(x) =$ $g(g^{-1}(x)) = g\left(\frac{1}{3}x - \frac{4}{3}\right) = 3\left(\frac{1}{3}x - \frac{4}{3}\right) + 4 = x$
3. $f \circ g(x) =$ $f(g(x)) = f(3x + 4) = 3x + 4 + 5 = 3x + 9$
4. $g \circ f(x) =$ $g(f(x)) = g(x + 5) = 3(x + 5) + 4 = 3x + 19$

c) Determine

1. $(f \circ g)^{-1}(x) =$ $\frac{1}{3}x - 3$
2. $(g \circ f)^{-1}(x) =$ $\frac{1}{3}x - \frac{19}{3}$
3. $g^{-1} \circ f^{-1}(x) =$ $\frac{1}{3}x - 3$
4. $f^{-1} \circ g^{-1}(x) =$ $\frac{1}{3}x - \frac{19}{3}$

d) What can you deduce? $(f \circ g)^{-1}(x) = g^{-1} \circ f^{-1}(x)$ and $(g \circ f)^{-1}(x) = f^{-1} \circ g^{-1}(x)$

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18. Consider the functions $f(x) = x^2 + 4x - 5$ and $g(x) = 2x - 1$.

a) Determine the rule of the composite $f \circ g$.

$f \circ g(x) = f(g(x)) = f(2x - 1) = (2x - 1)^2 + 4(2x - 1) - 5 = 4x^2 + 4x - 8$

b) Determine $f \circ g(2)$ in two different ways:

1. by finding $f(g(2)) =$ $f(3) = 16$
2. by using the rule found in a). $4(2)^2 + 4(2) - 8 = 16$

19. In Quebec, every purchase is taxable. The goods and services tax (GST) is 5 %. The Quebec sales tax (QST) is 7.5 %.

Let f be the function which associates a given purchase amount x to the amount y including GST.

Let g be the function which associates a given purchase amount x to the amount y including QST.

a) Determine the rule of the function

1. f: $y = 1.05x$ 2. g: $y = 1.075x$

b) 1. Determine the rule of the function $g \circ f$. $g \circ f(x) = 1.12875x$

2. Determine the rule of the function $f \circ g$. $f \circ g(x) = 1.12875x$

c) Compare the rules of the functions $g \circ f$ and $f \circ g$. What can you conclude?

The rules are equal. To calculate the final price of a product, it doesn't matter if you apply the GST first and then the QST, or the QST first and then the GST.

d) 1. What is the final price of a product with a \$39.80 price tag? **\$44.92**

2. What is the initial price tag of a product if the final cost paid is \$56.44? **\$50**

20. The weekly salary of a sporting goods store salesman includes a base salary of \$300 per week and a \$40 bonus for every item sold.

During the holidays, the owner of the store decides to give each employee a 4% bonus on their weekly salary.

Let f be the function which gives the regular weekly salary y as a function of the number of items sold x.

Let g be the function which gives the bonus holiday weekly salary y as a function of the regular weekly salary x.

a) Determine the rule of the function

1. f: $y = 40x + 300$ 2. g: $y = 1.04x$ 3. $g \circ f$: $y = 41.6x + 312$

b) What will an employee's salary be, during the holidays, if he sells 4 items during the week? **\$478.40**

c) How many items did an employee sell if he receives a weekly salary of \$561.60 during the holidays? ***6 items***

ACTIVITY 7 Operations between functions

Consider the functions $f(x) = x^2 - 9$ and $g(x) = x + 3$. Determine

a) $f(x) + g(x) =$ $x^2 + x - 6$

b) $f(x) - g(x) =$ $x^2 - x - 12$

c) $f(x) \times g(x) =$ $x^3 + 3x^2 - 9x - 27$

d) $\dfrac{f(x)}{g(x)} =$ $x - 3$

OPERATIONS BETWEEN FUNCTIONS

Given two real functions f and g, we have:

$(f + g)(x) = f(x) + g(x)$ $\quad$ $(f - g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \times g(x)$ $\quad$ $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)}$

Ex.: Given $f(x) = x^2 + 2x - 15$ and $g(x) = 2x - 6$, we have:

$(f + g)(x) = f(x) + g(x) = (x^2 + 2x - 15) + (2x - 6) = x^2 + 4x - 21.$

$(f - g)(x) = f(x) - g(x) = (x^2 + 2x - 15) - (2x - 6) = x^2 - 9.$

$(f \cdot g)(x) = f(x) \times g(x) = (x^2 + 2x - 15)(2x - 6) = 2x^3 - 2x^2 - 42x + 90.$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 + 2x - 15}{2x - 6} = \dfrac{(x-3)(x+5)}{2(x-3)} = \dfrac{x+5}{2}.$

21. Consider the four functions f, g, h, and i. Let $f(x) = x^2 + x - 6$, $g(x) = 2x - 4$, $h(x) = x^2 - 9$ and $i(x) = 3x^2 - 12$.

a) $(f + g + h)(x) =$ $2x^2 + 3x - 19$

b) $(f - g + h)(x) =$ $2x^2 - x - 11$

c) $(f \cdot g)(x) =$ $2x^3 - 2x^2 - 16x + 24$

d) $(g \cdot h)(x) =$ $2x^3 - 4x^2 - 18x + 36$

e) $(f - h - i)(x) =$ $-3x^2 + x + 15$

f) $\left(\dfrac{f}{g}\right)(x) =$ $\dfrac{x+3}{2}$ $(x \neq -3)$

g) $\left(\dfrac{f \cdot g}{i}\right)(x) =$ $\dfrac{2(x+3)(x-2)}{3(x+2)}$ $(x \neq 2)$

h) $\left(\dfrac{g \cdot h}{f}\right)(x) =$ $2(x - 3)$ $(x \neq -3 \text{ and } x \neq 2)$

22. The condominium association of a building establishes the following fees to be charged to each of its condo owners.

- Monthly condo fees: \$225
- Monthly fees for renovations: \$80
- Municipal taxes paid at the beginning of the year: \$1500

a) Determine the rule of the function f which gives the cost y of condo fees as a function of the number x of months. $y = 225x$

b) Determine the rule of the function g which gives the total cost y of renovation fees and municipal taxes as a function of the number x of months. $y = 80x + 1500$

c) Determine the rule of the function $f + g$ and interpret this rule. $y = 305x + 1500$

$f + g$ gives the total fees charged to a condo owner as a function of the number x of months.

d) What is the total amount of fees paid by a condo owner after 8 months of occupancy?

\$3940

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3.2 Polynomial functions

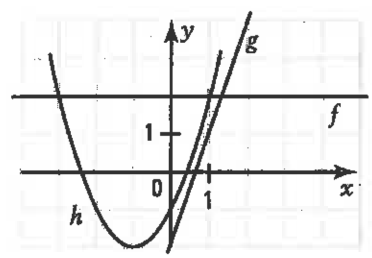
Activity 1 Polynomial functions

a) Among the following functions, indicate which ones are polynomial functions. If it is a polynomial function, indicate its degree.

1. $P(x) = -5x + 8$ ***Yes, 1st degree***
2. $P(x) = -4x^2 - 5x$ ***Yes, 2nd degree***
3. $P(x) = \frac{5}{x} + 3$ ***No***
4. $P(x) = -3$ ***Yes, degree 0***
5. $P(x) = \sqrt{x} - 7$ ***No***
6. $P(x) = x^3 + 4x^2 - 5x + 3$ ***Yes, 3rd degree***

b) Represent the following polynomial functions in the Cartesian plane on the right.

1. $f(x) = 2$ 2. $g(x) = 3x - 2$ 3. $h(x) = x^2 + 2x - 1$



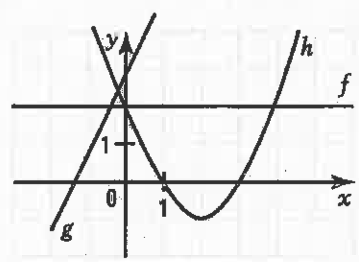
POLYNOMIAL FUNCTIONS

- A **polynomial function** is any function with a polynomial for a rule.

  Ex.: $f(x) = 2$ is a zero degree polynomial function.

  $g(x) = 2x + 3$ is a 1st degree polynomial function.

  $h(x) = x^2 - 4x + 3$ is a 2nd degree polynomial function.



- The following table classifies polynomial functions according to their **degree**.

Degree	Basic polynomial function	Transformed polynomial function	Name
0	$f(x) = 1$	$f(x) = b$ where $b \in \mathbb{R}$	constant function
1	$f(x) = x$	$f(x) = ax$ where $a \in \mathbb{R}^*$	direct variation linear function
		$f(x) = ax + b$ where $a, b \in \mathbb{R}^*$	partial variation linear function
2	$f(x) = x^2$	$f(x) = ax^2 + bx + c$ where $a \in \mathbb{R}^*$	quadratic function
3	$f(x) = x^3$	$f(x) = ax^3 + bx^2 + cx + d$ where $a \in \mathbb{R}^*$	cubic function

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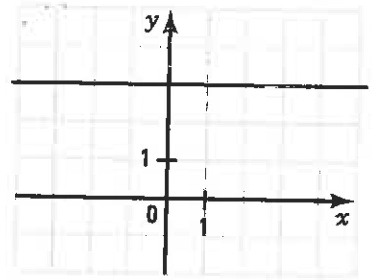
ACTIVITY 2 Study of a constant function

Consider the function f given by the rule $y = 3$.



a) Represent this function in the Cartesian plane.

b) Determine

1. dom $f =$ $\mathbb{R}$ 2. ran $f =$ $\{3\}$
3. the zeros of f if they exist. **No zeros**
4. the y-intercept. **3**
5. the sign of f $f(x) \geq 0$ **over** $\mathbb{R}$
6. the variation of f ***f* is a constant function** 7. the extrema of f $\max f = \min f = 3$

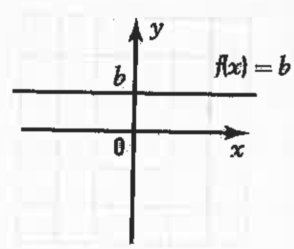
c) What is the rate of change between two random points on the graph of f? ***It is zero.***

CONSTANT FUNCTIONS

- A **constant function** is a zero degree polynomial function. It is described by a rule of the form: $\boxed{f(x) = b}, b \in \mathbb{R}$
- The Cartesian graph of a constant function is a horizontal line with the equation $y = b$.

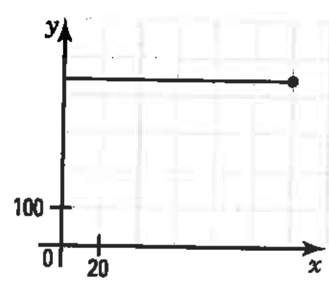
Study of a constant function

- dom $f = \mathbb{R}$
- ran $f = \{b\}$
- The constant function has no zero unless $b = 0$.
- $f(x) > 0$ over $\mathbb{R}$ if $b > 0$
  $f(x) < 0$ over $\mathbb{R}$ if $b < 0$
- $\max f = \min f = b$



- The rate of change of any constant function is zero.
- A **zero function** is a constant function described by the rule $f(x) = 0$. Its Cartesian graph is represented by the x-axis.

1. A ski resort is open 120 days during the ski season. The cost of a season pass is $450. Consider the function f which gives the total cost y as a function of the number x of days of skiing.



a) How much does it cost to ski for 12 days? **$450**

b) What is the rule of function f? $y = 450$

c) Represent function f in the Cartesian plane.

d) Determine

1. dom $f =$ $[0, 120]$ 2. ran $f =$ $\{450\}$

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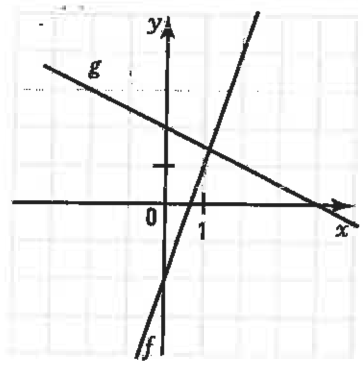
Activity 3 Study of a linear function

Consider the functions $f(x) = 3x - 2$ and $g(x) = -\frac{1}{2}x + 2$.

a) Represent the functions f and g in the Cartesian plane on the right.

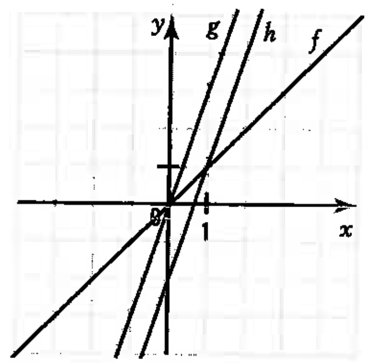
b) Study the functions f and g and complete the following table.



	Function f	Function g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R}$	$\mathbb{R}$
Zero	$\frac{2}{3}$	4
Initial value	−2	2
Sign	$f(x) \geq 0$ if $x \in \left[\frac{2}{3}, +\infty\right[$ $f(x) \leq 0$ if $x \in \left]-\infty, \frac{2}{3}\right]$	$f(x) \geq 0$ if $x \in]-\infty, 4]$ $f(x) \leq 0$ if $x \in [4, +\infty[$
Variation	*f* is increasing over $\mathbb{R}$	*g* is decreasing over $\mathbb{R}$

Activity 4 Transformations of the basic linear function

The basic 1st degree linear function $f(x) = x$ is represented on the right.



a) 1. Draw the image of function f by the vertical scale change $(x, y) \rightarrow (x, 3y)$ to obtain the graph of function g.

2. What is the rule of the function g? $g(x) = 3x$

b) 1. Draw the image of function g by the vertical translation $(x, y) \rightarrow (x, y - 2)$ to obtain the graph of function h.

2. What is the rule of the function h? $h(x) = 3x - 2$

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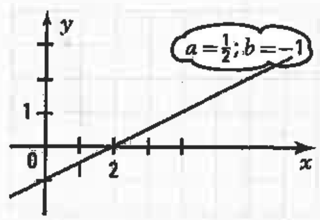
LINEAR FUNCTION

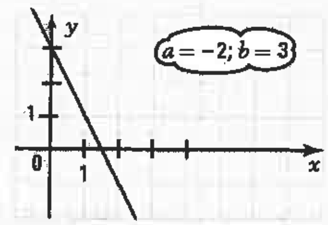
- A **linear function** is a 1st degree polynomial function. It is described by a rule of the form:

$$\boxed{f(x) = ax + b} \quad a \in \mathbb{R}^*$$

A linear function represents a situation where the rate of change is **constant**. a represents the rate of change and b represents the initial value (y-intercept).

Ex.: $f(x) = \frac{1}{2}x - 1$ | Ex.: $g(x) = -2x + 3$

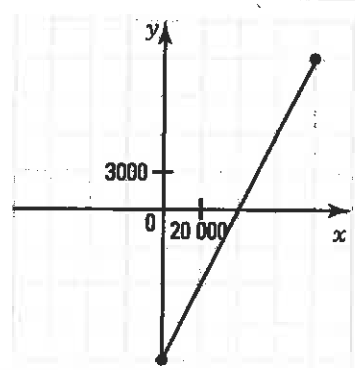




	Function f	Function g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R}$	$\mathbb{R}$
Zero	2	1.5
Initial value	−1	3
Sign	$f(x) \leqslant 0$ if $x \leqslant 2$ $f(x) \geqslant 0$ if $x \geqslant 2$	$f(x) \geqslant 0$ if $x \leqslant 1.5$ $f(x) \leqslant 0$ if $x \geqslant 1.5$
Rate of change	$\frac{1}{2}$	-2
Variation	increasing function	decreasing function

- – The function is **increasing** when the rate of change is **positive**.
  – The function is **decreasing** when the rate of change is **negative**.

2. A video game software company establishes that its monthly revenue corresponds to 30% of the amount of sales. The company's fixed monthly operating costs are \$12 000 and the company cannot sell for more than \$80 000 in one month.



a) What is the rule which gives the net revenue y as a function of the amount x of sales? ***y = 0.3 x – 12 000***

b) Represent the function in the Cartesian plane.

c) Determine for this function

1. the domain. ***[0, 80 000]***
2. the range. ***[-12 000, 12 000]***

d) Determine and interpret

1. the zero of the function. ***\$40 000. Amount of sales to have a zero net revenue.***
2. the initial value of the function. ***\$ -12 000. For zero sales, the loss is \$12 000.***

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e) Study the sign of this function. ***f(x) ≤ 0 over [0, 40 000]; f(x) ≥ 0 over [40 000, 80 000].***

f) Study the variation of this function. ***f ↗ over [0, 80 000].***

3. The graph of a linear function passes through the points A(3, 3) and B(5, –3). Determine the interval over which this function is positive. ***]–∞, 4]***

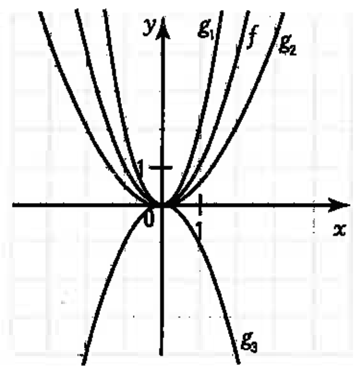
ACTIVITY 5 Transformation of the basic quadratic function

The basic quadratic function $f(x) = x^2$ can be transformed into a quadratic function with a rule of the form $g(x) = a(x - h)^2 + k$.

a) Consider the basic quadratic function $f(x) = x^2$ and the quadratic function $g(x) = ax^2$.

Represent, in the same Cartesian plane, the functions $g_1(x) = 2x^2$, $g_2(x) = \frac{1}{2}x^2$ and $g_3(x) = -x^2$ and explain how to deduce the graph of g from the graph of f when

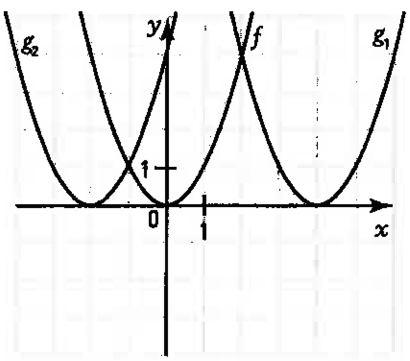
1. $a > 1$: ***by a vertical stretch.***
2. $0 < a < 1$: ***by a vertical reduction.***
3. $a = -1$: ***by a reflection about the x-axis.***



b) Consider the basic quadratic function $f(x) = x^2$ and the function $g(x) = (x - h)^2$.

Represent, in the same Cartesian plane, the functions $g_1(x) = (x - 4)^2$ and $g_2(x) = (x + 2)^2$ and explain how to deduce the graph of g from the graph of f when

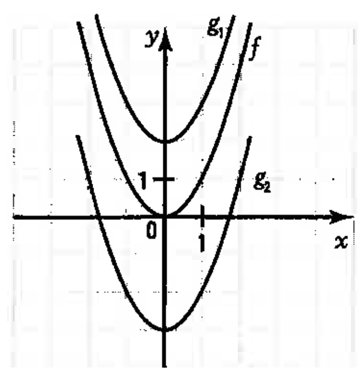
1. $h > 0$: ***by a horizontal translation to the right.***
2. $h < 0$: ***by a horizontal translation to the left.***



c) Consider the basic quadratic function $f(x) = x^2$ and the quadratic function $g(x) = x^2 + k$.

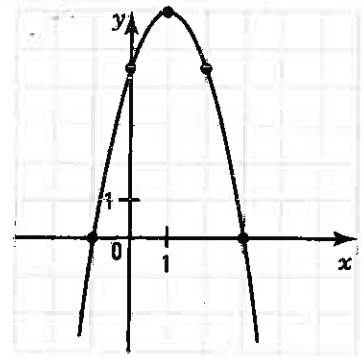
Represent, in the same Cartesian plane, the functions $g_1(x) = x^2 + 2$ and $g_2(x) = x^2 - 3$ and explain how to deduce the graph of g from the graph of f when

1. $k > 0$: ***by a vertical translation upward.***
2. $k < 0$: ***by a vertical translation downward.***



Activity 6 Study of a quadratic function (standard form)

Consider the function f given by the rule $y = -1.5(x - 1)^2 + 6$.



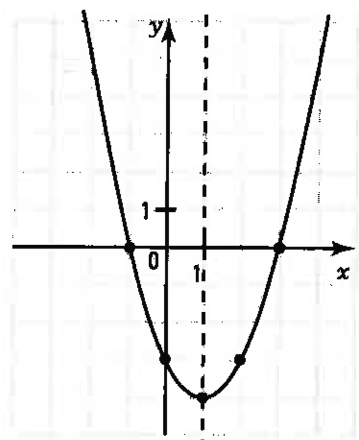
a) Represent this function in the Cartesian plane.

b) Determine

1. dom f = ℝ
2. ran f = $]-\infty, 6]$
3. the zeros of f. -1 and 3
4. the initial value of f. 4.5
5. the sign of f. $f(x) \geq 0$ over $x \in [-1, 3]$; $f(x) \leq 0$ over $x \in]-\infty, -1] \cup [3, +\infty[$
6. the variation of f. $f \nearrow$ over $x \in]-\infty, 1]$; $f \searrow$ over $x \in [1, +\infty[$
7. the extrema of f. max $f = 6$

Activity 7 Study of a quadratic function (general form)

Consider the function f given by the rule $y = x^2 - 2x - 3$.



a) Is the parabola representing f open upward or downward?
Upward, $a > 0$.

b) What are the coordinates of the vertex? $V(1, -4)$

c) Determine the zeros of the function f. $x_1 = -1$ and $x_2 = 3$

d) What is the initial value of f? -3

e) What is the equation of the axis of symmetry? $x = 1$

f) Represent this function in the Cartesian plane.

g) Determine:

1. dom f = ℝ
2. ran f = $[-4, +\infty[$
3. the sign of f. $f(x) \geq 0$ over $x \in]-\infty, -1] \cup [3, +\infty[$; $f(x) \leq 0$ over $x \in [-1, 3]$
4. the variation of f. $f \searrow$ over $x \in]-\infty, 1]$; $f \nearrow$ over $x \in [1, +\infty[$
5. the extrema of f. min $f = -4$

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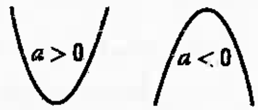
QUADRATIC FUNCTION

Standard form

$\boxed{f(x) = a(x - h)^2 + k}$



General form

$\boxed{f(x) = ax^2 + bx + c}$

- Vertex: $V(h, k)$
- Axis of symmetry: $x = h$
- Zeros: $h \pm \sqrt{-\frac{k}{a}}$

- Vertex: $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$
- Axis of symmetry: $x = -\frac{b}{2a}$
- Zeros: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- y-intercept: c

Factored form

$\boxed{f(x) = a(x - x_1)(x - x_2)}$

- Zeros: x_1 and x_2
- Axis of symmetry: $x = \frac{x_1 + x_2}{2}$

4. Determine the domain and range of the following functions.

a) $f(x) = -3(x - 2)^2 + 5$

dom f = ℝ

ran f =]−∞, 5]

b) $f(x) = 2x^2 + 4x - 9$

dom f = ℝ

ran f = [−11, +∞[

5. Determine the zeros of the function $f(x) = -3(x + 1)^2 + 12$. ***$x_1 = -3$ and $x_2 = 1$***

6. Determine the y-intercept of $f(x) = -\frac{1}{2}(x + 4)^2 + 9$. ***y = 1***

7. Determine over what interval the function $f(x) = 2x^2 - 5x - 3$ is positive.

$f(x) \geqslant 0$ over $\left]-\infty, -\frac{1}{2}\right] \cup [3, +\infty[$

8. Determine over what interval the function $f(x) = 3x^2 + 6x - 5$ is increasing. ***[−1, +∞[***

9. Determine the extrema of the function $f(x) = -2x^2 + 12x - 7$. ***max f = 11***

10. What is the axis of symmetry of the function $f(x) = -\frac{1}{4}x^2 + 3x + 1$? ***x = 6***

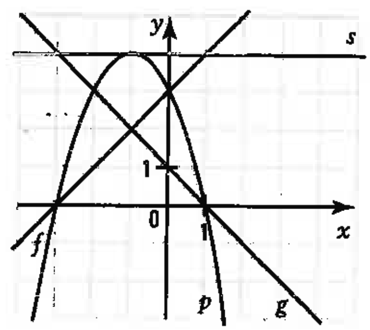
11. Determine the values of x for which the function $f(x) = -3(x + 4)^2 + 5$ is equal to −7.

x = −6 or x = −2

12. Find the rule of the quadratic function represented by a parabola with a vertex at V(−1, 5) and passing through the point P(1, 3).

$y = -\frac{1}{2}(x + 1)^2 + 5$

13. Consider the functions $f(x) = x + 3$ and $g(x) = -x + 1$ represented on the right.



a) Represent the function s given that $s(x) = f(x) + g(x)$.

$s(x) = 4$

b) Represent the function p given that $p(x) = f(x) \cdot g(x)$.

$p(x) = -x^2 - 2x + 3$

14. A stone is thrown upward from the top of a seaside cliff. The function which gives the stone's height h (in m) above sea level as a function of time t (in sec) since it was thrown has the rule: $h = -t^2 + 12t + 160$.

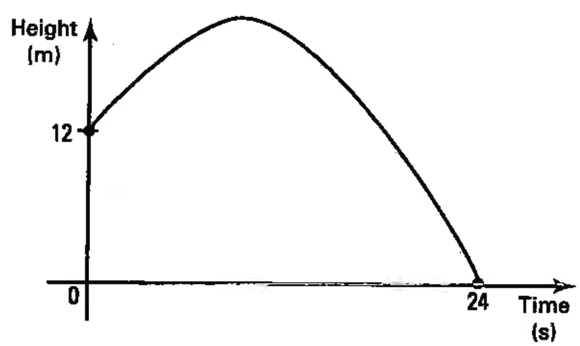
Find the interval of time over which the height of the stone is at least 180 m above sea level.

Between the instants $t = 2$ and $t = 10$ seconds after it was thrown.

15. The height h, in metres, of a diver relative to the water level is described by the rule $h = \frac{1}{2}t^2 - 6t + 10$ where t represents the elapsed time, in seconds, since the start of the dive. How long did the diver remain underwater?

During 8 seconds.

16. A projectile is thrown upward from a height of 12 m. After 10 seconds, it reaches its maximum height and after 24 seconds, it hits the ground. Knowing that its trajectory follows the rule of a quadratic function, find the elapsed time between the moment it reaches a height of 6.5 m, on its descent, and the time when it hits the ground.



$y = -\frac{1}{8}(x + 4)(x - 24)$.

It reaches, on its descent, a height of 6.5 m at the instant $t = 22$ sec. The elapsed time is therefore 2 sec.

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3.3 Absolute value function

ACTIVITY 1 Absolute value of a real number

On a winter day, the temperature (in °C), recorded at noon, has an absolute value of 5.

a) What is the recorded temperature that day if the temperature is:
1. above 0 °C? 5 °C
2. below 0 °C? -5 °C

b) We represent the absolute value of a number x by $|x|$. Determine:
1. $|+10| =$ 10
2. $|-10| =$ 10
3. $|0| =$ 0

c) Is it true to say that two opposite numbers have the same absolute value? Yes

ABSOLUTE VALUE OF A REAL NUMBER

The **absolute value** of a real number a, written $|a|$, is defined by:

$$|a| = \begin{cases} a & \text{if } a \geqslant 0 \\ -a & \text{if } a < 0 \end{cases}$$

Ex.: $|+4| = 4$; $|-3| = 3$; $|0| = 0$.

Note that the absolute value of a real number is never negative.

1. Determine the following absolute values.

a) $|+8| =$ 8 **b)** $|-4.7| =$ 4.7 **c)** $|0| =$ 0 **d)** $|\pi| =$ π

e) $|-6.53| =$ 6.53 **f)** $\left|+\frac{3}{4}\right| =$ $\frac{3}{4}$ **g)** $\left|-\frac{2}{3}\right| =$ $\frac{2}{3}$ **h)** $\left|-\frac{5}{18}\right| =$ $\frac{5}{18}$

ACTIVITY 2 Properties

Consider a real number a and a non-zero real number b. Answer true or false.

a) $|a| \geqslant 0$ True

b) $|a| = |-a|$ True

c) $|a + b| = |a| + |b|$ False

d) $|a - b| = |a| - |b|$ False

e) $|a \cdot b| = |a| \cdot |b|$ True

f) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ True

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PROPERTIES

For any real number a and any real number b, we have the following properties.

• $\lvert a\rvert \geqslant 0$	Ex.: $\lvert +5\rvert \geqslant 0$; $\lvert -4\rvert \geqslant 0$
• $\lvert a\rvert = \lvert -a\rvert$	$\lvert 4\rvert = \lvert -4\rvert$
• $\lvert a+b\rvert \leqslant \lvert a\rvert + \lvert b\rvert$	$\lvert 7+(-2)\rvert \leqslant \lvert 7\rvert + \lvert -2\rvert$
• $\lvert a-b\rvert \geqslant \lvert a\rvert - \lvert b\rvert$	$\lvert 5-(-3)\rvert \geqslant \lvert 5\rvert - \lvert -3\rvert$
• $\lvert ab\rvert = \lvert a\rvert \cdot \lvert b\rvert$	$\lvert 8\times(-3)\rvert = \lvert 8\rvert \times \lvert -3\rvert$
• $\left\lvert \frac{a}{b}\right\rvert = \frac{\lvert a\rvert}{\lvert b\rvert}$ $(b \neq 0)$	$\left\lvert \frac{-8}{2}\right\rvert = \frac{\lvert -8\rvert}{\lvert 2\rvert}$

2. Complete the following using the appropriate symbol =, >, < .

a) $\lvert x+5\rvert$ ___>___ 0 **b)** $\lvert x-3\rvert$ ___=___ $\lvert 3-x\rvert$ **c)** $\lvert 2(x-1)\rvert$ ___=___ $2\lvert x-1\rvert$

d) $\lvert 7-12\rvert$ ___>___ $\lvert 7\rvert - \lvert 12\rvert$ **e)** $\left\lvert \frac{x+2}{x-1}\right\rvert$ ___=___ $\frac{\lvert x+2\rvert}{\lvert x-1\rvert}$ **f)** $\lvert -6+9\rvert$ ___<___ $\lvert -6\rvert + \lvert 9\rvert$

ACTIVITY 3 Absolute value equations

a) Today's temperature x, in degrees Celsius, recorded at noon has an absolute value of 20. Determine this temperature if

1. it is warm. ***20°***
2. it is cold. ***–20°***

b) What are the solutions to the equation $\lvert x\rvert = 20$? ***–20 and 20***

c) Consider the equation $\lvert x\rvert = 0$. What is the unique real number that verifies this equation? ***0***

d) Consider the equation $\lvert x\rvert = -4$. Is there a real number that verifies this equation? Justify your answer.

No, since the absolute value of a real number is never negative.

ABSOLUTE VALUE EQUATIONS

The number of solutions to the equation:

$$\boxed{\lvert x\rvert = k}$$

depends on the sign of k.

If $k > 0$	If $k = 0$	If $k < 0$
The equation has 2 solutions.	The equation has 1 solution.	The equation has no solution.
$x = -k$ or $x = k$	$x = 0$	
Ex.: $\lvert x\rvert = 3$	**Ex.:** $\lvert x\rvert = 0$	**Ex.:** $\lvert x\rvert = -5$
$S = \{-3, 3\}$	$S = \{0\}$	$S = \varnothing$

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3. Solve the following equations.

a) $|x| = 12$ $S = \{-12, 12\}$

b) $|x| = -8$ $S = \varnothing$

c) $|x + 5| = 0$ $S = \{-5\}$

d) $|2x + 1| = 7$ $S = \{-4, 3\}$

e) $\left|\frac{1}{2}x - 5\right| = 4$ $S = \{2, 18\}$

f) $|6 - x| = -3$ $S = \varnothing$

4. Solve the following equations.

a) $2|x - 5| - 4 = 0$ $S = \{3, 7\}$

b) $-2|3x - 1| + 4 = -6$ $S = \left\{-\frac{4}{3}, 2\right\}$

c) $12 - |6 - 2x| = 3$ $S = \left\{-\frac{3}{2}, \frac{15}{2}\right\}$

d) $|x - 5| + 8 = 2$ $S = \varnothing$

e) $-3|2x + 5| + 6 = 6$ $S = \left\{-\frac{5}{2}\right\}$

f) $|4x - 5| + 6 = 9$ $S = \left\{\frac{1}{2}, 2\right\}$

Activity 4 Absolute value inequalities

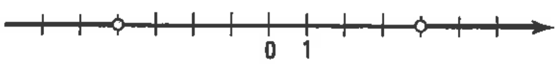
a) Consider the inequality $|x| \leqslant 3$.

On the real number line below, represent the set of all real numbers verifying this inequality and find the solution set.

S = $[-3, 3]$

b) Consider the inequality $|x| > 4$.

On the real number line below, represent the set of all real numbers verifying this inequality and find the solution set.



S = $]-\infty, -4[\cup]4, +\infty[$

ABSOLUTE VALUE INEQUALITIES

Given a **positive** real number k, we have:

$|x| \leqslant k$

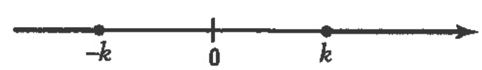
$\Leftrightarrow x \geqslant -k$ **and** $x \leqslant k$

$S = [-k, k]$

Ex.: The inequality $|x| \leqslant 5$ has the solution set: $S = [-5, 5]$.

$|x| \geqslant k$

$\Leftrightarrow x \leqslant -k$ **or** $x \geqslant k$



$S =]-\infty, -k] \cup [k, +\infty[$

Ex.: The inequality $|x| \geqslant 5$ has the solution set: $S =]-\infty, -5] \cup [5, +\infty[$.

5. For each of the following inequalities, determine the solution set and represent it on the real number line.

a) $|x| > 10$

S =]–∞, –10[∪]10, +∞[

b) $|x| \leqslant 4$

S = [–4, 4]

c) $|x| > -3$

S = ℝ

d) $|x| \leqslant -2$

S = ∅

e) $|x| \geqslant 0$

S = ℝ

f) $|x| \leqslant 0$

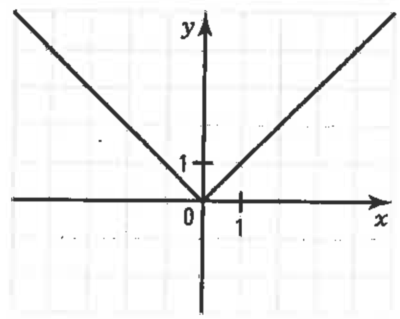
S = {0}

ACTIVITY 5 Basic absolute value function

Consider the function f defined by the rule $y = |x|$.

a) Complete the following table of values.

	–3	–2	–1	0	1	2	3
	3	*2*	*1*	*0*	*1*	*2*	*3*



b) Represent the function f in the Cartesian plane.

c) Determine

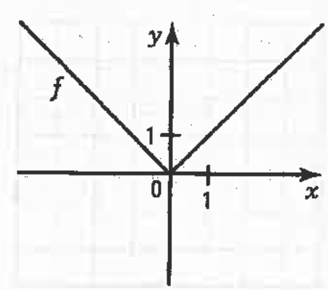
1. dom f. ℝ
2. ran f. $\mathbb{R}_+$
3. the zero of f. *0*
4. the initial value of f. *0*
5. the sign of f. *$f(x) \geqslant 0$ over ℝ.*
6. the variation of f. *f ↗ over [0, +∞[, f ↘ over]–∞, 0]*
7. the extrema of f. *min f = 0*

BASIC ABSOLUTE VALUE FUNCTION

- The function f defined by the rule:

$$f(x) = |x|$$

is called the basic **absolute value** function.



- We have:

dom $f = \mathbb{R}$ **ran** $f = \mathbb{R}_+$

The **zero** of f is 0. The **initial value** of f is 0.

Sign of f: $f(x) \geqslant 0$ over ℝ.

Variation of f: f is increasing over $\mathbb{R}_+$; f is decreasing over $\mathbb{R}_-$.

The function f has a **minimum** of 0.

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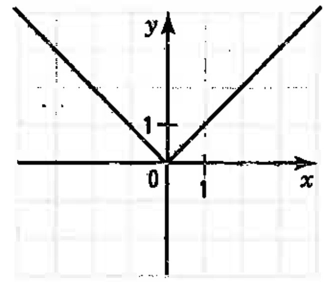
6. Consider the basic absolute value function $f(x) = |x|$ represented on the right.



a) Using the graph, find the values of x for which the function $f(x)$ is:

1. equal to 2. ***-2 and 2***
2. less than 2. ***$x \in]-2, 2[$***
3. less than or equal to 2. ***$x \in [-2, 2]$***
4. greater than 2. ***$x \in]-\infty, -2[\cup]2, +\infty[$***
5. greater than or equal to 2. ***$x \in]-\infty, -2] \cup [2, +\infty[$***

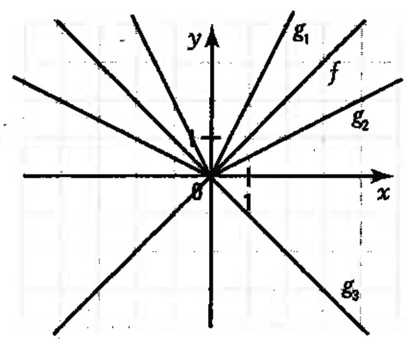
b) Using the graph, solve the following equations or inequalities:

1. $|x| = 1$ ***$S = \{-1, 1\}$*** 2. $|x| = 0$ ***$S = \{0\}$*** 3. $|x| = -1$ ***$S = \varnothing$***
4. $|x| \leqslant 1$ ***$S = [-1, 1]$*** 5. $|x| > 3$ ***$S =]-\infty, -3[\cup]3, +\infty[$*** 6. $|x| > -1$ ***$S = \mathbb{R}$***

ACTIVITY 6 Absolute value function $f(x) = a|b(x - h)| + k$

The basic absolute value function $f(x) = |x|$ can be transformed into an absolute value function defined by the rule

$$\boxed{g(x) = a|b(x - h)| + k}$$

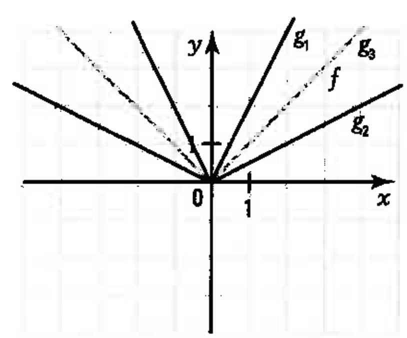


a) Consider the basic absolute value function $f(x) = |x|$ and the absolute value function $g(x) = a|x|$.

Represent, in the same Cartesian plane, the functions $g_1(x) = 2|x|$, $g_2(x) = \frac{1}{2}|x|$ and $g_3(x) = -|x|$ and explain how to deduce the graph of g from the graph of f when

1. $a > 1$: ***by a vertical stretch.***
2. $0 < a < 1$: ***by a vertical reduction.***
3. $a = -1$: ***by a reflection about the x axis.***
4. Complete: From the graph of $f(x) = |x|$, we obtain the graph of $g(x) = a|x|$ by the transformation $(x, y) \to$ ***(x, ay)***.
5. Is the graph of $g(x) = a|x|$ open upward or downward when
   1) $a > 0$? ***upward*** 2) $a < 0$? ***downward***

b) Consider the basic absolute value function $f(x) = |x|$ and the absolute value function $g(x) = |bx|$.



Represent, in the same Cartesian plane, the functions $g_1(x) = |2x|$, $g_2(x) = \left|\frac{1}{2}x\right|$ and $g_3(x) = |-x|$ and explain how to deduce the graph of g from the graph of f when

1. $b > 1$: ***by a horizontal reduction.***
2. $0 < b < 1$: ***by a horizontal stretch.***
3. $b = -1$: ***by a reflection about the y axis.***

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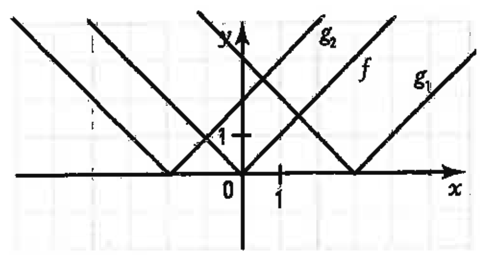
4. Complete: From the graph of $f(x) = |x|$, we obtain the graph of $g(x) = |bx|$ by the transformation $(x, y) \rightarrow$ $\left(\frac{x}{b}, y\right)$

5. Compare the graphs of the functions $y = 2|x|$ and $y = |2x|$ obtained in a) and b). Justify your answer. ***They are the same. In fact, $|2x| = |2| \cdot |x| = 2|x|$.***

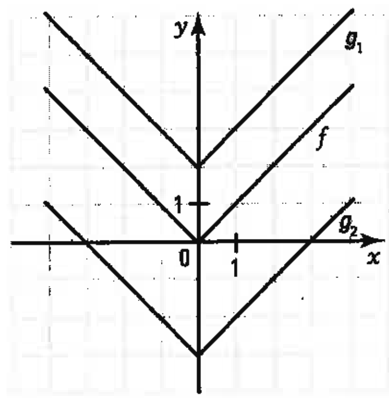
6. Compare the graphs $f(x) = |x|$ and $f(x) = |-x|$. Justify your answer.
They are the same. In fact, $|x| = |-x|$.

c) Consider the basic absolute value function $f(x) = |x|$ and the absolute value function $g(x) = |x - h|$.
Represent, in the same Cartesian plane, the functions $g_1(x) = |x - 3|$ and $g_2(x) = |x + 2|$ and explain how to deduce the graph of g from the graph of f when



1. $h > 0$: ***by a horizontal translation to the right.***
2. $h < 0$: ***by a horizontal translation to the left.***
3. Complete: From the graph of $f(x) = |x|$, we obtain the graph of $g(x) = |x - h|$ by the transformation $(x, y) \rightarrow$ ***$(x + h, y)$***.

d) Consider the basic absolute value function $f(x) = |x|$ and the absolute value function $g(x) = |x| + k$.
Represent, in the same Cartesian plane, the functions $g_1(x) = |x| + 2$ and $g_2(x) = |x| - 3$ and explain how to deduce the graph of g from the graph of f when



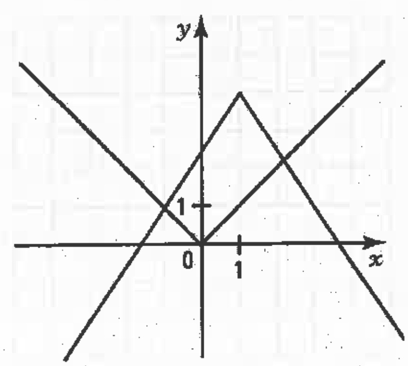
1. $k > 0$: ***by a vertical translation upward.***
2. $k < 0$: ***by a vertical translation downward.***
3. Complete: From the graph of $f(x) = |x|$, we obtain the graph of $g(x) = |x| + k$ by the transformation $(x, y) \rightarrow$ ***$(x, y + k)$.***

ABSOLUTE VALUE FUNCTION $f(x) = a|b(x - h)| + k$

The graph of the function $f(x) = a|b(x - h)| + k$ is deduced from the graph of the basic absolute value function $y = |x|$ by the transformation:

$$\boxed{(x, y) \rightarrow \left(\frac{x}{b} + h, ay + k\right)}$$

Ex.: The graph of the function $f(x) = -3\left|\frac{1}{2}(x - 1)\right| + 4$ is deduced from the graph of the basic absolute value function $g(x) = |x|$ by the transformation $(x, y) \rightarrow (2x + 1, -3y + 4)$



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7. The following functions have a rule of the form $f(x) = a|b(x - h)| + k$.
$f_1(x) = 3|x|$, $f_2(x) = |2x|$, $f_3(x) = |x + 4|$, $f_4(x) = |x| + 1$ and $f_5(x) = 2|3(x - 1)| - 4$.

Complete the table on the right by determining, for each function, the parameters a, b, h and k and by giving the rule of the transformation which enables you to obtain the function from **the basic absolute value** function $g(x) = |x|$.

	a	b	h	k	Rule
$f_1(x) = 3\|x\|$	*3*	*1*	*0*	*0*	$(x, y) \rightarrow (x, 3y)$
$f_2(x) = \|2x\|$	*1*	*2*	*0*	*0*	$(x, y) \rightarrow \left(\frac{x}{2}, y\right)$
$f_3(x) = \|x + 4\|$	*1*	*1*	*-4*	*0*	$(x, y) \rightarrow (x - 4, y)$
$f_4(x) = \|x\| + 1$	*1*	*1*	*0*	*1*	$(x, y) \rightarrow (x, y + 1)$
$f_5(x) = 2\|3(x - 1)\| - 4$	*2*	*3*	*1*	*-4*	$(x, y) \rightarrow \left(\frac{x}{3} + 1, 2y - 4\right)$

8. In each of the following cases, we apply a transformation to the basic absolute value function $y = |x|$. Find the rule of the function obtained by applying the given transformation.

a) $(x, y) \rightarrow (x, -y)$ *$y = -|x|$*

b) $(x, y) \rightarrow (x - 2, y + 4)$ *$y = |x + 2| + 4$*

c) $(x, y) \rightarrow \left(\frac{x}{2}, y\right)$ *$y = |2x|$*

d) $(x, y) \rightarrow (5x, y)$ *$y = \left|\frac{x}{5}\right|$*

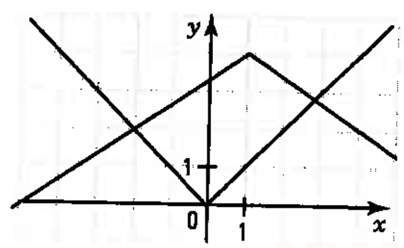
e) $(x, y) \rightarrow (3x, -7y)$ *$y = -7\left|\frac{1}{3}x\right|$*

f) $(x, y) \rightarrow \left(\frac{x}{3} + 1, 2y - 4\right)$ *$y = 2|3(x - 1)| - 4$*

9. From the basic absolute value function and using the transformation $(x, y) \rightarrow \left(\frac{x}{b} + h, ay + k\right)$, represent the function

$y = -2\left|\frac{1}{3}(x - 1)\right| + 4$ in the Cartesian plane.

For example, (1, 1) → (4, 2)



ACTIVITY 7 Graphing the function $f(x) = a|b(x - h)| + k$

Consider the function $f(x) = 4\left|-\frac{1}{2}(x - 1)\right| - 4$.

a) Identify the parameters a, b, h and k.

$a = 4$, $b = -\frac{1}{2}$, $h = 1$ and $k = -4$

b) Is the graph open upward or downward? Justify your answer.

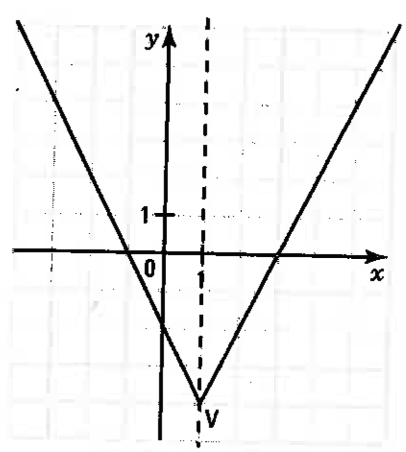
Upward, $a > 0$.

c) What are the coordinates of the vertex? *V(1, -4)*

d) Find the zeros of the function. *-1 and 3*

e) Represent the function f in the Cartesian plane after completing the following table of values.

x	−2	1	4
y	*2*	*-4*	*2*



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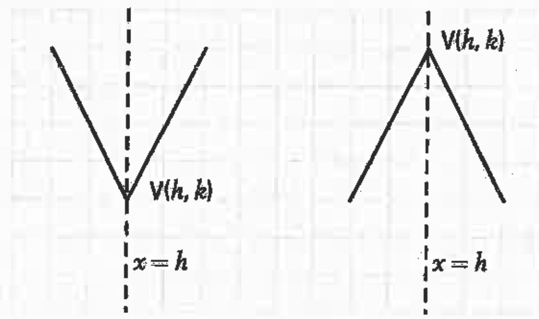
GRAPH OF AN ABSOLUTE VALUE FUNCTION

Consider the **absolute value function** defined by the rule:

$$\boxed{f(x) = a|b(x - h)| + k}$$

- The graph is open:
  - upward if $a > 0$.
  - downward if $a < 0$.
- The graph has the vertex: $\boxed{V(h,\ k)}$
- The graph has the following line as an axis of symmetry:

$$\boxed{x = h}$$



10. Write the rules of the following functions in the form $y = a|x - h| + k$ and identify the parameters a, h and k.

a) $y = -2|3x + 3| + 5$

$y = -6|x + 1| + 5;\ a = -6,\ h = -1,\ k = 5$

b) $y = 4|6 - 3x| + 5$

$y = 12|x - 2| + 5;\ a = 12,\ h = 2,\ k = 5$

c) $y = -\frac{1}{2}|8x - 4| + 3$

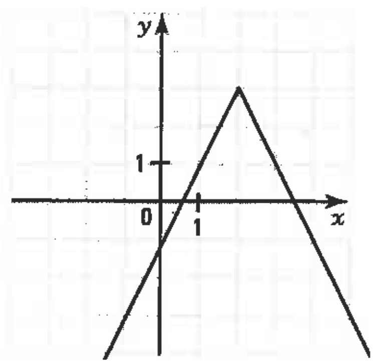
$y = -4\left|x - \frac{1}{2}\right| + 3;\ a = -4,\ h = \frac{1}{2},\ k = 3$

d) $y = -\frac{5}{6}\left|4 - \frac{1}{5}x\right| + 3$

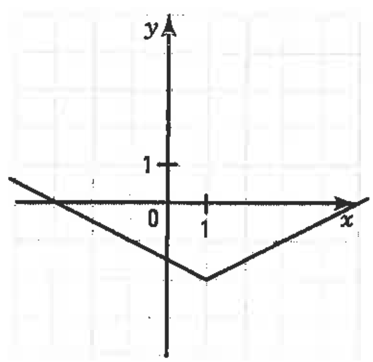
$y = -\frac{1}{6}|x - 20| + 3;\ a = -\frac{1}{6},\ h = 20,\ k = 3$

11. Graph the following functions.

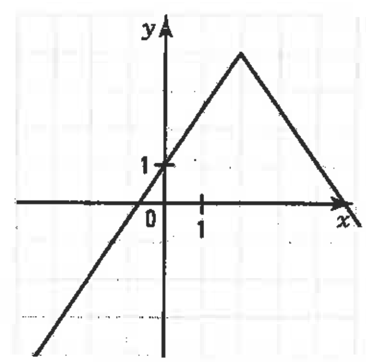
a) $y = -2|x - 2| + 3$



b) $y = \frac{1}{8}|4 - 4x| - 2$



c) $y = -\frac{1}{2}|3x - 6| + 4$



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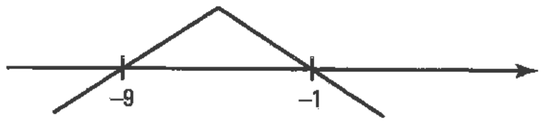
ACTIVITY 8 Determining the sign of an absolute value function

Consider the absolute value function $f(x) = -2|x + 5| + 8$.

a) What are the zeros of this function? *-9 and -1*

b) Determine the sign of this function using a sketch.

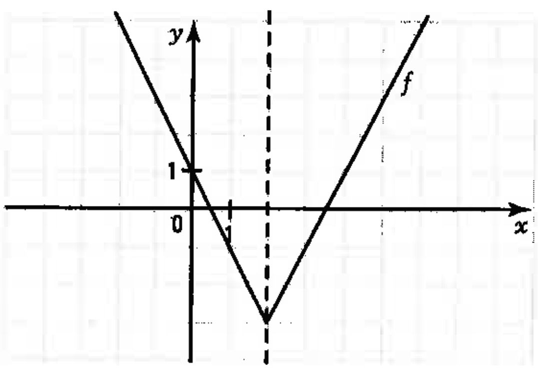


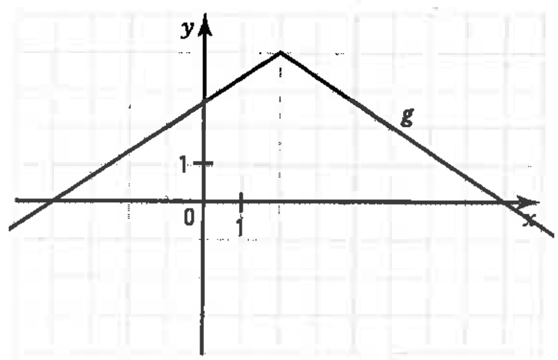
$f(x) \geq 0$ if $x \in [-9, -1]$ and $f(x) \leq 0$ if $x \in]-\infty, -9] \cup [-1, +\infty[$

ACTIVITY 9 Study of an absolute value function

Consider the functions $f(x) = \frac{1}{2}|8 - 4x| - 3$ and $g(x) = -\frac{1}{3}|2x - 4| + 4$.

a) Write each of the rules in the form $y = a|x - h| + k$ and represent the functions in the Cartesian plane.





$f(x) = 2|x - 2| - 3$

$g(x) = -\frac{2}{3}|x - 2| + 4$

b) Do a study of each of the preceding functions and complete the table below.

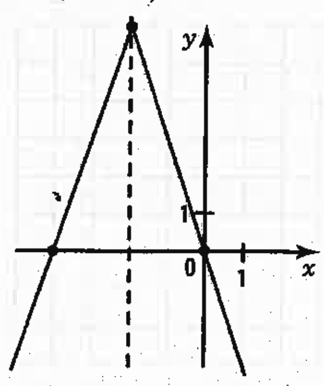
Properties	f	g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$[-3, +\infty[$	$]-\infty, 4]$
Zeros	$\frac{1}{2}$ and $\frac{7}{2}$	-4 and 8
Initial value	1	$\frac{8}{3}$
Sign	$f(x) \geq 0$ over $\left]-\infty, \frac{1}{2}\right] \cup \left[\frac{7}{2}, +\infty\right[$ $f(x) \leq 0$ over $\left[\frac{1}{2}, \frac{7}{2}\right]$	$f(x) \geq 0$ over $[-4, 8]$ $f(x) \leq 0$ over $]-\infty, -4] \cup [8, +\infty[$
Variation	$f \searrow$ over $]-\infty, 2]$; $f \nearrow$ over $[2, +\infty[$	$f \nearrow$ over $]-\infty, 2]$; $f \searrow$ over $[2, +\infty[$
Extrema	$\min f = -3$	$\max f = 4$

STUDY OF AN ABSOLUTE VALUE FUNCTION

Given the absolute value function: $f(x) = a|b(x - h)| + k$, we have:

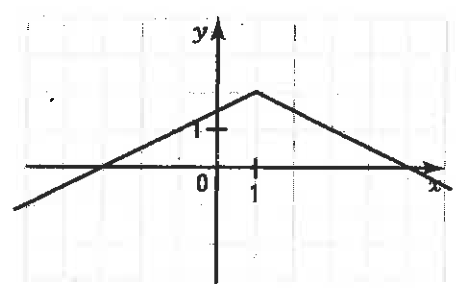
- **dom** $f = \mathbb{R}$.
- **ran** $f = [k, +\infty[$ if $a > 0$; $[-\infty, k[$ if $a < 0$.
- The **zero(s)** of f exist if a and k are opposite signs or if $k = 0$.
- To study the **sign** of f,
  - we find the zero(s) if they exist;
  - we establish the sign of f from a sketch of the graph.
- **Variation**
  If $a > 0$, f is decreasing over $]-\infty, h]$.
  f is increasing over $[h, +\infty[$.
  If $a < 0$, f is increasing over $]-\infty, h]$.
  f is decreasing over $[h, +\infty[$.
- **Extrema**
  If $a > 0$, f has a minimum. $\min f = k$.
  If $a < 0$, f has a minimum. $\max f = k$.

Ex.: Consider the function $f(x) = -3|x + 2| + 6$. ($a = -3$, $b = 1$, $h = -2$, $k = 6$)



- Open downward, $a < 0$.
- Vertex: V(−2, 6).
- Axis of symmetry: $x = -2$.
- Zeros: $-3|x + 2| + 6 = 0$
  $|x + 2| = 2$
  $\Leftrightarrow \quad x + 2 = -2 \quad$ or $x + 2 = 2$
  $x = -4 \quad$ or $\quad x = 0$
- Initial value: $y = 0$.
- dom $f = \mathbb{R}$; ran $f =]-\infty, 6]$
- Sign of f: $f(x) \geq 0$ over $[-4, 0]$; $f(x) \leq 0$ over $]-\infty, -4] \cup [0, +\infty[$.
- Variation of f: f is increasing over $]-\infty, -2]$; f is decreasing over $[-2, +\infty[$.
- $\max f = 6$.

12. Represent the graph and do a study of the function
$f(x) = -\frac{1}{4}|2(x - 1)| + 2$.



dom = $\mathbb{R}$; ran = $]-\infty, 2]$.
Zeros: −3 and 5.
Initial value: 1.5.
Sign: $f(x) \geq 0$ over $[-3, 5]$.
$f(x) < 0$ over $]-\infty, 2[\cup]5, +\infty[$.
Variation: $f \nearrow$ over $]-\infty, 1]$; $f \searrow$ over $[1, +\infty[$
Extrema: max = 2

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13. Determine the domain and range of each of the following functions.

a) $y = -2|x + 5| - 1$

dom = ℝ, ran =]−∞, −1]

b) $y = \frac{1}{4}|-2(x - 1)| + 5$

dom = ℝ, ran = [5, +∞[

14. Determine the zeros of the following functions.

a) $y = 3|x - 5| - 6$ *3 and 7*

b) $y = -\frac{1}{2}|6 - 3x| + 4$ $-\frac{2}{3}$ *and* $\frac{14}{3}$

c) $y = 4|2x + 1| + 8$ *No zero*

d) $y = -5|6 - x|$ *6*

15. Consider the linear function $f(x) = 2x - 3$ and the absolute value function $g(x) = 3|3x + 5| - 4$. Determine the initial value of the composite

a) $g \circ f$: *8*

b) $f \circ g$: *19*

16. Determine the interval over which each of the following functions is positive.

a) $y = -\frac{1}{3}|x - 5| + 2$

f(x) ⩾ 0 over [−1, 11]

b) $y = 2|3 - 2x| - 4$

$f(x) \geqslant 0$ *over* $\left]-\infty, \frac{1}{2}\right] \cup \left[\frac{5}{2}, +\infty\right[$

c) $y = \frac{3}{4}|-2x + 4| - 3$

f(x) ⩾ 0 over]−∞, 0] ∪ [4, +∞[

d) $y = 3|x - 5| + 6$

f(x) ⩾ 0 over ℝ

17. Determine the interval over which each of the following functions is increasing.

a) $y = 5|6 - 4x| + 2$

$f \nearrow$ *over* $\left[\frac{3}{2}, +\infty\right[$

b) $y = -3|2x + 4| + 5$

f ↗ over]−∞, −2]

18. Determine the solution set to each of the following inequalities.

a) $|x - 5| > 3$

S =]−∞, 2[∪]8, +∞[

b) $|6 - x| \leqslant 1$

S = [5, 7]

c) $|3x - 2| \geqslant 4$

$S = \left]-\infty, -\frac{2}{3}\right] \cup [2, +\infty[$

d) $|2x + 5| \leqslant 0$

$S = \left\{-\frac{5}{2}\right\}$

e) $-2|x + 1| + 5 > -5$

S =]−6, 4[

f) $3|2 - x| + 4 > 1$

S = ℝ

g) $6 - 3|x - 1| \leqslant 0$

S =]−∞, −1] ∪ [3, +∞[

h) $-|2x - 1| + 5 > 0$

S =]−2, 3[

i) $\left|\frac{x}{2} - 1\right| > 0$

S = ℝ \ {2}

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19. Study each of the following functions and complete the following table.

	$f(x) = -2\lvert x - 1\rvert + 4$	$f(x) = 3\lvert x + 2\rvert - 6$	$f(x) = \frac{1}{2}\lvert x - 4\rvert + 3$	$f(x) = -3\lvert 5 - x\rvert$
Dom f	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
Ran f	$]-\infty, 4]$	$[-6, +\infty[$	$[3, +\infty[$	$]-\infty, 0]$
Zero(s) if they exist	-1 and 3	-4 and 0	None	5
Initial value	2	0	5	-15
Sign	$f(x) \geq 0$ over $[-1, 3]$ $f(x) < 0$ over $]-\infty, -1[\cup]3, +\infty[$	$f(x) \geq 0$ over $]-\infty, -4] \cup [0, +\infty[$ $f(x) < 0$ over $]-4, 0[$	$f(x) \geq 0$ over $\mathbb{R}$ $f(x) < 0$ never	$f(x) \geq 0$ over $\{5\}$ $f(x) < 0$ over $\mathbb{R} \setminus \{5\}$
Variation	$f \nearrow$ over $]-\infty, 1]$ $f \searrow$ over $[1, +\infty[$	$f \nearrow$ over $[-2, +\infty[$ $f \searrow$ over $]-\infty, -2]$	$f \nearrow$ over $[4, +\infty[$ $f \searrow$ over $]-\infty, 4]$	$f \nearrow$ over $]-\infty, 5]$ $f \searrow$ over $[5, +\infty[$
Extrema	max $= 4$	min $= -6$	min $= 3$	max $= 0$

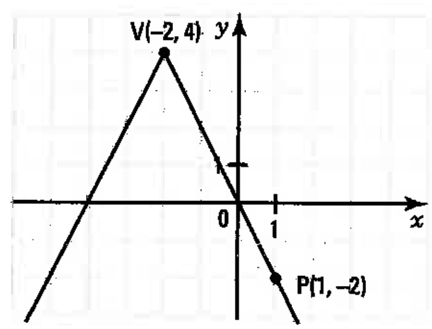
Activity 10 Finding the rule of an absolute value function

The rule of any absolute value function can be written in the form $f(x) = a\lvert x - h\rvert + k$.

a) Consider the function $f(x) = 3\lvert -2(x - 5)\rvert + 7$.
Write the rule of this function in the form $f(x) = a\lvert x - h\rvert + k$. $y = 6\lvert x - 5\rvert + 7$

b) Consider the absolute value function with the vertex V(−2, 4) and passing through the point P(1, −2).

1. Identify h and k. $h = -2$, $k = 4$
2. Determine a knowing that the coordinates of the point P(1, −2) verify the rule of the function.
   We have: $y = a\lvert x + 2\rvert + 4$
   $-2 = a\lvert 1 + 2\rvert + 4$
   $-6 = 3a$
   $a = -2$



3. What is the rule of the function? $f(x) = -2\lvert x + 2\rvert + 4$

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FINDING THE RULE OF AN ABSOLUTE VALUE FUNCTION

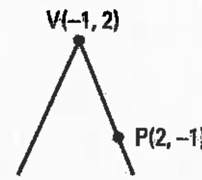
The rule of any absolute value function can be written in the form:

$$\boxed{f(x) = a|x - h| + k}$$

1st case: The vertex V and a point P are given.

1. Identify the parameters h and k.
2. Find a after replacing x and y in the rule by the coordinates of the given point P.
3. Deduce the rule.

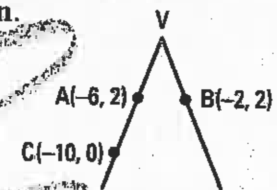
1. $h = -1$ and $k = 2$
   $y = a|x + 1| + 2$
2. $-1 = a|2 + 1| + 2$
   $-1 = 3a + 2$
   $a = -1$
3. $y = -|x + 1| + 2$



2nd case: Three points, of which two have the same y-coordinate, are given.

1. Identify h as half the sum of the x-coordinates of the points with the same y-coordinates.
2. Find the slope of the ray passing through two given points, and establish parameter a according to the opening of the graph.
3. Find k after replacing x and y in the rule by the coordinates of one of the given points.
4. Deduce the rule.

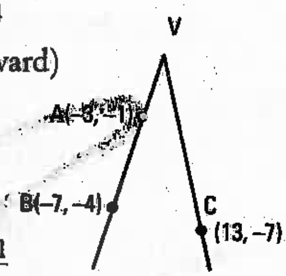
1. $h = \frac{(-6) + (-2)}{2} = -4$
2. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2}$
   $a = -\frac{1}{2}$ (open downward)
3. $y = -\frac{1}{2}|x + 4| + k$
   $2 = -\frac{1}{2}|-2 + 4| + k$
   $k = 3$
4. $y = -\frac{1}{2}|x + 4| + 3$



3rd case: Any three points are given.

1. Find the slope of the ray passing through two given points, and establish parameter a according to the opening of the graph.
2. Find the equation of each ray knowing that their slopes are opposite.
3. Find the coordinates (h, k) of the vertex V, which is the intersection of the two rays.
4. Deduce the rule.

1. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{4}$
   $a = \frac{-3}{4}$ (open downward)
2. $y_1 = \frac{3}{4}x + \frac{5}{4}$
   $y_2 = -\frac{3}{4}x + \frac{11}{4}$
3. $\frac{3}{4}x + \frac{5}{4} = -\frac{3}{4}x + \frac{11}{4}$
   $6x = 6$
   $x = 1 \Rightarrow y = 2$
   Thus, V(1, 2)
4. $y = -\frac{3}{4}|x - 1| + 2$



20. Find the rule of an absolute value function whose graph

a) has the vertex V(3, 4) and passes through the point P(7, 6). $y = \frac{1}{2}|x - 3| + 4$

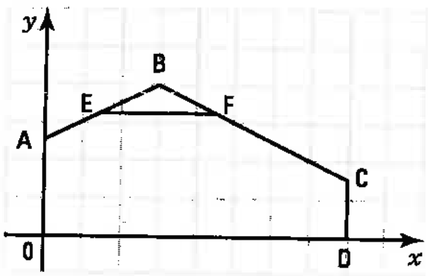
b) passes through the points A(2, –6), B(5, –8) and C(–4, –6). $y = -\frac{2}{3}|x + 1| - 4$

c) passes through the points A(1, –1), B(3, –5) and C(–4, –3). $y = -2|x + 1| + 3$

21. In order to draw the simulated trajectory of a toy airplane, Ethan uses the rule of an absolute value function that gives the airplane's height y, in metres, as a function of elapsed time x, in seconds. The rule of the function is $y = -\frac{5}{4}|x - 8| + 10$.

For how many seconds is the height of the airplane above 7 m? *4.8 seconds*

22. In the Cartesian plane on the right, a view of an airplane hangar is represented with the roof of the hangar corresponding to an absolute value function given by the rule $y = -\frac{1}{2}|x - 6| + 8$.

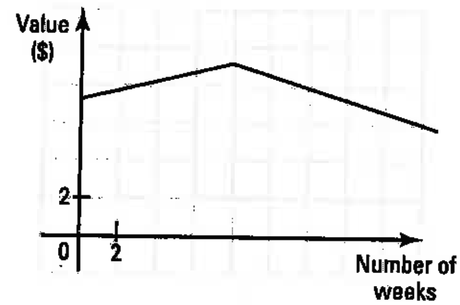


a) What is the height of the wall A0? *5 m*

b) What is the height of the wall CD if the width of the hangar is equal to 16 m? *3 m*

c) The ceiling EF is built at a height of 6.5 m. What is the width of the ceiling? *5.6 m*

23. The graph on the right represents the evolution of a share's value on the stock market. Eight weeks after its purchase, the share reaches its maximum value of $9. If it initially was worth $7, what will it be worth after 13 weeks?



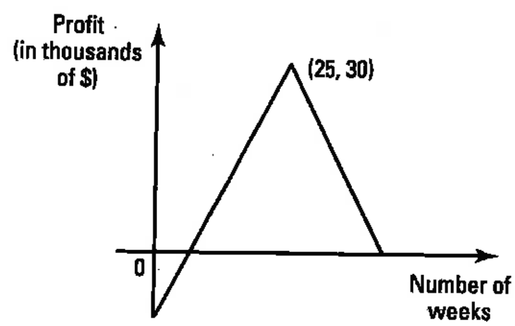
$y = a|x - 8| + 9$; $7 = 8a + 9$; $a = -\frac{1}{4}$

$y = -\frac{1}{4}|x - 8| + 9$.

It will be worth $7.75.

24. The graph on the right represents the profit of a recycling company during its first 40 weeks of operation.

During how many weeks was the profit greater than $15 000?



$y = -2|x - 25| + 30$

$-2|x - 25| + 30 = 15$; $x = 17.5$ or $x = 32.5$.

During 15 weeks.

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25. The air conditioning system in an office building has been programmed so that it turns on when the outside temperature reaches 23 °C and turns off when it reaches 20 °C.

The outside temperature varies according to the rule of the absolute value function given by $y = -3|x - 6| + 35$ where x represents the elapsed number of hours since 6 a.m. and y represents the outside temperature in °C.

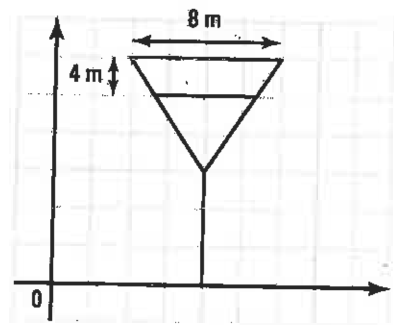
How many hours was the system on?

It turns on at 8 a.m. and turns off at 5 p.m. The system is on during 9 hours.

26. The lateral view of a channelling system is represented in the Cartesian plane on the right, scaled in metres.

The walls of this system are represented by an absolute value function with the rule: $y = 3|x - 8| + 12$.

A filtering net is placed 4 m below the ceiling of the canal. If the width of the canal is 8 m, what is the width of the filtering net?



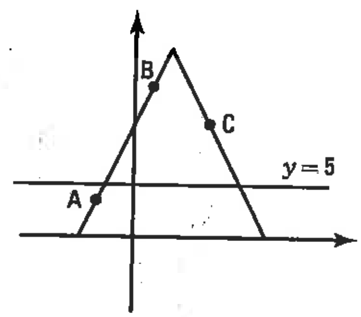
When $x = 12$, $y = 24$;

When $y = 20$, $x = \frac{16}{3}$ or $x = \frac{32}{3}$. The width of the net is 5.33 m.

27. The graph on the right represents the front of a house. The base of the roof corresponds to the line $y = 5$.

The sides of the roof form the graph of an absolute value function passing through the points A(–2, 3), B(2, 13) and C(8, 8).

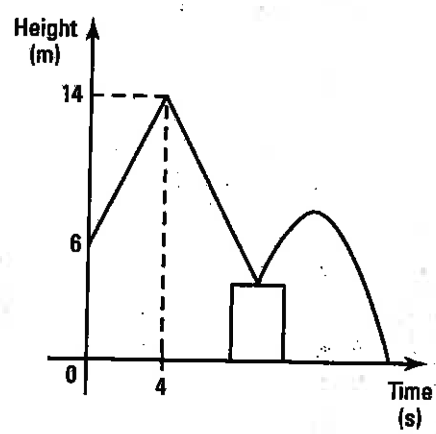
What is the area of the triangle limited by the roof and the line?



$y = -\frac{5}{2}|x - 4| + 18$; base $= \frac{52}{5}$; height $= 13$.

The area of the triangle is 67.6 u².

28. A projectile is thrown from a height of 6 m and follows the trajectory of an absolute value function. It reaches a maximum height of 14 m after 4 seconds. Five seconds after reaching its maximum height, it bounces off a cement block and follows the trajectory of a quadratic function. If the maximum height of the second bounce is 8 m and occurs three seconds after bouncing off the cement block, when will the projectile hit the ground? (Round your answer to the nearest second).



$y = -2|x - 4| + 14$, P(9, 4); $y = -\frac{4}{9}(x - 12)^2 + 8$;

The projectile hits the ground at $t = 16$ s.

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